QBUS2310 Management Science

BY JOY

优能教育

EXCELLENCE EDU　WECHAT：EXCELLENCEEDU2020

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Description	Weight
Final exam Written exam	50%
Outcomes assessed: LO1 LO2 LO3 LO4	
Assignment 1 n/a	10%
Outcomes assessed: LO1 LO2 LO3 LO4	
Mid-semester exam Short answer and MCQ	30%
Outcomes assessed: LO1 LO2 LO3 LO4	
Assignment 2 n/a	10%
Outcomes assessed: LO1 LO2 LO3 LO4	

Wk	Topic
Week 01	Management science: introduction
Week 02	Linear programming: model formulation and graphical solution
Week 03	Linear programming: computer solution and sensitivity analysis
Week 04	Linear programming: modelling examples
Week 05	Integer programming
Week 06	1. Transportation; 2. Transshipment; 3. Assignment problems
Week 08	Network flow models
Week 09	Project management
Week 10	Non-linear programming
Week 11	Queuing analysis
Week 12	Simulation
Week 13	Revision

Assignment 1
Not available until Mar 16 | **Due** Apr 6 at 11:59 | -/100 pts

Mid-term Exam
Due Apr 8 at 14:00 | -/100 pts

Assignment 2
Not available until May 4 | **Due** May 18 at 23:59 | -/100 pts

Assignment 1
Available until Apr 16 | **Due** Apr 6 at 11:59 | -/100 pts

Mid-term Exam
Due Apr 8 at 14:00 | -/100 pts

Assignment 2
Not available until May 4 | **Due** May 18 at 23:59 | -/100 pts

课程安排

每周知识点复习（共4节）：

Week1-3：week4

Week4-6：week7

Week8-10：week11

Week11-13：week13

讲解内容：知识点+题型练习+部分tutorial题目讲解

ASM题目练习及讲解：

- ASM1：往年ASM题目练手讲解+今年ASM题目讲解提示
- ASM2：往年ASM题目练手讲解+今年ASM题目讲解提示

考试复习：考试前1-2周内

- 期中考试：往年期中考试复习题+期中考试题
- 期末考试：往年复习题+题型复习及练习

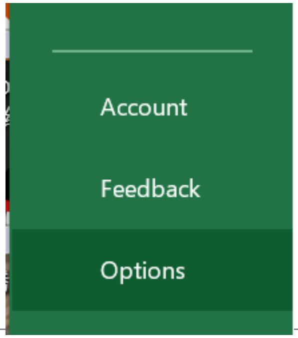
Week1

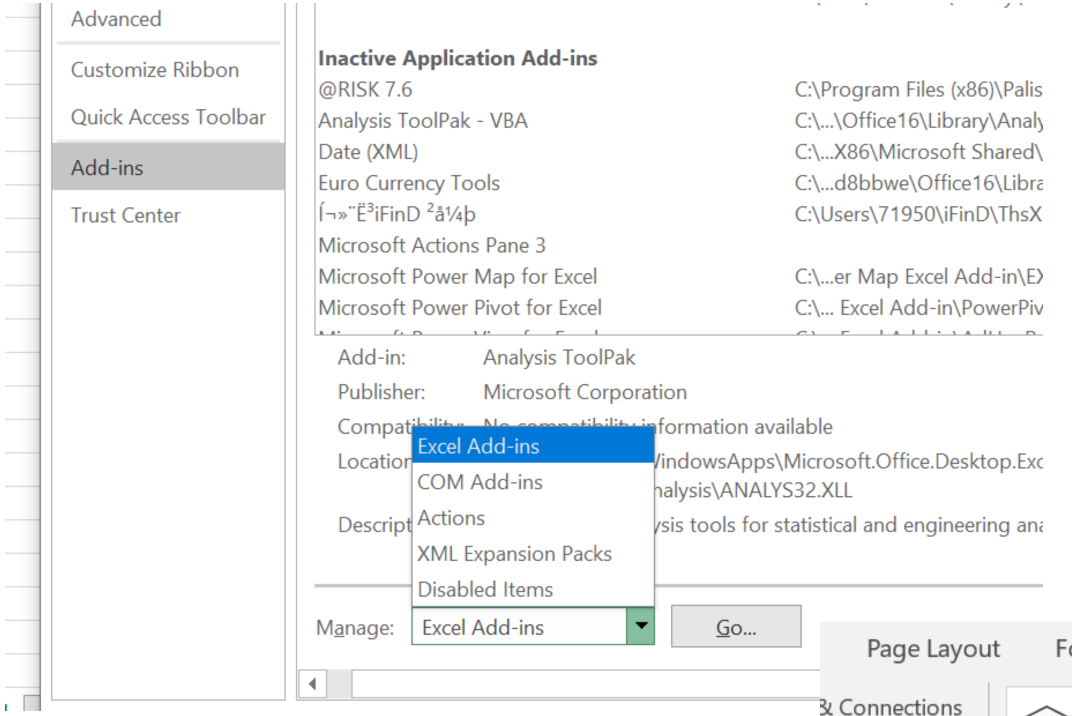
Python Setup & Material

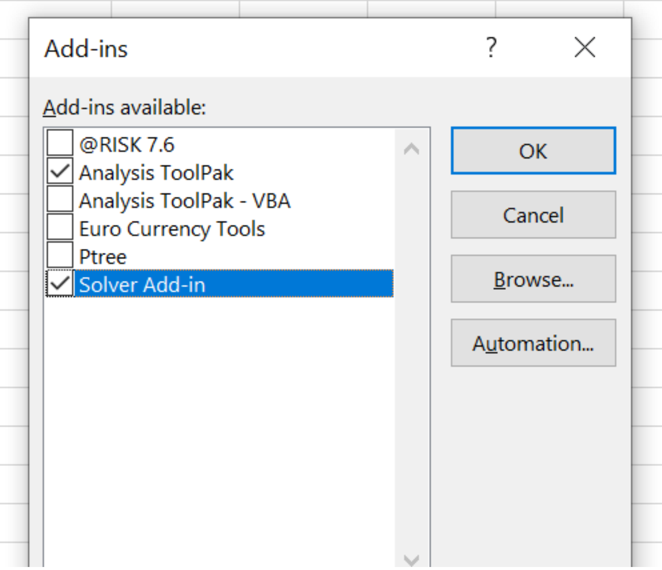
[Python Installation](#)

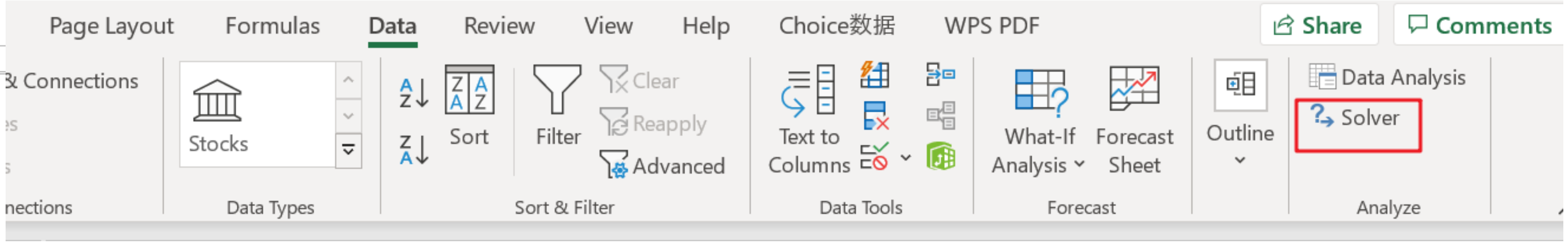
[Installation Slides](#)

