QBUS2820 Assignment2

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Task A

Introduction

In a traditional manner, sale prices of houses were predicted by comparing sale prices and costs in the real estate market. There was no general standard to estimate the value of houses. Machine learning techniques therefore play an important role to help establishing models for sale prices of house predictions. It has been stated that the availability of a house price prediction model helps fill up an essential information gap and improve the efficiency of the real estate market (Calhoun, 2003).

This project aims to develop predictive models for sale prices of house with machine learning techniques. With the sale price which is a numerical variable being the response of predictive models, five models are developed and validated.

By comparing the root mean squared errors of predictions, the lasso regression model and random forest model are found to have the best predictive performance for the housing data, compared to elastic net, ridge regression, k-nearest neighbor regression and stepwise regression with forward selection.

Data processing and exploratory data analysis

There are 36 numeric variables and 43 categorical variables in the housing data. By calculating the correlation coefficient, 12 numeric variables are found to be highly linearly related to sale price, as the absolute values of corresponding correlation coefficients are greater than 0.5. The distributions of these variables are visualized in figure 2. 'TotRms AbvGrd', 'Garage Area', '1st Fir SF' and 'SalePrice' are shown to be right-skewed while 'Garage Yr Blt' and 'Overall Qual' are left-skewed, but these distributions are significantly influenced by outliers in several columns. Moreover, some variables, such as 'TotRms AbvGrd', tend to have linear relationships with other variables except sale price, leading to multi-collinearity. This could violate the assumption of some predictive models, such as multiple linear regression, thus robustness to multi-collinearity should be carefully considered when developing predictive models.

Figure 3 shows the distribution of sale price with regard to different categorical features. For most categorical features, sale prices tend to largely different for different groups of the categorical feature, except 'BsmtFin Type 2' and 'Land Slope'. However, although medians sale prices look similar for different groups of 'BsmtFin Type 2' and 'Land Slope', the distribution of sale prices are not identical. Hence, it can still be worthwhile to include these two variables as features to predict sale price. In addition, the boxplots also highlight the outliers of sale price existing in different categorical groups.

Besides affecting the shapes of data distribution, the existing outliers of numeric variables can also post an effect on predictive performance and goodness of fit of models. However, as long as these outliers do not violate the assumptions of predictive models, it is acceptable to contain outliers when developing models.

Feature engineering

As shown in Figure 1, there are huge amounts of missing values in several columns: 'Alley', 'Fireplace Qu', 'Pool QC', 'Fence', 'Misc Feature', with more than 40% missing values within each column. With this issue, such variables are uninformative to be a feature of predictive models as there are too few observations. To deal with this, removing all rows with missing values can lead to significant loss of information, while imputation using small amount of observations can misrepresent the population for largely incomplete columns. Therefore, 'Alley', 'Pool QC', 'Fence', 'Misc Feature' are abandoned due to high missing rates.

Besides, there are 19 columns containing missing values but the percentages of missing values are less than 20%. This can be deal with by imputation. The missing values are imputed by using the most frequent value of each column.

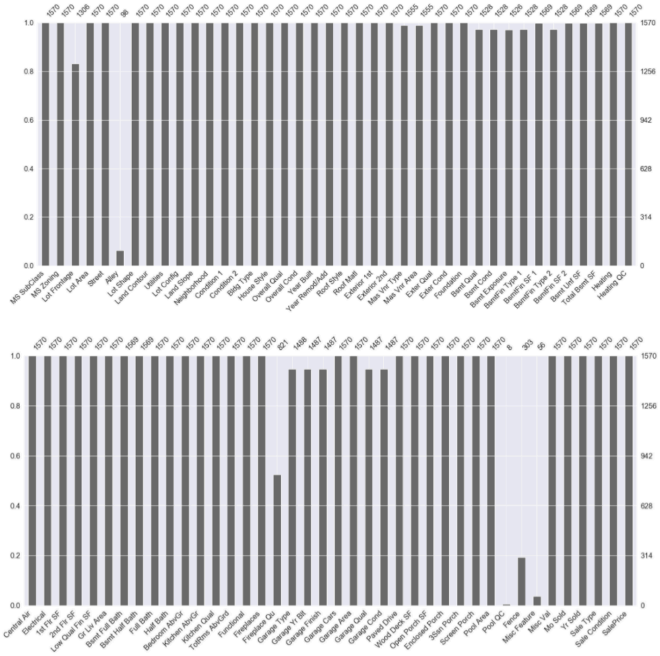

Fig. 1: *Visualizing missingness of housing data in training set.*

From exploratory data analysis, the numeric variables tend to have different scales. Therefore, data standardization is performed prior to model development for numeric variables by subtracting the mean, followed by dividing the standard deviation of the corresponding columns.

