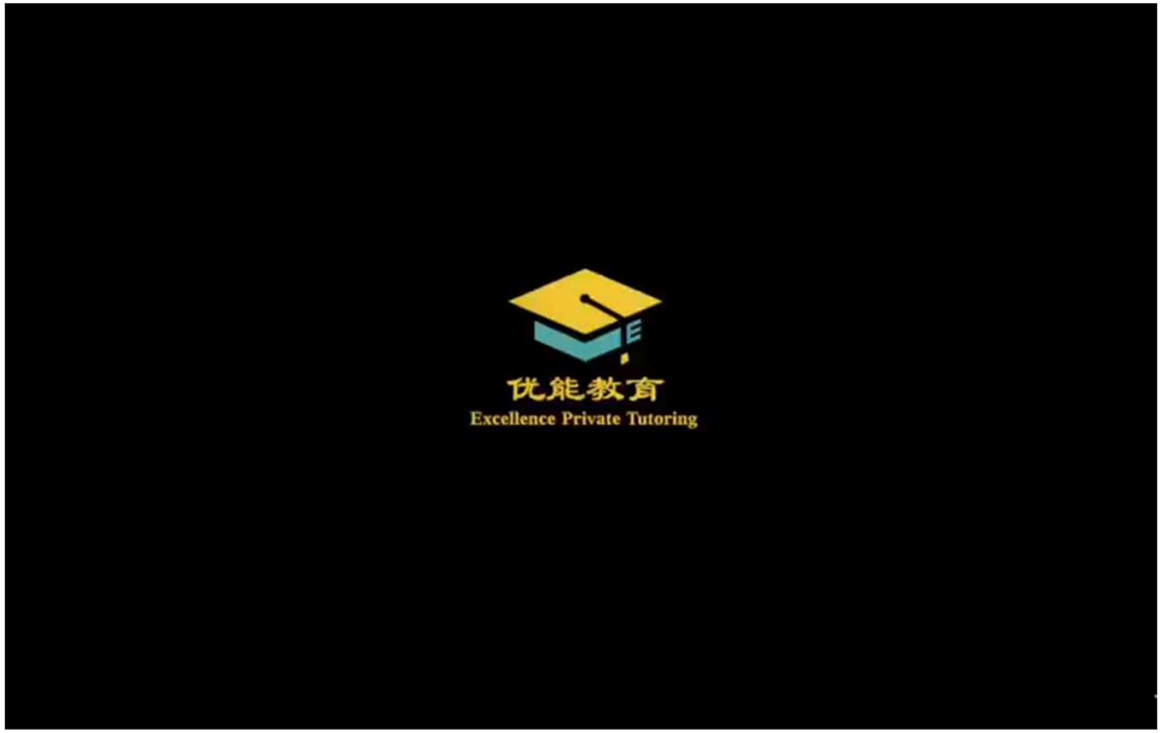
优能教育

2020 Semester 1

QBUS 2310 期中复习课2

TUTOR: Joy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课程安排

每周知识点复习（共4节）：

~~Week1-3：week4~~

Week4-6：week7 期中考试想拿高分的同学建议弄懂

Week8-10：week11

Week11-13：week13

讲解内容：知识点+题型练习+部分tutorial题目讲解

ASM题目练习及讲解：

- ASM1+2：往年ASM2题目练手讲解+今年ASM题目讲解提示

Online test题目讲解：

待定

考试复习：考试前1-2周内

- ~~期中考试：往年期中考试复习题+期中考试题~~
- 期中考试2：期中考试1中部分未讲解题目解析，期中复习题目所有疑问汇总
- 期末考试：往年复习题+题型复习及练习

By Joy

往年ASM1 题目练习及讲解

By Joy

Question 3 (40 marks):