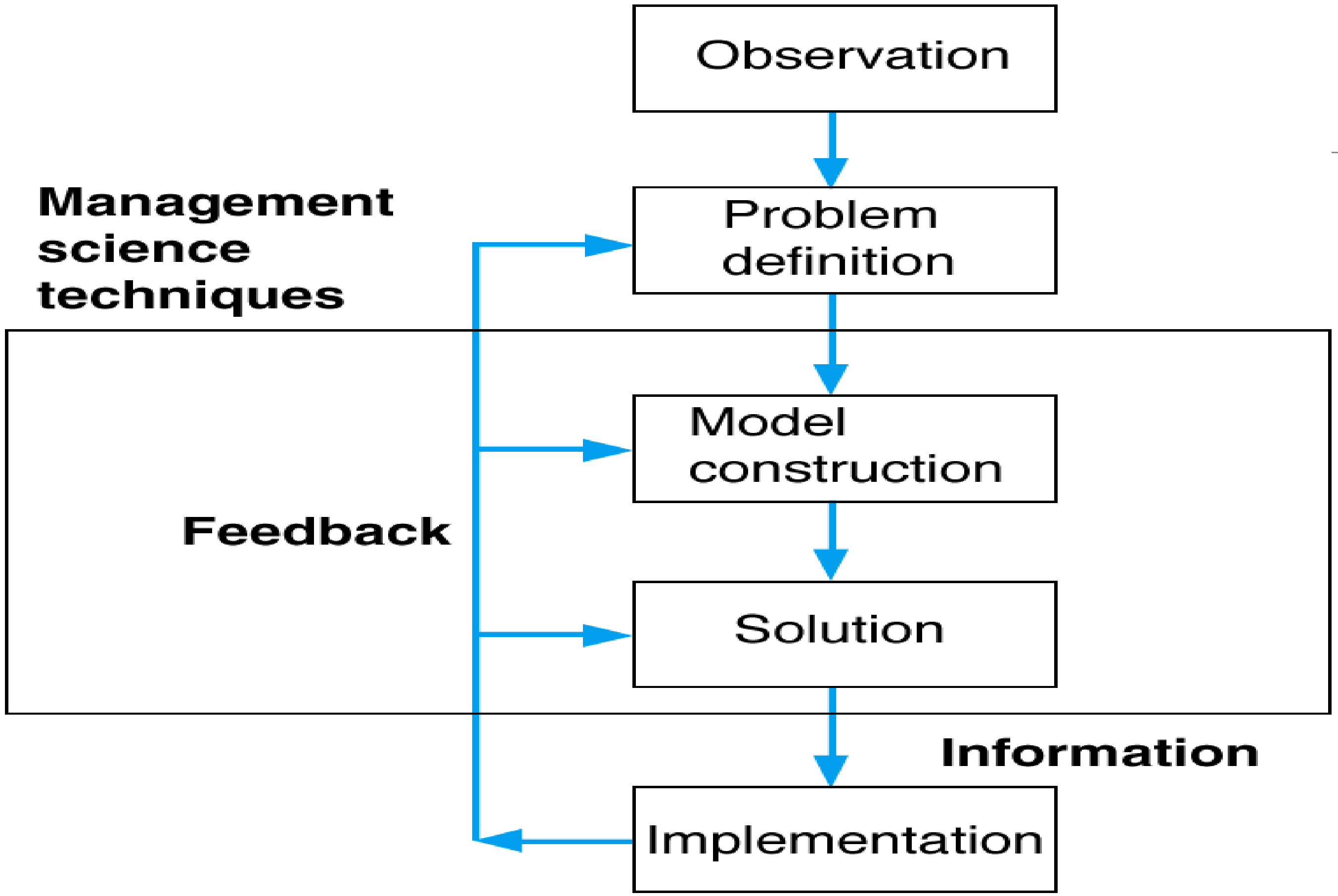
Alternatively, you can use the following link to install Python (instructions and files included): https://www.dropbox.com/sh/3ljjvwtbixd9r3q/AAADiBlxPZzUVbY-Esun6GPAa?dl=0











