优能教育

2020 Semester 1

QBUS 2310 wk4-6

TUTOR: Joy

课程安排

每周知识点复习（共4节）：

~~Week1-3：week4~~

Week4-6：week7 期中考试想拿高分的同学建议弄懂

Week8-10：week11

Week11-13：week13

讲解内容：知识点+题型练习+部分tutorial题目讲解

ASM题目练习及讲解：

- ASM1+2：往年ASM2题目练手讲解+今年ASM题目讲解提示

Online test题目讲解：

待定

考试复习：考试前1-2周内

- ~~期中考试：往年期中考试复习题+期中考试题~~
- ~~期中考试2：期中考试1中部分未讲解题目解析，期中复习题目问题汇总~~
- 期末考试：往年复习题+题型复习及练习

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Week1-3回顾:

解题框架及步骤

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-
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-
-

定义

- Variable:
- Objective:
- Model:

Excel\python

- Excel solver:
- Sensitive report:

- Python:

直角坐标系画图解题

- 只能用于解
- Constraints限制

- Objective function:

Week1-3回顾:

解题框架及步骤

- Assumptions
- Define decision variables
- Objective functions
- Constraints
- Optimal solution
- conclusion

定义

- Variable: 定义、i的取值范围
- Objective: parameters、max/min
- Model: constraints、non-negative、integer

Excel\python

- Excel solver: 三要素
- Sensitive report: allowance increase &decrease、reduced cost、shadow price
- Python: variable 角标从0开始

直角坐标系画图解题

- 只能用于解两个未知数
- Constraints限制未知数范围: feasible region
- Objective function: slope 不变上下移动找点/固定点旋转找slope

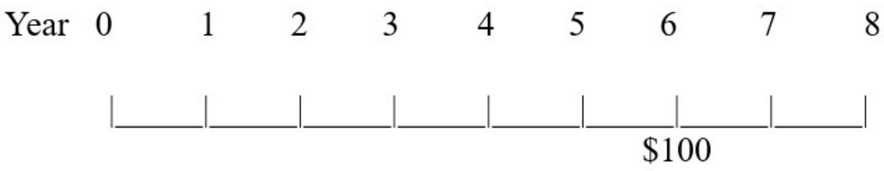
Week4-6:

- Cash flow: cash flow excel计算，NPV 概念
- Integer problem
- Binary variables & Logical Conditions
- Fixed charge
- Contract Award
- The Transshipment Model
- The Assignment Model

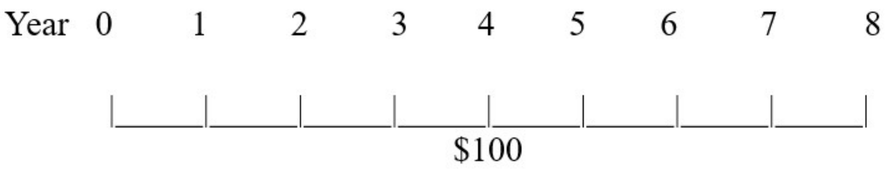
Relevant readings: pages 205-225 Chapter 5 Pages 258-277 & 315-324 in the textbook

Time value of money

Asset 1



Asset 2



FV= the accumulated (future) value
PV= the initial amount invested or borrowed
i= the interest rate each period
n= the number of periods

Simple interest

$$FV = PV + \text{interest}$$

$$FV = PV[1 + (i \times n)]$$

Compound interest

$$FV = PV(1+i)^n$$

Continuous compounding

$$FV = PVe^{it}$$

Present value:

$$PV = \frac{FV}{(1+i)^n}$$

FV= the future cash flow to be received
PV= the present value of the future cash flow
i= the compound interest rate on an alternative comparable investment
n= the number of periods before FV is received

Example：
You are offered an investment that promises $1000 in the first year, $2000 in the second year, $3000 in the third year and $500 in the fourth year. If an investment opportunity of similar risk pays 10% p.a. compounded annually, what is the maximum amount that you would pay for this investment?