Accessories & co. is producing three kinds of covers for Apple products: one for iPod, one for iPad, and one for iPhone. The company's production facilities are such that if we devote the entire production to iPod covers, we can produce 6000 of them in one day. If we devote the entire production to iPhone covers or iPad covers, we can produce 5000 or 3000 of them in one day. The production schedule is one week (5 working days), and the week's production must be stored before distribution. Storing 1000 iPod covers (packaging included) takes up 40 cubic feet of space. Storing 1000 iPhone covers (packaging included) takes up 45 cubic feet of space, and storing 1000 iPad covers (packaging included) takes up 210 cubic feet of space. The total storage space available is 6000 cubic feet. Due to commercial agreements with Apple, Accessories & co. has to deliver at least 5000 iPod covers and 4000 iPad covers per week in order to strengthen the product's diffusion. The marketing department estimates that the weekly demand for iPod covers, iPhone, and iPad covers does not exceed 10000 and 15000, and 8000 units, therefore the company does not want to produce more than these amounts for iPod, iPhone, and iPad covers. Finally, the net profit per each iPod cover, iPhone cover, and iPad cover is $4, $6, and $10, respectively.

The aim is to determine a weekly production schedule that maximizes the total net profit.

By Joy

Module 2:

1. Write a Linear Programming formulation for the problem. Start by stating any assumptions that you make. For this first formulation, the decision variables should represent the proportion of time spent each day on producing each of the two items:

   x1 = proportion of time devoted each day to iPod cover production,

   x2 = proportion of time devoted each day to iPhone cover production,

   x3 = proportion of time devoted each day to iPad cover production.

   (Different formulations will be required for parts (2) and (3).)

By Joy

We assume:

(a) that the production can be split between the two products in any desired way (that is, fractional values for x1, x2, and x3 are acceptable), and

(b) that the number of produced item of each type is directly proportional to the time devoted to producing the item.

Given these assumptions, we can formulate the problem as an LP as follows:

Max $\{120000x1 + 150000x2 + 150000x3\}$

s.t.:

Max daily production: $x1 + x2 + x3 \leq 1$

Storage: $1200x1 + 1125x2 + 3150x3 \leq 6000$

Min iPod production: $30000x1 \geq 5000$

Min iPad production: $15000x3 \geq 4000$

Max iPod demand: $30000x1 \leq 10000$

Max iPhone demand: $25000x2 \leq 15000$

Max iPad demand: $15000x3 \leq 8000$

Non-negativity constraints: $x1, x2, x3 \geq 0$ (note that all variables have lower bounds so it is not a mistake if you forget to add non-negativity constraints. Nevertheless, it is good practice to always include them).

By Joy

2. Write a second Linear Programming formulation for the problem. Label each constraint (except nonnegativity). For this second formulation, the decision variables should represent the number of items of each type produced over the week:

   y1 = number of iPod covers produced over the week,

   y2 = number of iPhone covers produced over the week,

   y3 = number of iPad covers produced over the week.

   The problem data is the same but you must make sure that everything matches the new decision variables.

We make the same assumptions as for part (1). We can formulate the problem as an LP as follows:

Max $\{4y1 + 6y2 + 10y3\}$
s.t.:
Max weekly production: $1/6000y1 + 1/5000y2 + 1/3000y1 \leq 5$
Storage: $0.04y1 + 0.045y2 + 0.21y3 \leq 6000$
Min iPod production: $y1 \geq 5000$
Min iPad production: $y3 \geq 4000$
Max iPod demand: $y1 \leq 10000$
Max iPhone demand: $y2 \leq 15000$
Max iPad demand: $y3 \leq 8000$
Non-negativity constraints: $y1, y2, y3 \geq 0$
Because y1 already has a lower bound, omitting the nonnegativity constraint for y1 is not considered an error.

By Joy

Q3:

3. Write a third Linear Programming formulation for the problem. Assume that each working day has 8 working ours. For this third formulation, the decision variables should be:

   $z1$ = number of hours devoted to the production of iPod smart covers in one week,

   $z2$ = number of hours devoted to the production of iPhone smart covers in one week,

   $z3$ = total number of production hours employed during the week.

   Express the objective function in thousands of dollars. The problem data is the same but you must make sure that everything matches the new decision variables.