答题框架

- Assumptions
- Define decision variables
- Objective functions
- Constraints
- Optimal solution
- conclusion

定义及解析

- Assumptions: 关于题目含义做出的合理假设，注意细节
- Variables: 设置变量X_1 X_2 Y_1 Y_2 Z …… 注意python中角标数字从0开始
- Parameters: 参数，变量前乘的系数，此处注意单位一致性
- Model: Objective functions and Constraints，constraints比较容易遗漏细节，最后还需要添加non-negative等条件
- Formal Specification of Model: e.g. maximize/minimize Z (profit/loss/reward/cost)（在objective function步骤中表明）
- Optimal solution: 对于前面model结出的答案，可能存在答案不唯一情况
- Conclusion: 对于解题及optimal solution的总结说明

The Seven-Step Model-Building Process

Step 1: Formulate the Problem

Step 2: Observe the System

Step 3: Formulate a Mathematical Model of the Problem

Step 4: Verify the Model and Use the Model for Prediction

Step 5: Select a Suitable Alternative

Step 6: Present the Results and Conclusion of the Study to the Organization

Step 7: Implement and Evaluate Recommendations

Matrices

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \end{bmatrix}$$

If a matrix A has m rows and n columns, we call A an $m \times n$ matrix. We refer to $m \times n$ as the **order** of the matrix. A typical $m \times n$ matrix A may be written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

*A*乘以常数c，c*A*即是A中的每一个数字乘c
A与B均为m*n 的matrics，两者相加，即为每个位置上的数字对应相加
Matrices 相乘，只能是m***n**与**n***p的matric相乘，并且得到的matrices是m*p的

Matrices

Given any $m \times n$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

the **transpose** of A (written A^T) is the $n \times m$ matrix

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Compute $C = AB$ for

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Because A is a 2×3 matrix and B is a 3×2 matrix, AB is defined, and C will be a 2×2 matrix. From Equation (2),

$$c_{11} = [1 \quad 1 \quad 2] \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 1(1) + 1(2) + 2(1) = 5$$

$$c_{12} = [1 \quad 1 \quad 2] \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = 1(1) + 1(3) + 2(2) = 8$$

$$c_{21} = [2 \quad 1 \quad 3] \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 2(1) + 1(2) + 3(1) = 7$$

Excel solver

	A	B	C	D	E	F
1	MatrixMultiplication					
2				1	1	2
3			A	2	1	3
4						
5			B	1	1	
6				2	3	
7				1	2	
8						
9				5	8	
10			C	7	11	
11						

= MMULT(D2:F3,D5:E7)

Then hit **Control Shift Enter** (not just Enter), and the desired matrix product will be computed. Note that MMULT is an *array* function and not an ordinary spreadsheet function. This explains why we must preselect the range for AB and use Control Shift Enter.

Example

Clock type	Machine hours	Labour hours	Special production hrs.	Profit per unit
Reg.	6	2	0	3
Alarm	2	4	1	8
Total	1800	1600	350	

x_1 – *number of regular clocks to produce*

x_2 – *number of alarm clocks to produce*

$Max\ 3x_1 + 8x_2$ *objective function* (max *profit*)

st.

