matrix_cheat_sheet

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1 Matrix Cheat Sheet

1.1 Glossary

- **Scalar** - A scalar is just a number like 1.2, -5, 0, or 239. When we use the word **scalar** it's usually to highlight that we are *not* talking about a **vector** or a **matrix**.
- **Matrix** - A matrix is a rectangular grid of numbers. For example, this is a matrix with **2 rows** and **3 columns** so we would call it a 2×3 "2 by 3" matrix.:

$$\begin{bmatrix} 1.5 & -9.2 & 0 \\ 5.4 & 7 & 2.2 \end{bmatrix}$$

- **Row / Column** - These terms describe the horizontal (row) and vertical (column) sequences of numbers in a matrix. For example, the first row in the matrix above is $\begin{bmatrix} 1.5 & -9.2 & 0 \end{bmatrix}$.

- **Vector** - A vector is a matrix where either the width or the height is 1. When the height is 1, it's called a **row vector**. When the width is 1 it's called a **column vector**.

- **Matrix Element** - The **element** in the first row and first column of the matrix given above is the number 1.5

- **Square Matrix** - A matrix is square when its height is equal to its width.

- **Main Diagonal** - The main diagonal of a **square matrix** is the sequence of **elements** from the top left to bottom right. For the matrix below the main diagonal refers to the numbers 2 and 6.

$$\begin{bmatrix} 2 & 9 \\ -4 & 6 \end{bmatrix}$$

* **Identity Matrix** - This is a special **square matrix** where all of the **elements** are equal to zero except those on the **main diagonal**, which are equal to 1. This is a 3×3 identity matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.2 Matrix Notation

1.2.1 Indexing with $\mathbf{A}_{ij}$

The numbers i and j are used to refer to the row number and column number of a matrix (respectively).

NOTE: The top left element of a matrix is typically given by $\mathbf{A}_{11}$ and **not** $\mathbf{A}_{00}$.**

```
In [1]: # python demo

        A = [
            ["A_1,1", "A_1,2", "A_1,3"],
            ["A_2,1", "A_2,2", "A_2,3"],
            ["A_3,1", "A_3,2", "A_3,3"],
            ["A_4,1", "A_4,2", "A_4,3"],
        ]

        print("the bottom right entry in the A matrix is:", A[3][2])
```

```
('the bottom right entry in the A matrix is:', 'A_4,3')
```

1.2.2 Summation with $\sum$

Summation is best described by example.

$$\sum_{i=1}^{n} a_{nn}$$

This equation can be read as "The sum from i equals one to n of the matrix element at row n, column n."

When you see $\sum$ you should think "for loop". The code below demonstrates the following mathematical equation:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij}$$

```
In [2]: def sum_all_matrix_elements(A):
            """
            Computes the sum of ALL elements in some matrix A.
            """
            n = len(A)
            m = len(A[0])
            total = 0.0
            for i in range(n):
                for j in range(m):
                    total = total + A[i][j]
            return total

        example_matrix = [
```

```
    [1, 3, 5],
    [4, 2, 2],
]

print(sum_all_matrix_elements(example_matrix))
```

```
17.0
```

1.3 Matrix Equations

1.3.1 Addition / Subtraction

Matrix addition and subtraction is an element by element operation. Two matrices must have the same dimensions in order to be added or subtracted.

$$\mathbf{A}+\mathbf{B} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \dots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \dots & a_{2n}+b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \dots & a_{mn}+b_{mn} \end{bmatrix}$$

1.3.2 Scalar Multiplication

When multiplying a matrix $\mathbf{A}$ by a scalar c, all of the entries in $\mathbf{A}$ are multiplied by c:

$$c\mathbf{A} = c\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = \begin{bmatrix} ca_{11} & ca_{12} & \dots & ca_{1n} \\ ca_{21} & ca_{22} & \dots & ca_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ca_{m1} & ca_{m2} & \dots & ca_{mn} \end{bmatrix}$$

1.3.3 Matrix Multiplication

Multiplication of Matrix $\mathbf{A}$ with matrix $\mathbf{B}$ is only possible if the width of $\mathbf{A}$ is equal to the height of $\mathbf{B}$

If $\mathbf{A}$ is an $m \times n$ matrix and $\mathbf{B}$ is an $n \times p$ matrix, their product $\mathbf{AB}$ is an $m \times p$ matrix.

When multiplying two matrices, we can calculate the value of the element at row i and column j with the following equation:

$$(\mathbf{AB})_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

1.3.4 Transpose

The transpose of a matrix $\mathbf{A}$ is given by $\mathbf{A}^{\mathbf{T}}$ and can be thought of in several ways:

- The **rows** of $\mathbf{A}^{\mathbf{T}}$ are the **columns** of $\mathbf{A}$.
- The **columns** of $\mathbf{A}^{\mathbf{T}}$ are the **rows** of $\mathbf{A}$.

Mathematically, the element at row i and column j of the transpose is given by:

$$[\mathbf{A}^{\mathbf{T}}]_{ij} = [\mathbf{A}]_{ji}$$

1.3.5 Trace

The trace of an $n \times n$ square matrix $\mathbf{A}$ is the sum of the elements on the **main diagonal** of the matrix.

$$\operatorname{tr}(\mathbf{A}) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots + a_{nn}$$

1.3.6 Determinant

The determinant is a useful value when describing a matrix. It can be denoted in one of three ways:

1. $\det(\mathbf{A})$
2. $\det \mathbf{A}$
3. $|\mathbf{A}|$

1x1 Matrices

The determinant of a 1×1 matrix is just the value of the matrice's only element. For example if $\mathbf{A} = |4|$, then the determinant of $\mathbf{A}$ is given by:

$$|\mathbf{A}| = 4$$

2x2 Matrices

The determinant of a 2×2 matrix is given by:

$$|\mathbf{A}| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Larger Matrices

You will not be required to calculate the determinant for larger matrices. If you are interested in learning more you should look at the Wikipedia article: Determinant.

1.3.7 Inverse

The inverse of a matrix $\mathbf{A}$ is given by $\mathbf{A}^{-1}$

A matrix $\mathbf{A}$ is **invertible** if there exists a matrix $\mathbf{B}$ such that the product of $\mathbf{A}$ and $\mathbf{B}$ is the **identity matrix I**:

$$\mathbf{AB} = \mathbf{BA} = \mathbf{I}$$

1x1 Matrices
For a 1×1 matrix with a single element with value a, the inverse is simlpy $\frac{1}{a}$
2x2 Matrices
The inverse of a 2×2 matrix is given by the following equation:

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \left[(\operatorname{tr} \mathbf{A}) \, \mathbf{I} - \mathbf{A} \right]$$

Larger Matrices
You will not be required to invert larger matrices. If you are interested in learning more you should look at the Wikipedia article Invertible Matrix.

```
In [ ]:
```