After feature engineering, there are 74 informative features, with 36 features being numeric and 38 features being categorical. There are 1570 observations in the training set and 1210 observations in the testing set. For regression models involving categorical features, dummy variables are created for each categorical feature.

Methodology

Five regression models are trained by two sets of data where one training set contains numeric features only and the other set includes both categorical and numeric features. With regards to the issue of multi-collinearity mentioned before, all of the five regression techniques used are capable for addressing multi-collinearity. Therefore there is no need to eliminate features suffering from multi-collinearity, and all features surviving in the feature engineering stage are involved in predictive models development.

To develop the best parameter set for each regression models, hyperparameters are tuned with 5-fold cross validation. The performance of models with different values of hyperparameters is estimated by negative mean squared error, where a larger score indicates better predictive performance. For each machine learning model, the value of hyperparameter resulting in the best performance is selected to be optimal.

Random forest regression. A random forest regression model, which is an extension of decision tree, is developed. It is a supervised learning techniques is developed. which applies ensemble learning method for regression. With this technique, decision trees are created in parallel by bagging (i.e. reduce the variance of predictions by resampling). In this case, the mean predicted sale price of the individual trees is reported.

As random forest applies bootstrap sampling, multi-collinearity is not a concern because it is simply selecting different features from training set to develop models. In addition, the splits of each tree are randomly sampled from the training set so that with the randomness, overfitting can be avoided.

The number of features being spitted on at each leaf node which is a hyperparameter is the main focus to find the optimal random forest model. Another determinant parameter is the number of trees in random forest. To tun the random forest model, 10 values, which are 200, 400, 600, 800, 1000, 1200, 1400, 1600, 1800 and 2000, for number of trees ('n_estimators') are fed into models. The maximum number of features being spitted, which is presented as 'max_feature' in python, is obtained by taking a square root of the number of feature.

The optimal number of features being spitted is 1200 for the training set with numeric features only. The optimal number changes to 200 when the model involves both categorical and numeric features. By design, the random forest model is a black box method, making it hard to interpret compared to other models in this study.

Lasso Regression. A lasso regression model is developed as it is able to deal with multi-collinearity as well as feature selection. As such it is a highly automate technique with satisfactory predictive performance and interpretability.

The key component of the lasso model is to perform L1 regularization. Hence the objective is to minimize $\sum_{i=1}^{n}(y_i - \beta_0 - \sum_j x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^{p} |\beta_j|$, where λ is the hyperparameter which is tuned to control the strength of L1 regularization penalize. Increasing λ results in higher level of L1 penalty and thus more features are eliminated. This also affects the bias-variance trade-off as a increase of λ leads to a increase in bias and a decrease in variance.

It might be challenging to initialize a list of lambdas to tun the model as there is no strict upper boundary of lambdas. Fortunately, the `LassoCV` package in python can fit the data and automatically find out the optimal λ among 100 different values ranging from 65 to 64990. For the numeric training set, the optimal value of λ is 921.2 while it is 130.6 for model including both numeric and categorical features.

36 features are selected by the numeric model, with "Gr Liv Area", "Overall Qual", "BsmFin SF 1", "Total Bsmt SF" and "Misc Val" being the top 5 significant predictors for sale price of house. 98 features are selected by the model involving numeric and categorical features while "Roof Matl_WdShngl", "Exter Qual_Ex", "Neighborhood_NoRidge", "Neighborhood_NridgHt" and "Gr Liv Area" constribute the most to predict sale price.

Ridge Regression. A ridge regression which is similar with the lasso regression, is constructed. In stead of L2 regularization, the ridge regression applies L2 regularization which does not help selecting features but it can still overcome the issue of multi-collinearity.

The hyperparameter is λ, being similar with that of lasso regression, to minimize $\sum_{i=1}^{n}(y_i - \beta_0 - \sum_j x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^{p} \beta_j^2$. The `RidgeCV` package in python helps to fit data and select the optimal value of lambda from 100 values. The optimal lambdas for numeric training set and full training set are 113.2 and 9.6 respectively, which are largely different.

Elastic Nets. An elastic nets model performs both L1 and L2 regularization, and integrates the strength of ridge regression and lasso regression. As such, it is able to perform certain level of feature selection as well as placing no restriction on the number of selected variables.