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As requested, we use two decision variables: z1 is the number of hours devoted to iPod cover production, z2 is the number of hours devoted to iPhone cover production, and z3 is the total number of production hours employed for the week. It follows that the number of hours devoted to iPad cover production is $z3 - z1 - z2$, which should be a nonnegative number (i.e. we must impose $z1 + z2 \leq z3$). We can therefore formulate the problem as follows:

$$\text{Max } \{3000z1 + 3750z2 + 3750(z3 - z1 - z2)\}$$

s.t.:

iPad Production: $z1 \leq 40$

iPhone Production: $z2 \leq 40$

Storage: $30z1 + 28.125z2 + 78.75(z3 - z1 - z2) \leq 6000$

Min iPod production: $750z1 \geq 5000$

Min iPad production: $375(z3 - z1 - z2) \geq 4000$

Max iPod demand: $750z1 \leq 10000$

Max iPhone demand: $625z2 \leq 15000$

Max iPad demand: $375(z3 - z1 - z2) \leq 8000$

Total working hours per week: $z3 \leq 40$

Non-negativity constraints: $z1, z2, z3 \geq 0$

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4. What is the relationship between the variables z1,z2,z3 of part (3) and the variables x1,x2,x3 of part (1) of this problem? Give a formula to compute z1, z2, z3 from x1, x2, x3.

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The relationship between z1, z2, z3 of part (3) and x1, x2, and x3 of part (1) is the following. Z1=40x1 because x1 represents the percentage of time of each day dedicated to iPad cover production: multiplying x1 by the total number of hours in a production period (= 40) yields the number hours spent on iPods. Similarly, we have z2=40x2. Then, z 3= 40 (x1+x2+x3) because x1+x2+x3 represents the fraction of time used during the day; multiplying this by the total number of hours in a production period yields the total number of hours employed during the week.

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5. Solve all three formulations using Python & Gurobi.
6. Provide a summary of the optimal production strategy, the optimal profit and the use of resources.

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The table below summarizes the results of parts (1) – (3):

	x-variable	y-variable	z-variable
meaning	Proportion of time spent on production	Units produced	Working hours
iPod	0.1666	5000	6.6666
iPhone	0.3	7500	22.666
iPad	0.5333	8000	40* (total number of hours)
Profit	145,000	145,000	145,000

All three formulations yield the same profit. The y-variables provide us with the number of units of each product, whereas the x-variables provide us with the proportion of time spent on each product. The z-variables are a bit more complicated. z1 and z2 provide us with the weekly hours devoted to the first two products. z3 provides us with the total number of weekly hours used (40). The difference between z3 and (z1+z2) is the total number of weekly hours devoted to producing product 3 (10.66 hours).

By Joy

往年期中考试题练习及讲解（共三套）

By Joy

Problem 2: (25 points)

The Candid Camera Company manufactures three lines of cameras: the Cub, the Quickiematic and the VIP, whose contributions to profit are \$3, \$9, and \$25, respectively. The distribution centre requires that at least 250 Cubs, 375 Quickiematics, and 150 VIPs be produced each week. Each camera requires a certain amount of time in order to: (1) manufacture the body parts; (2) assemble the parts (lenses are purchased from outside sources and can be ignored in the production scheduling decision); and (3) inspect, test, and package the final product. The Cub takes 0.1 hours to manufacture, 0.2 hours to assemble, and 0.1 hours to inspect, test, and package. The Quickiematic needs 0.2 hours to manufacture, 0.35 hours to assemble, and 0.2 hours for the final set of operations. The VIP requires 0.7, 0.1, and 0.3 hours, respectively. In addition, there are 250 hours per week of manufacturing time available, 350 hours of assembly, and 150 hours total to inspect, test, and package.

(a) (10 points) Formulate and solve this scheduling problem as a linear program that maximises the profit.

(b) (8 points) Formulate and solve the Lagrange dual problem. Verify that strong duality holds.

(c) (2 points) What is the shadow price of each type of resource (manufacturing time, assembly time, and inspection, testing and packaging time)?

(d) (2 points) Provide interpretation of the dual variable associated with the constraint on the number of 'inspect, test and package' hours.

