Question 3

(a)

Minimizing the total toll

Let X_{ij} be the number of trucks travel from node i to j, where $i, j = A, B, C, E, D, F$

Objective: $Minimize\ 8X_{AB} + 5X_{AC} + 7X_{BD} + 17X_{BE} + 11X_{CD} + 14X_{CE} + 18X_{DF} + 16X_{EF}$

Constraints:

$$X_{ij} \geq 0\ for\ all\ i, j$$
$$X_{ij}\ is\ integer\ for\ all\ i, j$$
$$5 \leq X_{ij} \leq 15\ for\ all\ i, j$$
$$-X_{AB} - X_{AC} = -25\ (node\ A)$$
$$X_{AB} - X_{BD} - X_{BE} = 0\ (node\ B)$$
$$X_{AC} - X_{CD} - X_{CE} = 0\ (node\ C)$$
$$X_{BD} + X_{CD} - X_{DF} = 0\ (node\ D)$$
$$X_{CE} + X_{BE} - X_{EF} = 0\ (node\ E)$$
$$X_{DF} + X_{EF} = 25\ (node\ F)$$

Maximizing the average safety rating

Let X_{ij} be the number of trucks travel from node i to j, where $i, j = A, B, C, E, D, F$

Objective: $Maximize\ 8X_{AB} + 5X_{AC} + 8X_{BD} + 10X_{BE} + 6X_{CD} + 4X_{CE} + 9X_{DF} + 5X_{EF}$

Constraints:

$$X_{ij} \geq 0\ for\ all\ i, j$$
$$X_{ij}\ is\ integer\ for\ all\ i, j$$
$$5 \leq X_{ij} \leq 15\ for\ all\ i, j$$
$$-X_{AB} - X_{AC} = -25\ (node\ A)$$
$$X_{AB} - X_{BD} - X_{BE} = 0\ (node\ B)$$
$$X_{AC} - X_{CD} - X_{CE} = 0\ (node\ C)$$
$$X_{BD} + X_{CD} - X_{DF} = 0\ (node\ D)$$
$$X_{CE} + X_{BE} - X_{EF} = 0\ (node\ E)$$
$$X_{DF} + X_{EF} = 25\ (node\ F)$$

(b) The minimum total toll is \$880.

The maximum accumulative safety rating is 535 with $X_{AB} = 15, X_{AC} = 10, X_{BD} = 10, X_{BE} = 5, X_{CD} = 5, X_{CE} = 5, X_{DF} = 15, X_{EF} = 10$

(c) Let Q be the maximum weighted proportional deviation from optimal target values.

Let X_{ij} be the number of trucks travel from node i to j, where $i, j = A, B, C, E, D, F$

Objective: $Minimize\ Q$

Constraints:

$$X_{ij} \geq 0\ for\ all\ i, j$$
$$X_{ij}\ is\ integer\ for\ all\ i, j$$
$$5 \leq X_{ij} \leq 15\ for\ all\ i, j$$
$$-X_{AB} - X_{AC} = -25\ (node\ A)$$
$$X_{AB} - X_{BD} - X_{BE} = 0\ (node\ B)$$
$$X_{AC} - X_{CD} - X_{CE} = 0\ (node\ C)$$

$$X_{BD} + X_{CD} - X_{DF} = 0 \ (node\ D)$$
$$X_{CE} + X_{BE} - X_{EF} = 0 \ (node\ E)$$
$$X_{DF} + X_{EF} = 25 \ (node\ F)$$
$$\frac{(8X_{AB} + 5X_{AC} + 7X_{BD} + 17X_{BE} + 11X_{CD} + 14X_{CE} + 18X_{DF} + 16X_{EF}) - 880}{880} \leq Q$$
$$2 \times \frac{535 - (8X_{AB} + 5X_{AC} + 8X_{BD} + 10X_{BE} + 6X_{CD} + 4X_{CE} + 9X_{DF} + 5X_{EF})}{535} \leq Q$$
$$Q \geq 0$$