The hyperparameter λ is tuned to minimize $\sum_{i=1}^{n}(y_i-\beta_0-\sum_j x_{ij}\beta_j)^2+\lambda\sum_{j=1}^{p}(\alpha\beta_j^2+(1-\alpha)|\beta_j|)$. Being similar to `RidgeCV` and `LassoCV`, the `ElasticNetCV` package in python selects the optimal value of λ from 100 different values for the fitted data. The optimal lambdas are both 65.6 for numeric set and full set.

Stepwise regression. The stepwise regression is simply a multiple linear regression with feature selection. The forward selection approach starts from a null model which contains only the constant, and iterate to add different number of features to the model. For each iteration, all predictors are add individually to the model to construct $p-k-1$ models, where p is the total number of predictors available, and k is the number of predictors involved in models. For a specific value of k, the best model is selected based on residual sum of square (RSS). Finally, the optimal stepwise model is selected by comparing the negative root mean squared error from cross validation, in order to estimate predictive performance of models involving different number of predictors.

The optimal model with numeric features contains 17 predictors while the best model including both categorical and numeric variables contains 73 features.

Cross validation results

The predictive performance of models developed are validated by root mean squared error (RMSE) from 5-fold cross validation. Table 1 summaries the predictive performances of five regression models for the two training sets. For models including only numeric features, random forest regression has the best performance with the lowest RMSE while other 4 models have similar performance. For models with both numeric and categorical variables, random forest regression still has the most accurate predictions, followed by lasso regression.

Model / Features	Numeric	Numeric and Categorical
Forward stepwise	34470.77	5.21×10^{15}
Lasso	34543.61	29628.07
Ridge	34475.68	30082.79
Elastic net	35380.58	32844.26
Random forest	28006.66	28797.24

Table 1: *Summary of predictive performance for regression models. The performance assessment metric is the root mean squared error from cross validation.*

It is noticeable from table 1 that the stepwise regression with forward selection has a poor predictions with a very large RMSE for training set involving categorical variables, whereas the predictive performance for numeric set is relatively satisfactory. This is because when performing stepwise regression, the dummy variables with values of 0 and 1 are treated as numeric variable when fitting multiple linear regression. This can pose a detrimental influence on predictive performance when forward selection does not handle this issue.

Validation set results from kaggle

Table 2 summarizes the validation set results from kaggle, assessing the predictive performance of five regression models by root mean squared errors. It is inline with table 1 that random forest and lasso model regression have the best predictive performance for both training sets. Moreover, random forest tends to have more accurate predictions for numeric sets compared to models with both categorical and numeric variables.

Model / Features	Numeric	Numeric and Categorical
Forward stepwise	40672.34	46984.44
Lasso	39910.50	29628.07
Ridge	40141.87	38481.91
Elastic net	38249.77	37679.12
Random forest	25405.28	28276.96

Table 2: *Summary of validation set results from kaggle. The performance assessment metric is the root mean squared error from cross validation.*

Conclusion and discussion

In conclusion, regardless of whether including categorical variables, lasso regression and random forest regression has the best predictive performance in terms of root mean squared error. Furthermore, lasso regression, ridge regression and elastic net regression tend to perform better with both categorical and numeric features while forward stepwise and random forest regression work better with numeric features.

Suggested by the best predictive model, to estimate the sale price of house, stakeholders should pay more attention to “Roof Matl_WdShngl”, “Exter Qual_Ex”, “Neighborhood_NoRidge”, “Neighborhood_NridgHt” and “Gr Liv Area”.

As discussed before, the forward stepwise regression performs poorly when involving both categorical and numeric feature. This raises a limitation that it might be inappropriate to include categorical variables in stepwise regression. Furthermore, outliers can pose a significant effect on both predictive performance and goodness of fit, but the potential effect of outliers is not evaluated in the study. To extend this study, further study could crave for evaluating the influence of outliers in performance of predictive models by filtering outliers or applying logarithm transformation. In addtion, to estimate how categorical affect the sale price of house, ANOVA and might help in further study. With integration of categorical and numeric features, random forest and support vector machine can be potential candidates.