$6x_1 + 2x_2 \le 1800$ *machine const.*

$2x_1 + 4x_2 \le 1600$ *labour const.*

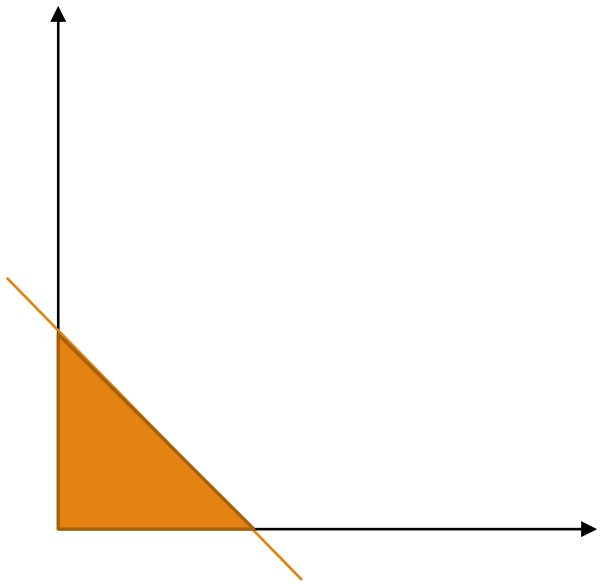
$0x_1 + x_2 \le 350$ *special prod. const.*

$x_1, x_2 \ge 0$ *non – negativity const.*

Graphical Solution

2个变量：直角坐标系
3个变量：立体坐标系

$y = ax + b$, 当a=-1 ,b =1时， 点(0,0) , (1,1) 分别处于直线下方和上方



$y = -x + 1$

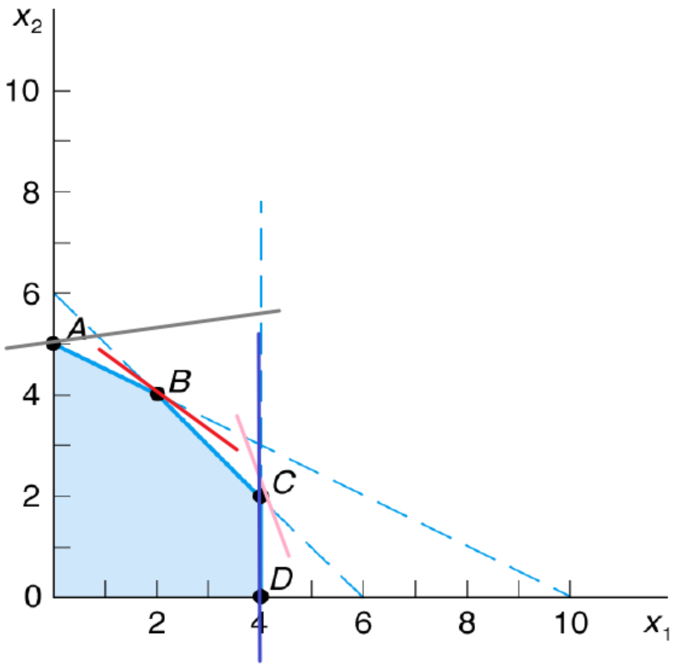
(0,0): 0<1 $y < -x + 1$

(1,1): 1>0 $y > -x + 1$

直线下方：$x + y < 1$

直线上方：$x + y > 1$

阴影部分：$x + y < 1$ 且x和y是non-negative



Week2&3

python

In [25]:
```
import gurobipy as grb
```

In [26]:
```
A = grb.Model('Pottery Barn')
```

In [27]:
```
# create variables
X={}
X[0] = A.addVar(vtype = grb.GRB.CONTINUOUS, obj = 40, name='bowls')
X[1] = A.addVar(vtype = grb.GRB.CONTINUOUS, obj = 50, name='mugs')
```

In [28]:
```
# update the model
A.update()
```

In [29]:
```
# objective
A.modelSense = grb.GRB.MAXIMIZE
```

python

```
In [7]: A.setObjective(40*x[0]+50*x[1], grb.GRB.MAXIMIZE)
```

```
In [30]: # constraints
         A.addConstr(X[0]+2*X[1] <= 40, name='Labour constraint')
         A.addConstr(4*X[0]+3*X[1] <= 120, name='Clay constraint')

Out[30]: <gurobi.Constr *Awaiting Model Update*>
```