Years	0	1	2	3	4
Cash flows	?	1000	2000	3000	500

Present value:

$$PV = \frac{FV}{(1+i)^n}$$

Example：

You are offered an investment that promises $1000 in the first year, $2000 in the second year, $3000 in the third year and $500 in the fourth year. If an investment opportunity of similar risk pays 10% p.a. compounded annually, what is the maximum amount that you would pay for this investment?

$$0 \qquad \frac{1000}{(1+.10)} \qquad \frac{2000}{(1+.10)^2} \qquad \frac{3000}{(1+.10)^3} \qquad \frac{500}{(1+.10)^4}$$

Present value $\quad$ $5157

NPV (net present value):

Net Present Value$_p$ =Present Value$_p$ -Cost$_p$

increase in wealth of owner from taking on project

Decision Rule

Accept project if NPV > 0
Reject project if NPV < 0

Example:
A company has a potential project requiring an outlay of $20m that will produce cash flows of $7m per year for the next 6 years. What is the NPV of the project if the required rate of return is 10% pa.

$$NPV = -\$20m + \$7m\left[\frac{1-(1.10)^{-6}}{0.10}\right]$$

$$NPV = \$10.49m \therefore accept$$

Computing NPV with Excel：

	C	D	E	F	G	H	I	J	K	L
t	0	1	2	3	4	5		npv		
	- 50,000	10,000	12,000	12,000	12,000	12,000		$43,671.26	=NPV(D6,D2:H2)	
	- 100,000	-	30,000	30,000	40,000	50,000		$105,699.44	=NPV(D6,D3:H3)	
	- 80,000	60,000	10,000	-	-	32,000		$82,679.40	=NPV(D6,D4:H4)	
	i	10%								
		9,090.91	9,917.36	9,015.78	8,196.16	7,451.06		43,671.26		
		=D2/1.1	=E2/1.1^2	=F2/1.1^3	=G2/1.1^4	=H2/1.1^5		=SUM(D8:H8)		

npv			
$43,671.26	=NPV(D6,D2:H2)		-$6,328.74
$105,699.44	=NPV(D6,D3:H3)		$5,699.44
$82,679.40	=NPV(D6,D4:H4)		$2,679.40

Example:

Star Oil Company is considering five different investment opportunities. The cash outflows and net present values (in millions of dollars) are given in Table 7. Star Oil has \$40 million available for investment now (time 0); it estimates that one year from now (time 1) \$20 million will be available for investment. Star Oil may purchase any fraction of each investment. In this case, the cash outflows and NPV are adjusted accordingly. For example, if Star Oil purchases one-fifth of investment 3, then a cash outflow of $\frac{1}{5}(5) = \$1$ million would be required at time 0, and a cash outflow of $\frac{1}{5}(5) = \$1$ million would be required at time 1. The one-fifth share of investment 3 would yield an NPV of $\frac{1}{5}(16) = \$3.2$ million. Star Oil wants to maximize the NPV that can be obtained by investing in investments 1–5. Formulate an LP that will help achieve this goal. Assume that any funds left over at time 0 cannot be used at time 1.

TABLE 7

Cash Flows and Net Present Value for Investments in Capital Budgeting

	Investment (\$)				
	1	2	3	4	5
Time 0 cash outflow	11	53	5	5	29
Time 1 cash outflow	3	6	5	1	34
NPV	13	16	16	14	39

decision variables

Star Oil must determine what fraction of each investment to purchase. We define

$$x_i = \text{fraction of investment } i \text{ purchased by Star Oil} \qquad (i = 1, 2, 3, 4, 5)$$

Objective functions

Star's goal is to maximize the NPV earned from investments. Now, (total NPV) = (NPV earned from investment 1) + (NPV earned from investment 2) + ⋯ + (NPV earned from investment 5). Note that

$$\begin{aligned}\text{NPV from investment 1} &= (\text{NPV from investment 1})(\text{fraction of investment 1 purchased}) \\ &= 13x_1\end{aligned}$$

Applying analogous reasoning to investments 2–5 shows that Star Oil wants to maximize

$$z = 13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5 \qquad (25)$$

Constraints

Star Oil's constraints may be expressed as follows:

Constraint 1 Star cannot invest more than \$40 million at time 0.

Constraint 2 Star cannot invest more than \$20 million at time 1.

Constraint 3 Star cannot purchase more than 100% of investment i ($i = 1, 2, 3, 4, 5$).