Appendix

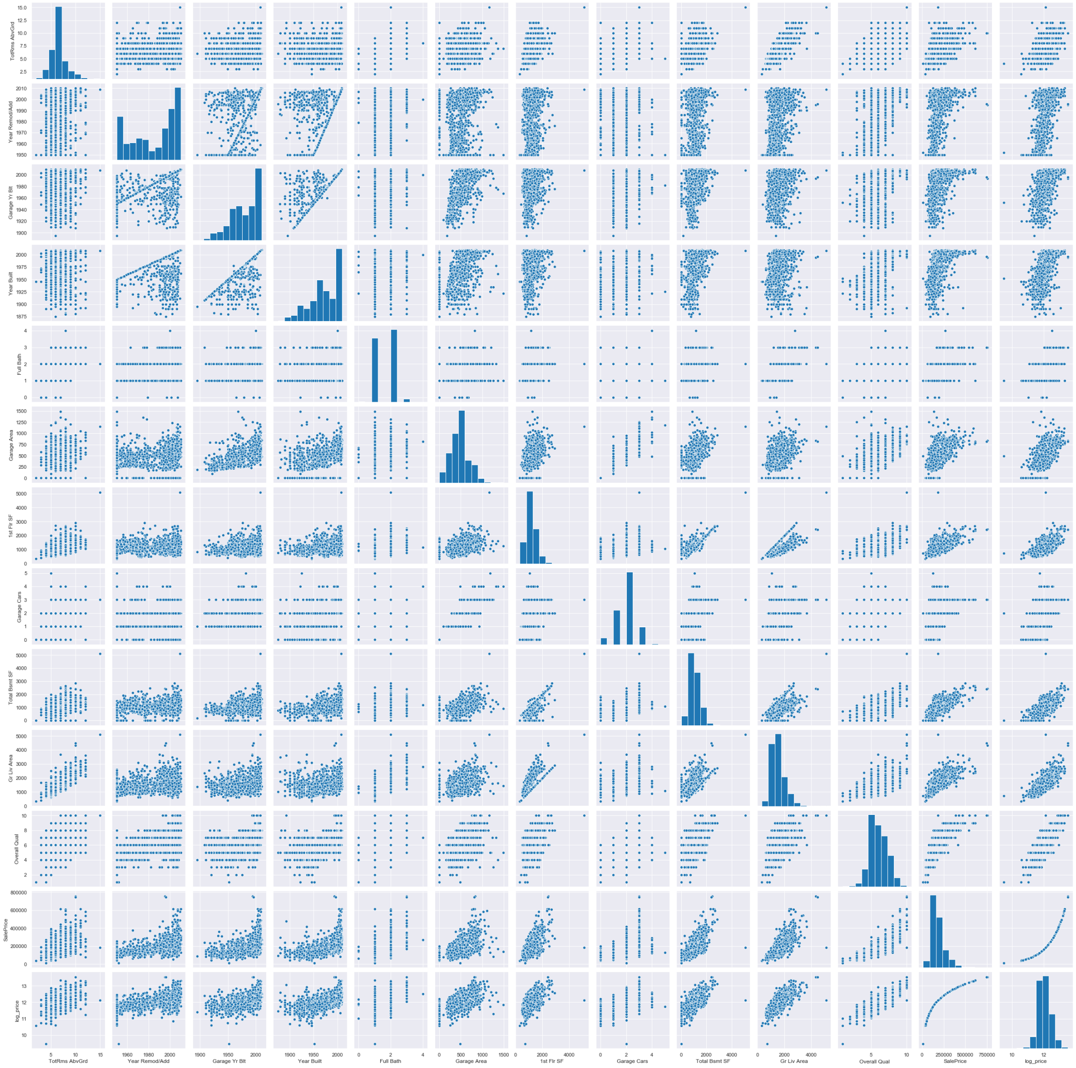

Fig. 2: *Distribution of numeric variables in housing data.*

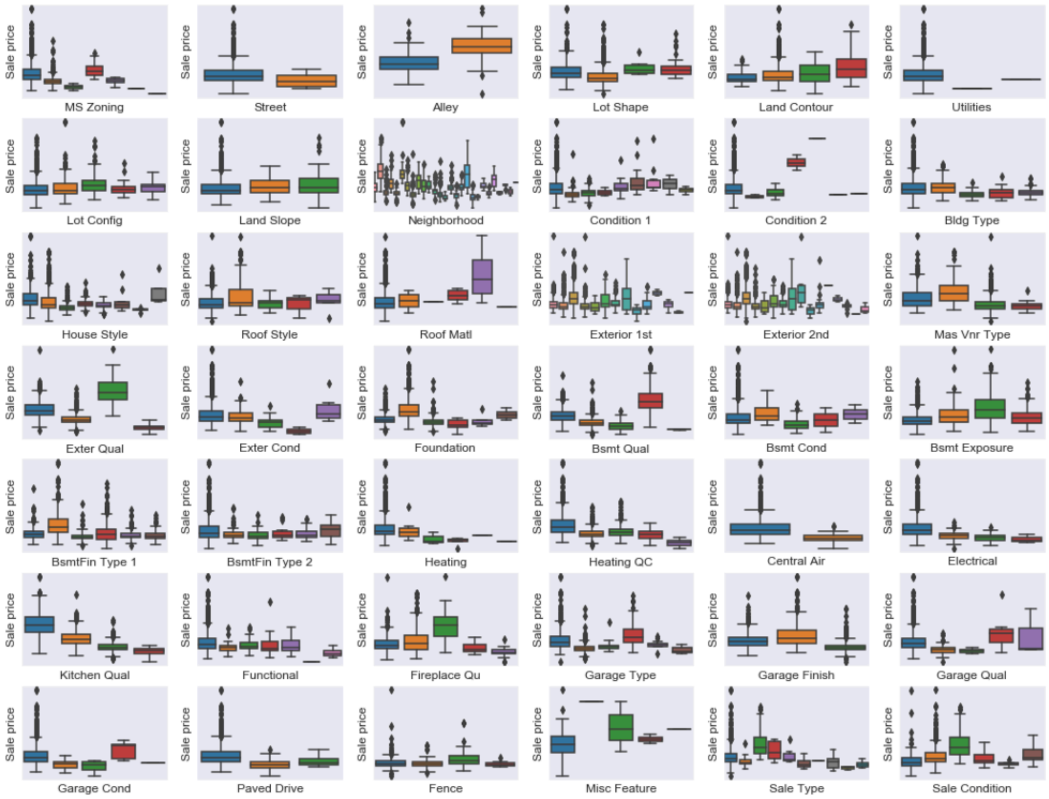

Fig. 3: *Boxplots demonstrating distribution of sale price for houses with different categorical features in housing data.*

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Task B

Exploratory data analysis

There are 312 observations of number of visitors recorded in the `Visitors` data. The number of visitors is recorded once a month from January 1991 to December 2016. The descriptive statistics of number of visitors show a mean of at 419407.37 and a range from 161400 to 971800.

To show the variation of number of visitors through the 25-year period. Figure 4 shows the time series decomposition of number of visitors. As systematic changes occur in short periods which are fixed, it demonstrates an upward trend with a seasonal pattern. Furthermore, the variation of number of visitors within the fixed period becomes greater as time moves. As such, a multiplicative forecasting model may be more suitable for this data compared to an additive model.