python

In [31]:
```
# Solve
A.optimize()
```

```
Optimize a model with 2 rows, 2 columns and 4 nonzeros
Coefficient statistics:
  Matrix range     [1e+00, 4e+00]
  Objective range  [4e+01, 5e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [4e+01, 1e+02]
Presolve time: 0.01s
Presolved: 2 rows, 2 columns, 4 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    9.0000000e+31   3.250000e+30   9.000000e+01      0s
       2    1.3600000e+03   0.000000e+00   0.000000e+00      0s

Solved in 2 iterations and 0.03 seconds
Optimal objective  1.360000000e+03
```

python

```
In [34]: print 'x1 = ', X[0].x
         print 'x2 = ', X[1].x
```

```
 x1 =  24.0
x2 =  8.0
```

Display the objective value of the model 'm' with the string 'obj val =' in front of it.

```
In [35]: print 'obj val = ', A.objVal
```

```
obj val =  1360.0
```

例题1

$\max Z = 3x_1 + 5x_2$

$x_1 \leq 4$

$2 * x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

$x_1, x_2 \geq 0$

分别用画图、excel solver、python 解答

例题2 Sensitive report

Questions 1-5 refer to the following problem description and Excel output:

The production manager for the Whoppy soft drink company is considering the production of two kinds of soft drinks: regular (R) and diet (D). The company operates one 8-hour shift per day. Therefore, the production time is 480 minutes per day. During the production process, one of the main ingredients, syrup, is limited to maximum production capacity of 675 litres per day. Production of a regular case requires 2 minutes and 5 litres of syrup, while production of a diet case needs 4 minutes and 3 litres of syrup. Profits for regular soft drink are \$3.00 per case and profits for diet soft drink are \$2.00 per case.

The formulation for this problem is given below.

MAX Z = \$3R + \$2D

s.t.

$2R + 4D \leq 480$

$5R + 3D \leq 675$

The Excel answer report is given below.

Adjustable Cells (Variables)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$6	Regular =	90.00	0.00	3	0.33	2
\$C\$6	Diet =	75.00	0.00	2	4	0.2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$3	Production (minutes)	480.00	0.07	480	420	210
\$E\$4	Syrup (litres)	675.00	0.57	675	525	315

Adjustable Cells (Variables)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B6	Regular =	90.00	0.00	3	0.33	2
C6	Diet =	75.00	0.00	2	4	0.2

例题

Question 1 (6 marks):

What is the value of the optimal daily profit (to the closest dollar)? Explain your calculation in one or two sentences.

The value of R is 90 and that of D is 75. The daily profit is therefore $3 \times 90 + 2 \times 75 = \$420$

3 marks for answer + 3 marks for brief explanation

Question 2 (6 marks):

Denote by C_2 the profit of one case of diet soft drink. For which values of C_2 will the optimal policy remain unchanged? Explain your calculation in one or two sentences.

The allowable increase and the allowable decrease of C_2 are 4 and 0.2, respectively. Hence the solution will not change as long as $2 - 0.2 = 1.8 \leq C_2 \leq 6 = 2 + 4$.

3 marks for answer + 3 marks for brief explanation

例题

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E3	Production (minutes)	480.00	0.07	480	420	210
E4	Syrup (litres)	675.00	0.57	675	525	315

Question 3 (6 marks):

What is the impact on the profit if the company decides to increase the amount of syrup it uses during production of these soft drinks to 975 litres? Explain your calculation in one or two sentences.

The shadow price of syrup is \$0.57. The increase to 975 requires adding $975 - 675 = 300$ litres. This is within the allowable increase (which is 525). Therefore, the increase in profit will be $300 \times 0.57 = \$171$.

3 marks for answer + 3 marks for brief explanation

Question 4 (6 marks):

What is the impact on the profit if the company decides to decrease the duration of the daily shift to 4 hours (240 minutes) instead of 8 (480 minutes)? Provide a brief explanation.