To express Constraint 1 mathematically, note that (dollars invested at time 0) = (dollars invested in investment 1 at time 0) + (dollars invested in investment 2 at time 0) + $\cdots$ + (dollars invested in investment 5 at time 0). Also, in millions of dollars,

$$\begin{aligned}\begin{matrix}\text{Dollars invested in investment 1}\\ \text{at time 0}\end{matrix} &= \begin{pmatrix}\text{dollars required for}\\ \text{investment 1 at time 0}\end{pmatrix}\begin{pmatrix}\text{fraction of}\\ \text{investment 1 purchased}\end{pmatrix}\\ &= 11x_1\end{aligned}$$

Similarly, for investments 2–5,

$$\text{Dollars invested at time 0} = 11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5$$

Then Constraint 1 reduces to

$$11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \le 40 \qquad \text{(Time 0 constraint)} \tag{26}$$

Constraint 2 reduces to

$$3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 \le 20 \qquad \text{(Time 1 constraint)} \tag{27}$$

Constraints 3–7 may be represented by

$$x_i \le 1 \qquad (i = 1, 2, 3, 4, 5) \tag{28–32}$$

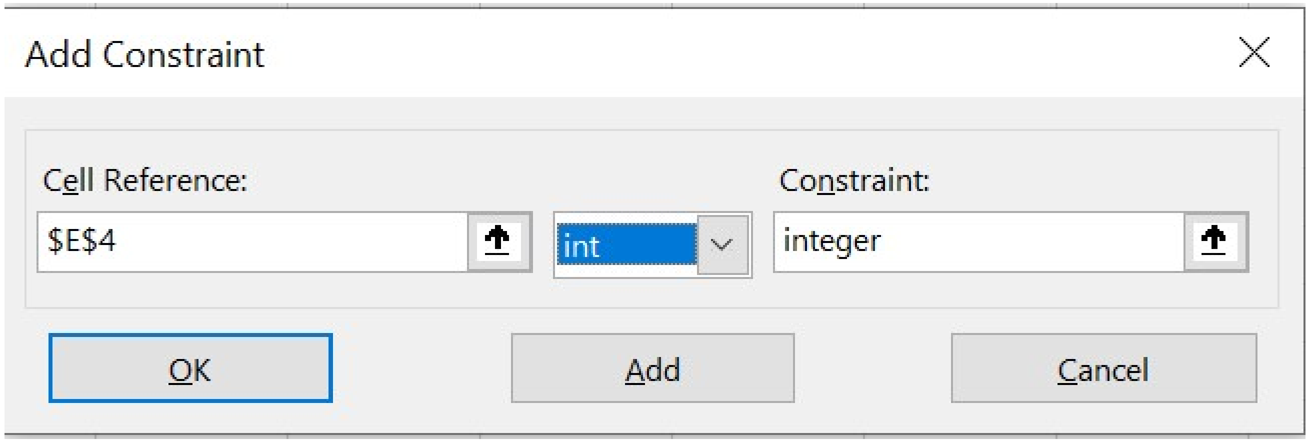
Combining (26)–(32) with the sign restrictions $x_i \geq 0$ ($i = 1, 2, 3, 4, 5$) yields the following LP:

$$
\begin{aligned}
\max z = {} & 13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5 \\
\text{s.t.} \quad & 11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 40 \qquad \text{(Time 0 constraint)} \\
& 3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 \leq 20 \qquad \text{(Time 1 constraint)} \\
& x_1 \leq 1 \\
& x_2 \leq 1 \\
& x_3 \leq 1 \\
& x_4 \leq 1 \\
& x_5 \leq 1 \\
& x_i \geq 0 \qquad (i = 1, 2, 3, 4, 5)
\end{aligned}
$$

The optimal solution to this LP is $x_1 = x_3 = x_4 = 1$, $x_2 = 0.201$, $x_5 = 0.288$, $z = 57.449$. Star Oil should purchase 100% of investments 1, 3, and 4; 20.1% of investment 2; and 28.8% of investment 5. A total NPV of $57,449,000 will be obtained from these investments.