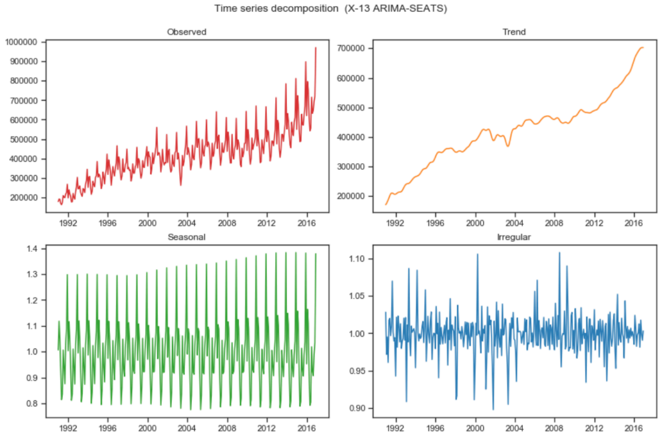


Fig. 4: Time series decomposition of number of visitors from 1991 to 2016.

Forecasting models

To forecast the number of visitors, two exponential smoothing models are developed.

The basic idea of exponential smoothing is to assign different weights to recent and past observations by the hyperparamter α. With the forecast equation

$$\hat{y}_{t+1} = l_1,$$

$$l_t = \alpha y_t + (1-\alpha)t_{t-1}$$

where l_t represents the level of the time series. A larger α gives greater weight to recent observations, and therefore the forecasts are more adaptive to recent changes in the series. In contrast, a lower α assign greater weight to the past observations, making the forecast smoother.

Holt-Winters exponential smoothing. As an extension of simple exponential smoothing, the Holt-Winters exponential smoothing capture not only the trend but also the seasonal pattern of number of visitors. It is available for both additive and multiplicative models. In this case, as mentioned before, the variance of number of visitors increases as time goes by, a multiplicative model is considered.