(e) (3 points) Provide sensitivity analysis for the RHS of the constraint on the total number of 'inspect, test and package' hours, i.e., provide and interpret the minimum and maximum RHS value. (*Hint*: get 'SARHSLow' and 'SARHSUp' attributes of your primal constraints)

(b) Let y_1, y_2 and y_3 be the dual variables associated with the first, second and third constraint, respectively. In addition, let z_1, z_2 and z_3 be the dual variables associated with the forth, fifth and sixth constraint, respectively. Then, the Lagrange dual problem is

$$\begin{aligned}
\text{Minimize} \quad & 250y_1 + 350y_2 + 150y_3 + 250z_1 + 375z_2 + 150z_3 && (21)\\
\text{subject to} \quad & 0.1y_1 + 0.2y_2 + 0.1y_3 + z_1 \geq 3 && (22)\\
& 0.2y_1 + 0.35y_2 + 0.2y_3 + z_2 \geq 9 && (23)\\
& 0.7y_1 + 0.1y_2 + 0.3y_3 + z_3 \geq 25 && (24)\\
& y_i \geq 0 \quad \forall i \in \{1,2,3\} && (25)\\
& z_i \leq 0 \quad \forall i \in \{1,2,3\} && (26)\\
& && (27)
\end{aligned}$$

numerical result: the optimal objective value is 8291.67 with

- $y^\star = (0.0, 0.0, 83.33)$
- $z^\star = (0.0, -7.67, -5.33)$

Since the objective value of the dual is equal to the objective value of the primal, strong duality holds.

 By Joy

某工厂拥有一定的生产原材料时，该工厂考虑时自己进行产品的生产所赚的利润最大还是将原材料直接出售给其他工厂所得利润最大？

利润最大化

$$\mathrm{Max}Z = 2x_1 + 3x_2$$

$$\text{s.t.}\begin{cases} 2x_1 + 2x_2 \le 12 \\ 4x_1 \le 16 \\ 5x_2 \le 15 \\ x_1, x_2 \ge 0 \end{cases}$$

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对应对偶问题是，在平衡劳动力及原材料成本后，所确定的价格系统最具有竞争力

$$\text{MinW} = 12y_1 + 16y_2 + 15y_3$$

$$\text{s.t.}\begin{cases} 2y_1 + 4y_2 \geq 2 \\ 2y_1 + 5y_3 \geq 3 \\ y_1, y_2, y_3 \geq 0 \end{cases}$$

用于生产第i种产品的资源转让收益，不小于生产该产品时获得的利润

经济意义为解释对工时及原材料的单位定价

By Joy

Dual variable 的对偶问题

若工厂自己不生产ABC 产品，将工时及原材料转让给外来加工时，这样的价格系统才能保证不亏本又最富有竞争力

当原问题与对偶问题都取得最优解时，这一对线性规划对应的目标函数值应相等，即 max Z= min W

$$\mathrm{Max}Z = 2x_1 + 3x_2$$

$$\text{s.t.}\begin{cases} 2x_1 + 2x_2 \le 12 \\ 4x_1 \le 16 \\ 5x_2 \le 15 \\ x_1, x_2 \ge 0 \end{cases}$$

$$\mathrm{Min}W = 12y_1 + 16y_2 + 15y_3$$

$$\text{s.t.}\begin{cases} 2y_1 + 4y_2 \ge 2 \\ 2y_1 + 5y_3 \ge 3 \\ y_1, y_2, y_3 \ge 0 \end{cases}$$

By Joy

Question 13 (6 marks):

Your company has the capacity to produce at most 300 units of a certain product. The per unit selling price of this product is \$50 for the first 100 units, and \$60 per unit for any additional unit sold. For example, if you sell 150 units, the first 100 will sell at \$50 each, while the last 50 units will produce a per unit revenue of \$60. Let X_1 denote the number of units you sell at \$50 per unit, and X_2 denotes the number of units sold at \$60 per unit. Clearly, your objective function is MAX $50X_1+60X_2$. Write the constraint(s) that will ensure that your objective function calculates the correct total revenue. (You are permitted to define additional variables if required).

Let Y be a binary variable.

$X_1 \geq 100Y$

$X_2 \leq 200Y$

$X_1 + X_2 \leq 300$.

6 marks – 2 for each constraint.