The shadow price of time is \$0.07. The decrease to 240 is beyond the allowable decrease of 210. Taking away the first 210 minutes will result is a loss of $210 \times 0.07 = \$14.7$. At this stage, we don't know the new shadow price of time, so will not be able to correctly calculate the effect of a further decrease of 30 minutes.

2 marks for answer + 4 marks for brief explanation: 2 marks for calculating the effect of the first 210 minutes & 2 marks for explaining the shadow price then changes

例题

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E3	Production (minutes)	480.00	0.07	480	420	210
E4	Syrup (litres)	675.00	0.57	675	525	315

Question 5 (6 marks):

You are considering producing and selling a new drink, the "Tropical Bomb". A case of the "Tropical Bomb" requires 3 minutes of production time and 4 litres of Syrup. What is the minimum profit per case that would make the "Tropical Bomb" drink worthwhile producing?

The cost of producing a case of Tropical Bomb is equal to $3 \times 0.07 + 4 \times 0.57 = \$2.49$. Therefore, a profit of $2.49 or more would make it worthwhile producing the new drink.

3 marks for answer + 3 marks for brief explanation

例题3-小挑战

某厂生产 Ⅰ、Ⅱ、Ⅲ 三种产品，都分别经 A、B 两道工序加工。设 A 工序可分别在设备 A1 和A2 上完成，有 B1 、B2 、B3 三种设备可用于完成 B 工序。已知产品 Ⅰ 可在A 、B 任何一种设备上加工；产品 Ⅱ 可在任何规格的 A 设备上加工，但完成 B 工序时，只能在B1 设备上加工；产品 Ⅲ 只能在A2 与B2 设备上加工。加工单位产品所需工序时间及其他各项数据如下表，试安排最优生产计划，使该厂获利最大。

设备	产品			设备有效台时	满负荷时的设备加工费（单位小时）
	Ⅰ	Ⅱ	Ⅲ		
A1	5	10		6000	300
A2	7	9	12	10 000	321
B1	6	8		4000	250
B2	4		11	7000	783
B3	7			4000	200
原料费（每件）	0.25	0.35	0.5		
售价（每件）	1.25	2.00	2.8		

例题3解析

解：设x_{ijk}表示产品i在工序j的设备k上加工的数量。约束条件有：

$5x_{111}+10x_{211}\le 6000$ (设备A1)

$7x_{112}+9x_{212}+12x_{312}\le 10000$（设备A2）

$6x_{121}+8x_{221}\le 4000$ （设备B1）

$4x_{122}+11x_{322}\le 7000$ (设备B2)

$7x_{123}\le 4000$ (设备B3)

$x_{111}+x_{112}=x_{121}+x_{122}+x_{123}$ （产品I在工序A，B上加工的数量相等）

$x_{211}+x_{212}=x_{221}$ （产品II在工序A，B上加工的数量相等）

$x_{312}=x_{322}$ （产品III在工序A，B上加工的数量相等）

$x_{ijk}\ge 0$ $(i=1,2,3;\ j=1,2;\ k=1,2,3)$

例题3解析

目标是利润最大化，得出公式：

$$\begin{aligned}\max\ & 0.75x_{111} + 0.775x_{112} + 1.15x_{211} + 1.36x_{212} \\ & + 1.915x_{312} - 0.375x_{121} - 0.5x_{221} - 0.448x_{122} \\ & - 1.23x_{322} - 0.35x_{123}\end{aligned}$$

例题

Question 1

The following diagram shows the constraints for a LP model. Assume the point (0,0) satisfies constraint (B,J) but does not satisfy constraints (D,H) or (C,I). Which set of points on this diagram defines the feasible solution space?