The model involves three hyperparameters α, β, δ, ranging from 0 to 1, which control the level, trend and seasonal indices respectively for the time series of number of visitors. The forecast equation of a multiplicative model is

$$\hat{y}_{t+h} = (\hat{l}_t + h\hat{b}_t) \times S_{t-L+(h\ mod\ L)},$$

$$l_t = \alpha(y_t/S_{t-L} + (1-\alpha)(l_{t-1} + b_{t-1}),$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1},$$

$$S_t = \delta(y_t/l_t) + (1-\delta)S_{t-L}.$$

The corresponding objective is, by tuning α, β, δ, to minimize

$$\sum_{t=1}^{N}(y_t - (l_{t-1} + b_{t-1}) \times S_{t-L})^2.$$

To estimate the appropriate values of hyperparameters, α, β, δ are initially set to 0.1, 0.1 and 0.05 respectively, and are fed into the `minimize` function in python. This will automatically selects the values of hyperparameters which generate the smallest residual sum of squared of predictions.

The optimal hyperparameters of Holt-Winters exponential smoothing are $\hat{\alpha} = 0.31, \hat{\beta} = 0.012, \hat{\delta} = 0.362$ for this data. Figure 5 visualizes the smoothed time series by the optimal values of hyperparameters. The smoothed series which nearly converge to the actual time series of number of visitors accurately capture most of the trend and seasonal pattern.

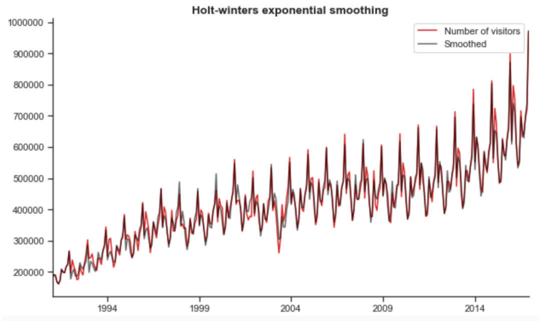


Fig. 5: Smoothed time series by Holt-Winter exponential smoothing.

Damped trend exponential smoothing. The damped trend exponential smoothing models seasonal pattern and trend of time series based on the Holt-Winters model. Additionally, it helps preventing the potentially implausible forecasts caused by extrapolating trends indefinitely into future. To do this, the trend is made smaller and a damping parameter ϕ ranging from 0 to 1 is considered on top of the Holt-Winters model. The forecast equation of a damped trend model is

$$\hat{y}_{t+h} = l_t + \phi b_t + \phi^2 b_t + \phi^3 b_t + \ldots + \phi^h b_t,$$

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + \phi b_{t-1},$$

$$b_t = \beta^*(l_t - l_{t-1} + (1-\beta^*)\phi b_{t-1}.$$

The objective of the model is, by tuning α, β, δ and ϕ, to minimize

$$\sum_{t=1}^{N}(y_t - (l_{t-1} + \phi b_{t-1} + \phi^2 b_{t-1} + \phi^3 b_{t-1} + \ldots + \phi^h b_{t-1}))^2.$$

To estimate the appropriate values of hyperparameters, $\alpha, \beta, \delta, \phi$ are initially set to 0.1, 0.1, 0.05 and 0.98 respectively, and are fed into the `minimize` function in python. This will automatically selects the values of four hyperparameters which generate the smallest residual sum of squared of predictions.

The optimal values of hyperparameters are $\alpha = 0.482, \beta = 0, \delta = 0.413$ and $\phi = 0.792$. Figure 6 shows the smoothed series of damped trend model which is highly similar with that of Holt-Winters smoothing. This is reasonable as two models follow similar rationale to fit models.