Integer problem

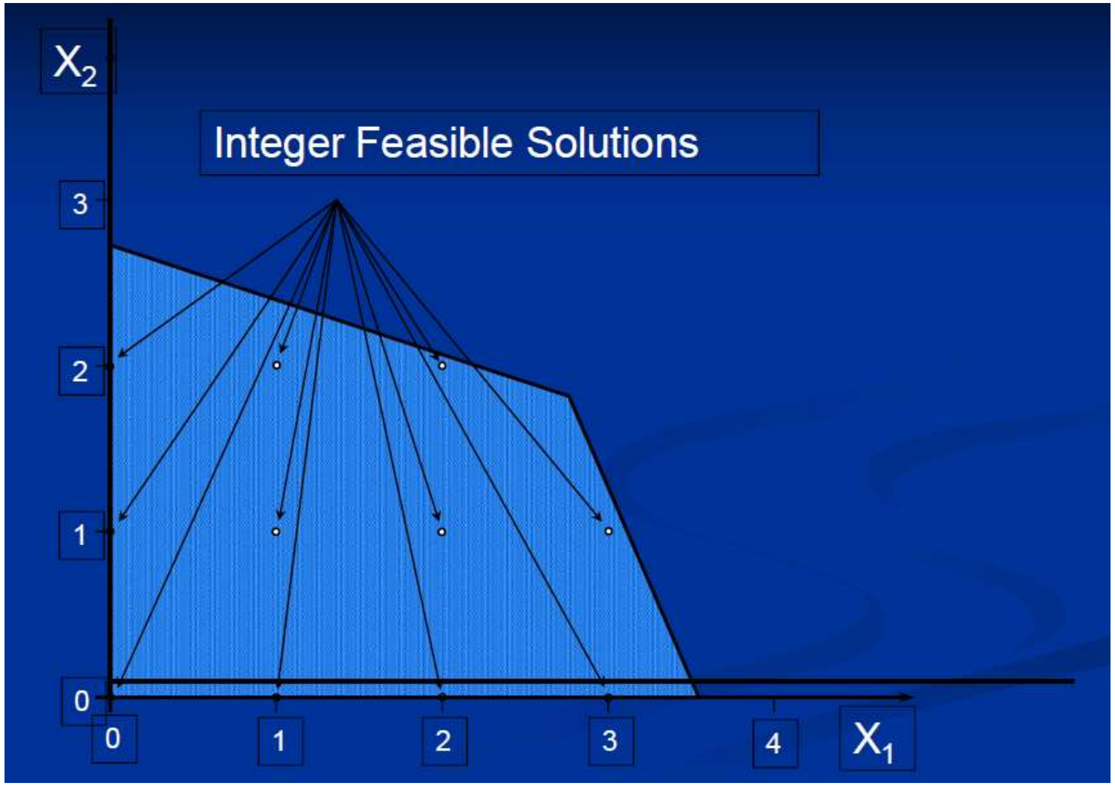
Excel solver



python

```
y = {}
y[0] = B.addVar(vtype = grb.GRB.INTEGER, name = 'y1')
y[1] = B.addVar(vtype = grb.GRB.INTEGER, name = 'y2')
y[2] = B.addVar(vtype = grb.GRB.INTEGER, name = 'y3')
```

Integer problem



???

思考：为什么不能计算完再四舍五入求整

人力资源分配问题

某公交车路线每天各时间段内所需司机和乘务人员人数如下，若工作人员再各时间段开始上班后需连续工作8小时，该如何安排工作使需要配备的司机和乘务人员人数最少？

班次	时间	所需人员
1	6:00——10:00	60
2	10:00——14:00	70
3	14:00——18:00	60
4	18:00——22:00	50
5	22:00——2:00	20
6	2:00——6:00	30

解：设x_i表示第i班次时开始上班的司机和乘务人员人数。

$$\min \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

$$s.t.\begin{cases} x_1 + x_6 \geq 60 \\ x_1 + x_2 \geq 70 \\ x_2 + x_3 \geq 60 \\ x_3 + x_4 \geq 50 \\ x_4 + x_5 \geq 20 \\ x_5 + x_6 \geq 30 \\ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{cases}$$

	variables	上班人数	所需人数
x1	60	60	60
x2	10	70	70
x3	50	60	60
x4	0	50	50
x5	30	30	20
x6	0	30	30
obj	150		

	60	10	50	0	30	0	150	
	x1	x2	x3	x4	x5	x6		
1	1					1	60	60
2	1	1					70	70
3		1	1				60	60
4			1	1			50	50
5				1	1		30	20
6					1	1	30	30

Binary variables & Logical Conditions

Binary variables are integer variables that can assume only two values: 0 or 1.

$$X_i = \begin{cases} 1, \text{if project } i \text{ selected} \\ 0, \text{ otherwise} \quad i = 1, 2, \ldots, 6 \end{cases}$$

$$Y_i = \begin{cases} 1, \text{if } X_i > 0 \\ 0, \text{ if } X_i = 0 \quad , i = 1, 2, 3 \end{cases}$$