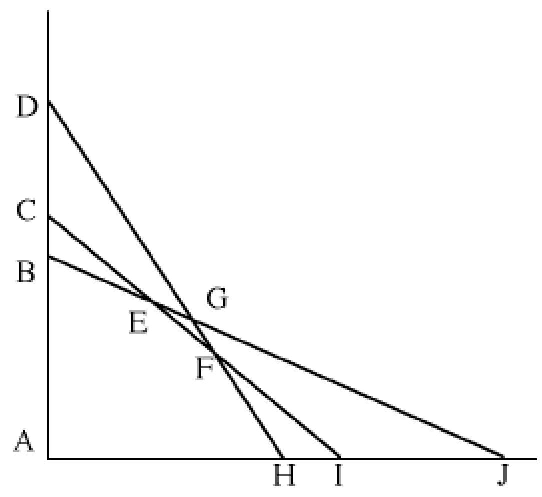
(constraint (B,J) refers to the line connecting points B and J, constraint (D,H) refers to the line connecting points D and H, etc.)

a. A, B, E, F, H

b. A, D, G, J

c. F, G, H, J

d. F, G, I, J



例题

Question 2

A company uses 4 pounds of resource 1 to make each unit of X_1 and 3 pounds of resource 1 to make each unit of X_2. There are only 150 pounds of resource 1 available. Which of the following constraints reflects the relationship between X_1, X_2 and resource 1?

a. $4 X_1 + 3 X_2 \geq 150$

b. $4 X_1 + 3 X_2 \leq 150$

c. $4 X_1 + 3 X_2 = 150$

d. $4 X_1 \leq 150$

例题

Question 3

A diet is being developed which must contain at least 100 mg of vitamin C. Two fruits are used in this diet. Bananas contain 30 mg of vitamin C and Apples contain 20 mg of vitamin C. The diet must contain at least 100 mg of vitamin C. Which of the following constraints reflects the relationship between Bananas, Apples and vitamin C?

a. $20\,A + 30\,B \geq 100$

b. $20\,A + 30\,B \leq 100$

c. $20\,A + 30\,B = 100$

d. $20\,A = 100$

例题

Question 4

In a maximization problem if the allowable increase for a constraint is 100 and we add 110 units of the resource what happens to the objective function value?

a. increase of 100

b. increase of 110

c. decrease of 100

d. increases but by unknown amount

例题

Question 5

How many constraints are there in a transportation problem which has 5 supply points and 4 demand points? (Ignore the non-negativity constraints)

a. 4

b. 5

c. 9

d. 20

例题

Questions 6-7 refer to the following LP:

Maximize 2x1+4x2

st

x1+x2<100

2x1+3x2<150

Question 6

What is the reduced cost of X1?

a. 0

b. -2/3

c. -4/3

d. -3/4

Question 7

What is the reduced cost of X2?

a. 0

b. 2/3

c. 4/3

d. 3/4

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B2	x1	0	-0.666666667	2	0.666666667	1E+30
C2	x2	50	0	4	1E+30	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D4	st	50	0	100	1E+30	50
D5		150	1.333333333	150	150	150

Questions 8-9 refer to the following problem description:

The Wiethoff Company has a contract to produce 10,000 garden hoses for a customer. Wiethoff has four different machines that can produce this kind of hose. Because these machines are from different manufacturers and use differing technologies, their specifications are not the same.

Machine	Fixed Cost to Setup Production Run	Variable Cost per Hose	Capacity
1	750	1.25	6000
2	500	1.50	7500
3	1000	1.00	4000
4	300	2.00	5000

Let X_i denote the number of garden hoses produced on machine i, and let Y_i be a binary variable obtaining a value of 1 if machine i is used and 0 otherwise $(i = 1,2,3,4)$.

Question 8 (6 marks):

Write the linking constraint for machine 1 (the constraint which ensures that production can take place on machine 1 only if it is used).

$X_1 - 6000 Y_1 \leq 0$

6 marks (don't deduct points if students used a larger value of M. deduct 2 points if they used M instead of a real number).

Question 9 (6 marks):

Write a constraint to ensure that if machine 4 is used, machine 1 will not be used.

$\mathbf{Y_1 + Y_4 \leq 1}$

6 marks.