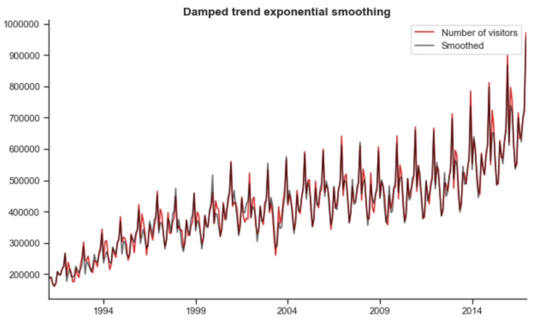


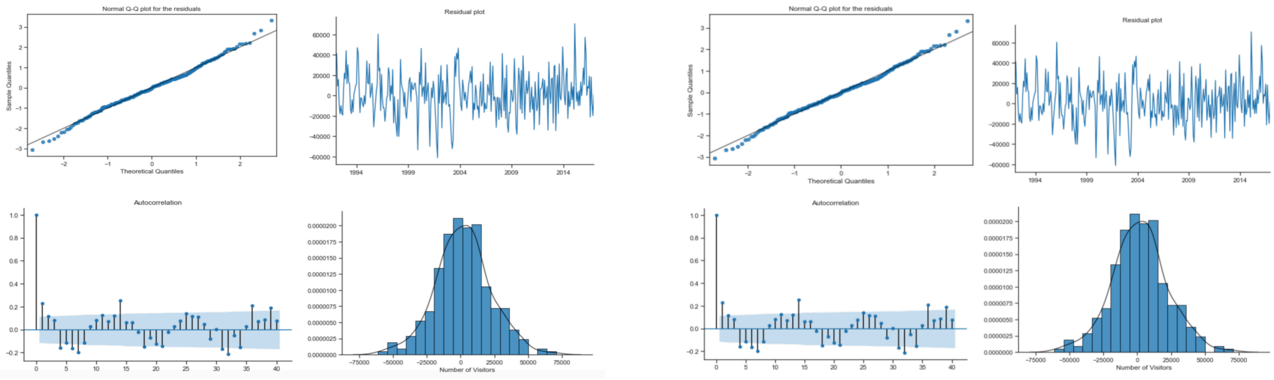
Fig. 6: Smoothed time series by Holt-Winter exponential smoothing.

Model diagnostic. To check the assumptions of exponential smoothing, residuals are visualized by four kinds of plots: residual plot, residual autocorrelation function plot, residual distribution and normal Q-Q plot. As shown in figure 7, with highly similar smoothed series for Holt-Winters and Damped trend exponential smoothing, residuals of two models are also similar.

The normal Q-Q plots illustrate a reasonable normal distribution of residuals as the scatters closely follow the actual Q-Q line excepting a few outliers. With a large number of observations, the central limit theorem can be applied, illustrating a normal distribution of data. This is in line with the distributions of residuals which show a symmetric bell shape. Moreover, the residual skewness and kurtosis are 0.059 and 0.509 respectively which are small, again proving that the assumption of normality is satisfied.

The residual plots does not show a clear pattern, representing homoscedastic variances for both models.

In autocorrelation function plots of residuals, the first coefficient prominently reach 1, which is reasonable as the first value is comparing to itself. Other coefficients show a mean at approximately 0 demonstrating relatively low and insignificant sample auto-correlations in the residual series. This is consistent with a white nose process.



***(a)** Holt-Winters exponential smoothing.*

***(b)** Damped trend exponential smoothing.*

***Fig. 7:** Residual plot, residual ACF, residual distribution plot and normal Q-Q plot for exponential smoothing.*

Model validation. Real time forecasting is performed to validate the predictive performance of Holt-Winters and damped trend exponential smoothing. Historical data of number of visitors before January 2010 is used to train the models. The models developed are then used to forecast the number of visitors until December 2016. These predictions are compared to actual historical data to calculate root mean squared errors and scaled errors.

Model / Statistic	RMSE	SE
Holt-Winter	23485.87	2702.33
Damped trend	24321.72	2724.25