	A	B	C
1	X	Y=1 if X>36, otherwise 0	
2	40	1	=IF(A2>36,1,0)
3	60	1	=IF(A3>36,1,0)
4	55	1	=IF(A4>36,1,0)
5	30	0	=IF(A5>36,1,0)
6	28	0	=IF(A6>36,1,0)
7	48	1	=IF(A7>36,1,0)
8	69	1	=IF(A8>36,1,0)
9			

1. 同时选择时 例：X1+X2+X3+X4=2
2. 先后顺序 例：X1>=X2
3. 倍数 例：2X1>=X2+X3
4. 其他 例：X1>=MY

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Bindery :

本质是帮助做分类函数问题

1. 分析题目， 判断是否需要分段， 是否需要使用bindery variables
2. 需要使用几个bindery variable
3. 判断是upper/lower limit，及不等式符号
4. 题目是否告知M(parameter)
5. 结合题目试着列式子
6. 代入0、1验算

Question 9

A company wants to select no more than 2 projects from a set of 4 possible projects. Let X_i be a binary variable obtaining a value of 1 if project *i* is selected and 0 otherwise. Write a constraint which ensures that no more than 2 projects are selected.

Answer: $X_1 + X_2 + X_3 + X_4 \leq 2$

Question 11

A company must invest in project 1 in order to invest in project 2. Let X_i is a binary variable obtaining a value of 1 if project *i* is selected and 0 otherwise . Formulate a constraint which will enforce the company's requirement.

Answer: $X_1 - X_2 \geq 0$

Linking Constraints (with "Big M")

M_i imposes the bounds on X_i.

The Wiethoff Company has a contract to produce 10,000 garden hoses for a customer. Wiethoff has four different machines that can produce this kind of hose. Because these machines are from different manufacturers and use differing technologies, their specifications are not the same.

Machine	Fixed Cost to Setup Production Run	Variable Cost per Hose	Capacity
1	750	1.25	6000
2	500	1.50	7500
3	1000	1.00	4000
4	300	2.00	5000

Let X_i denote the number of garden hoses produced on machine i, and let Y_i be a binary variable obtaining a value of 1 if machine i is used and 0 otherwise ($i = 1,2,3,4$).

Write the linking constraint for machine 1 (the constraint which ensures that production can take place on machine 1 only if it is used).

$$X_1 - 6000Y_1 \leq 0$$

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Question 13 (6 marks):

A company produces and sells a single product A. The first 20 units of product A generate a per unit profit of \$100. Any unit beyond the first 20 units generates a profit of \$110. Denote by A_1 the number of units generating a per unit profit of \$100 and by A_2 the number of units generating a per unit profit of \$110. Clearly, your objective function is MAX $100X_1+110X_2$. Write the constraint(s) that will ensure that your objective function calculates the correct total revenue. (You are permitted to define additional variables if required).

Let Y be a binary variable.

$A_1 \geq 20Y$

$A_2 \leq MY$ where M is a very large number.

6 marks – 3 for each constraint (large numbers may substitute M).

Question 14 (6 marks):

In a capital budgeting problem, let x_i be a binary variable with a value of 1 if project i is chosen and 0 otherwise. Formulate a single constraint which will ensure that if project 1 and/or project 2 are selected then project 5 cannot be selected.

$$\boldsymbol{2x_5 + x_1 + x_2 \leq 2}$$

6 marks (give 1 point if coefficient of X5 is wrong or if RHS is wrong.).

Question 15 (6 marks):

In a capital budgeting problem, let x_i be a binary variable with a value of 1 if project i is chosen and 0 otherwise. Formulate **a set of two constraints** which will ensure that project 5 can only be selected if exactly 2 out of the 3 projects 1, 2 & 3 are selected.

$$\boldsymbol{x_5 + x_1 + x_2 + x_3 \leq 3}$$

$$\boldsymbol{2x_5 - x_1 - x_2 - x_3 \leq 0}$$

6 marks – 3 for each constraint.

Question 16 (6 marks):

A company produces a single product that is sold for $25 per unit. The company has decided to restrict itself to one of the following production strategies: (1) produce at most 40 units of the product; or (2) produce exactly 50 units of the product. Formulate a set of constraints that will impose the above restriction (you are permitted to introduce additional variable(s)).