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Question 12 (10 marks):

In Hope County there are 6 main towns but no hospitals. Since Hope County is isolated and the closest hospital is more than 100KM away, officials have decided to build hospitals within the county. The cost of building a hospital in each of the county's towns is given in the following table:

Town	Cost (in millions of $)
1	28
2	25
3	19
4	33
5	21
6	27

By Joy

The distance in KM between any 2 towns in Hope county is given in the following table:

	Town1	Town2	Town3	Town4	Town5	Town6
Town1	--	23	26	45	13	33
Town2		--	17	33	18	22
Town3			--	22	23	17
Town4				--	26	21
Town5					--	23
Town6						--

The county wants to minimize the cost of building the new hospital while ensuring that every town has at least 2 hospitals that are located within 25KM of the town.

Formulate an ILP model for the problem. (Do not attempt to solve!)

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Town3			--	22	23	17
Town4				--	26	21
Town5					--	23
Town6						--

The county wants to minimize the cost of building the new hospital while ensuring that every town has at least 2 hospitals that are located within 25KM of the town.

Formulate an ILP model for the problem. (Do not attempt to solve!)

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Answer:

Let X_i = binary variable that is equal to 1 if a hospital is built in town i and 0 otherwise, i=1,2,…,6.

MIN: $28X_1 + 25X_2 + 19X_3 + 33X_4 + 21X_5 + 27X_6$

Subject to:

$X_1 + X_2 + X_5 \geq 2$

$X_1 + X_2 + X_3 + X_5 + X_6 \geq 2$

$\boxed{X_2 + X_3 + X_4 + X_5 + X_6 \geq 2}$

$X_3 + X_4 + X_6 \geq 2$

$X_1 + X_2 + X_3 + X_5 + X_6 \geq 2$

$X_2 + X_3 + X_4 + X_5 + X_6 \geq 2$

X_i binary

10 marks – 2 for defining variables; 2 for OF; 1 point for each constraint (3 points deducted if all “2”s are “1”s).

 By Joy

往年期中复习题练习及讲解

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Question 10

A company can choose one of two policies: (i) not produce Product 1; or (ii) produce at least 150 units of Product 1. Let X_1 be the number of units of Product 1 to produce. Formulate a constraint to enforce the company's requirement (You may define additional variables if needed).

Answer: $X_1 - 150Y_1 \geq 0$

练习题答案

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Questions 6-7 refer to the following problem description:

In a blending problem, X_1, X_2, and X_3 represent the weight in Kilograms of three types of fertilizers. The price of the fertilizers is $1.5, $2.5, and $3 per Kilogram, respectively. The decision maker has up to $250 for the purchase of fertilizers.

Question 6 (6 marks):

There two requirements imposed on the decision maker: (1) no more that $100 is spent on fertilizer 2; and (2) the combined weight of fertilizers 1 and 2 does not exceed 75 Kilograms. How would these conditions be formulated as linear programming constraints?

$\mathbf{2.5X_2 \le 100}$

$\mathbf{X_1 + X_2 \le 75}$

3 marks for each constraint

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Question 7 (6 marks):

It turns out that when mixing fertilizers, some of their weight is lost in the process. When fertilizer 1 is mixed with other fertilizers it loses 10% of its weight. This figure increases to 15% and 20% for fertilizers 2 and 3, respectively. The decision maker is now required to ensure that the weight of fertilizer 1 in the **mixed product** is at least twice that of fertilizer 2. How would this condition be formulated as a linear constraint?

$0.9X_1 \geq 2(0.85X_2)$

Which is equivalent to:

$0.9X_1 \geq 1.7X_2$

$0.9X_1 - 1.7X_2 \geq 0$

2/6 marks for students who gave one of the following answers:

$X_1 - 2X_2 >= 0$

$0.9X_1 - 0.85 >= 0$

Question 16 (6 marks):

A company produces a single product that is sold for $25 per unit. The company has decided to restrict itself to one of the following production strategies: (1) produce at most 40 units of the product; or (2) produce exactly 50 units of the product. Formulate a set of constraints that will impose the above restriction (you are permitted to introduce additional variable(s)).

Let Y_1 and Y_2 be a binary variables.

$$\boldsymbol{X \leq 40Y_1 + 50Y_2}$$

$$\boldsymbol{X \geq 50Y_2}$$

$$\boldsymbol{Y_1 + Y_2 = 1}$$

6 marks – 2 for each constraint.

By Joy