***Table 3:** Summary of predictive performance for number of visitors forecasting by exponential smoothing.*

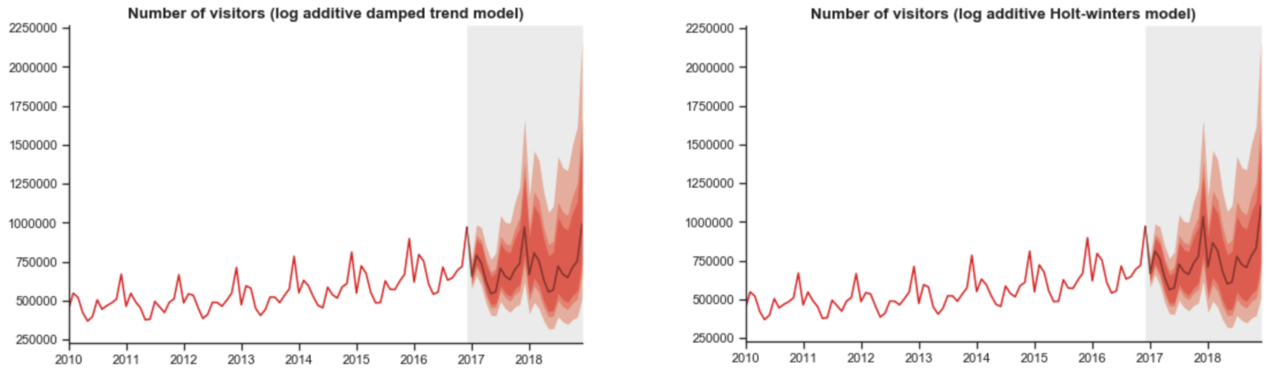
As shown in table 3, the Holt-Winters model has higher root mean squared errors and scaled error. This indicates a better predictive performance for Holt-Winters model compared to damped trend exponential smoothing. The differences of errors between two models may result from the "damped" trend. As long as the historical data shows a continuing growing trend, the damped trend predictions tends to be more deviated to actual data, comparing to the Holt-Winters predictions.

Forecasting results. The exponential smoothing models forecast the number of visitors for the following 24 months since the last month in historical data. This generates predictions for January 2017 to December 2018.

As a simple expression of variance is not available for the multiplicative Holt-Winters exponential smoothing, it can be challenging to generate range predictions of number of visitors. Fortunately, log-additive exponential is basically the same with the multiplicative models therefore it could be an alternative to make range predictions.

Figure 8 visualizes the number of visitors predicted by the two log-additive exponential smoothing models. Both Holt-Winters and damped trend models forecast the number of visitors to increase and reach the local maximum nearly in the Christmas holiday, which follows trend and seasonal pattern for historical data.

To compare the predictions of Holt-Winters and damped trend exponential smoothing, figure 9 shows the point predictions generated by two models. Although predictions of two models follow a highly similar seasonal pattern and a growing trend, the predicted trend of damped trend model is obviously smaller than that of Holt-Winter model. This is what it means by "damped" trend, and it is designed to prevent implausible forecast by gentling the trend.



(a) *Holt-Winters exponential smoothing.*

(b) *Damped trend exponential smoothing.*

Fig. 8: *Predictions of number of visitors from January 2017 to December 2018.*

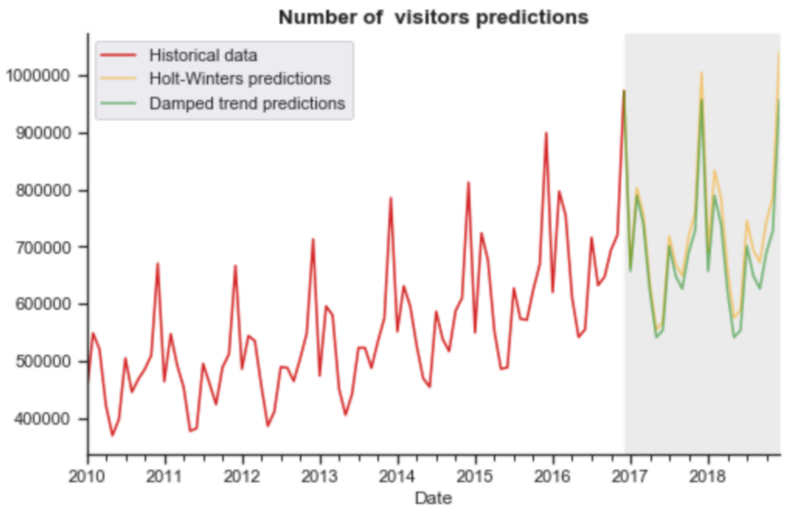


Fig. 9: *Point predictions of number of visitors from January 2017 to December 2018 by Holt-Winter and damped trend exponential smoothing. The red, yellow and green lines represent historical data, Holt-Winters predictions and damped trend predictions respectively.*

Conclusion

To conclude, the number of visitors kept growing with a seasonal pattern from 1991 to 2016. The seasonal pattern shows that the number of visitors tends to reach the peak of a year during Christmas holiday. In addition, the variation between high season and low season in a year was greater and greater.

Both Holt-Winters and damped trend exponential smoothing predict the number of visitors to increase in the following two years. The seasonal pattern of Christmas holiday being the high season is also followed. Although the two exponential smoothing models generate similar predictions, damped trend model illustrates a more gentle upward trend (i.e. predicted number of customers are smaller) compared to the Holt-Winters, as it “damps” the trend.