Let Y_1 and Y_2 be a binary variables.

$$X \le 40Y_1 + 50Y_2$$

$$X \ge 50Y_2$$

$$Y_1 + Y_2 = 1$$

6 marks – 2 for each constraint.

Question 12

If a company selects either of Project 1 or Project 2 (or both), then either Project 3 or Project 4 (or both) must also be selected. Let X_i is a binary variable obtaining a value of 1 if project *i* is selected and 0 otherwise . Formulate a constraint which will enforce the company's requirement.

Answer: $\mathbf{X_1 + X_2 \leq 2(X_3 + X_4)}$

Question 16 (6 marks):

Your company has the capacity to produce at most 200 units of a certain product. The per unit selling price of this product is $8 per unit. Let X denote the number of units you produce. Your boss has suddenly decided that you should either produce according to one of the following policies: (1) at most 50 units; or (2) at least 100 units. Write the constraint(s) that will ensure that you your production level is consistent with this requirement. (You are permitted to define additional variables if required).

Let Y be a binary variable.

$$\boldsymbol{X \leq 50 + 150Y}$$

$$\boldsymbol{X \geq 100Y}$$

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Question 13 (6 marks):

Your company has the capacity to produce at most 300 units of a certain product. The per unit selling price of this product is \$50 for the first 100 units, and \$60 per unit for any additional unit sold. For example, if you sell 150 units, the first 100 will sell at \$50 each, while the last 50 units will produce a per unit revenue of \$60. Let X_1 denote the number of units you sell at \$50 per unit, and X_2 denotes the number of units sold at \$60 per unit. Clearly, your objective function is MAX $50X_1+60X_2$. Write the constraint(s) that will ensure that your objective function calculates the correct total revenue. (You are permitted to define additional variables if required).

Let Y be a binary variable.

$\mathbf{X_1 \geq 100Y}$

$\mathbf{X_2 \leq 200Y}$

$\mathbf{X_1 + X_2 \leq 300}$.

6 marks – 2 for each constraint.

The Transportation Model Characteristics：

- 产品以尽可能低的成本从多个来源运输到多个目的地
- 每个货源都能提供固定数量的产品，每个目的地对商品的需求都固定产品
- 线性规划模型对每个来源的供应和每个目的地的需求都有约束
- 在供给等于需求的平衡运输模型中，所有约束都是平等的
- 在供应不等于需求的不平衡模型中，约束包含不平等

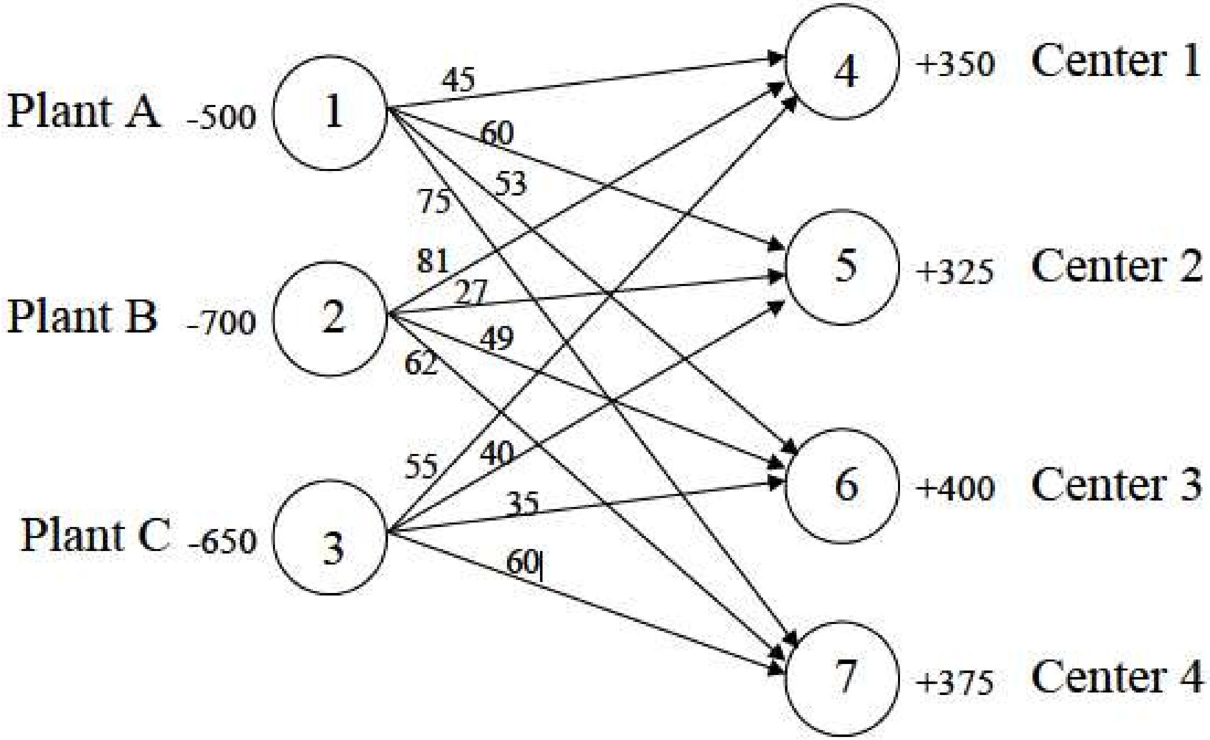
运输模型的扩展

- 中间转运点被添加到来源和目的地。
- 物品可从以下地点运输：
  - Sources through transshipment points to destinations
  - One source to another
  - One transshipment point to another
  - One destination to another
  - Directly from sources to to destinations
  - Some combination of these

X_{ij} = the amount being shipped (or flowing) from node i to node j

For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule At Each Node:
Total Supply > Total Demand:	Inflow-Outflow >= Supply or Demand
Total Supply < Total Demand:	Inflow-Outflow <=Supply or Demand
Total Supply = Total Demand:	Inflow-Outflow = Supply or Demand

The following network depicts a transportation/distribution problem for Clifton Distributing. Formulate the LP for Clifton assuming they wish to minimize the total product-miles incurred.



Let X_{ij} = flow from plant i (A, B, or C) to distribution center j (Center 1, 2, 3, or 4).

MIN: $45 X_{14} + 60X_{15} + 53X_{16} + 75X_{17} + 81X_{24} + 27X_{25} + 49X_{26} + 62X_{27} + 55X_{34} + 40X_{35} + 35X_{36} + 60X_{37}$

Subject to:

$$-X_{14} - X_{15} - X_{16} - X_{17} \geq -500$$
$$-X_{24} - X_{25} - X_{26} - X_{27} \geq -700$$
$$-X_{34} - X_{35} - X_{36} - X_{37} \geq -650$$
$$X_{14} + X_{24} + X_{34} \geq 350$$
$$X_{15} + X_{25} + X_{35} \geq 325$$
$$X_{16} + X_{26} + X_{36} \geq 400$$
$$X_{17} + X_{27} + X_{37} \geq 375$$
$$\text{All } X_{ij} \geq 0$$

The Assignment Model Characteristics:

Special form of linear programming model similar to the transportation model.

Supply at each source and demand at each destination limited to one unit.

In a balanced model supply equals demand.
In an unbalanced model supply does not equal demand.

例题:

现有ABCDE，共五个任务，交给小红小绿小兰小明韩梅梅五人完成，如下表是她们各自完成任务的分数，问如何分配到总分数最高。

	小红	小绿	小兰	小明	韩梅梅
A	51	61	98	63	100
B	89	51	84	73	53
C	28	29	46	89	98
D	98	47	68	61	66
E	78	72	77	73	83

H	I	J	K	L	M	N	O	P
	i	1	2	3	4	5		
j	Xij	小红	小绿	小兰	小明	韩梅梅	total	
1	A	0	0	0	0	1	1	1
2	B	0	0	1	0	0	1	1
3	C	0	0	0	1	0	1	1
4	D	1	0	0	0	0	1	1
5	E	0	1	0	0	0	1	1
	total	1	1	1	1	1		
		1	1	1	1	1		

obj	443	=SUMPRODUCT(B3:F7,J3:N7)

Wk	Topic
Week 01	Management science: introduction
Week 02	Linear programming: model formulation and graphical solution
Week 03	Linear programming: computer solution and sensitivity analysis
Week 04	Linear programming: modelling examples
Week 05	Integer programming
Week 06	1. Transportation; 2. Transshipment; 3. Assignment problems
Week 08	Network flow models
Week 09	Project management
Week 10	Non-linear programming
Week 11	Queuing analysis
Week 12	Simulation
Week 13	Revision