优能教育

2020 Semester 2

QBUS 2310 wk8-10

TUTOR: Joy

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课程安排

每周知识点复习（共4节）：

~~Week1-3：week4~~

~~Week4-6：week7~~

Week8-10：week11

Week11-13：week13

讲解内容：知识点+题型练习+部分tutorial题目讲解

ASM和online task题目练习及讲解：

- ~~Online task往年题目讲解 ASM1：往年题目+今年题目讲解 10月2日due~~
- ASM：今年ASM题目讲解提示+往年ASM题目讲解 11月20日due→ 11月10号上课

考试复习：考试前1-2周内

- ~~期中考试：往年期中考试复习题+复习题+往年ASM题目讲解 10月13日~~
- 期末考试：往年复习题+题型复习及练习

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Week4-6:

- Cash flow: cash flow excel计算，NPV 概念
- Integer problem
- Binary variables & Logical Conditions
- Fixed charge
- Contract Award
- The Transshipment Model
- The Assignment Model

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The Transportation Model Characteristics：

- 产品以尽可能低的成本从多个来源运输到多个目的地
- 每个货源都能提供固定数量的产品，每个目的地对商品的需求都固定产品
- 线性规划模型对每个来源的供应和每个目的地的需求都有约束
- 在供给等于需求的平衡运输模型中，所有约束都是平等的
- 在供应不等于需求的不平衡模型中，约束包含不平等

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The Assignment Model Characteristics:

Special form of linear programming model similar to the transportation model.

Supply at each source and demand at each destination limited to one unit.

In a balanced model supply equals demand.
In an unbalanced model supply does not equal demand.

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Week8-10知识点

Week8
Network flow models:
- The shortest path problem
- The minimal spanning tree
- The maximal flow problem
- Minimum cost network flow models

Text: chapter 7

Week9
- Goal programming
- Multi-objective LP

Text: chapter 9

Week10
Project management
- The Elements of Project Management
- CPM/PERT Networks Project Crashing and Time-Cost Trade-Off
- Formulating the CPM/PERT Network as a Linear Programming Model
- Probabilistic Activity Times
- Time-Cost Trade-Off with Linear Programming

Text: chapter 8

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Week 8

Network flow models:

- The shortest path problem
- The minimal spanning tree
- The maximal flow problem
- Minimum cost network flow models

Text: chapter 7

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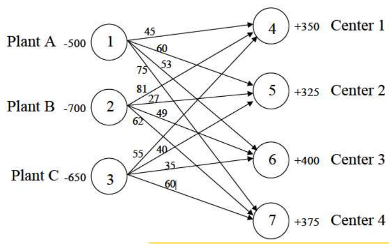
Network 网络图

A network is an arrangement of paths (branches分支) connected at various points (nodes节点) through which one or more items move from one point to another.

The network is drawn as a diagram providing a picture of the system thus enabling visual representation and enhanced understanding. 可视化

A large number of real-life systems can be modelled as networks which are relatively easy to conceive and construct. 简化题型



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- The shortest path problem

Many decision problems boil down to determining the shortest (or least costly) route or path through a network.

This is a special case of a transshipment problem where: 转运题

- There is one supply node with a supply of -1
- There is one demand node with a demand of +1
- All other nodes have supply/demand of +0

There are two possible objectives for this problem:

- Finding the quickest route (minimizing travel time) 最小时间/路径/成本
- Finding the most scenic route(maximizing the scenic rating points) 最大评分/点数

The Equipment Replacement Problem 更换设备问题

The problem of determining when to replace equipment is another common business problem

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- The minimal spanning tree

For a network with *n* nodes, a *spanning tree* is a set of *n-1* arcs that connects all the nodes and contains no loops.

The minimal spanning tree problem involves determining the set of arcs that connects all the nodes at minimum cost.

1. Select any node. Call this the current subnetwork.
2. Add to the current subnetwork the cheapest arc that connects any node within the current subnetwork to any node not in the current subnetwork. (Ties for the cheapest arc can be broken arbitrarily.) Call this the current subnetwork.
3. If all the nodes are in the subnetwork, stop; this is the optimal solution. Otherwise, return to step 2.

对于有n个节点的网络，生成树是一组连接所有节点且不包含循环的n-1弧。最小生成树问题涉及到确定一组以最小代价连接所有节点的弧。1. 选择任意节点。称之为当前子网。2. 在当前子网中添加连接当前子网中的任何节点到当前子网中的任何节点的最便宜的弧。（最便宜的弧的连接可以任意断开）称之为当前子网。3. 如果所有节点都在子网中，停止，这是最佳解决方案。否则，返回步骤2。

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- The maximal flow problem

In some network problems, the objective is to determine the maximum amount of flow that can occur through a network. The arcs in these problems have upper and lower flow limits.

Examples
- How much water can flow through a network of pipes?
- How many cars can travel through a network of streets?

Arbitrarily select any path in the network from origin to destination. Adjust the capacities at each node by subtracting the maximal flow for the path selected in step 1. Add the maximal flow along the path to the flow in the opposite direction at each node. Repeat steps 1, 2, and 3 until there are no more paths with available flow capacity.

任意选择网络中从原点到目的地的任何路径。通过减去步骤1中选择的路径的最大流量来调整每个节点的容量。将沿路径的最大流添加到每个节点的反向流中。重复步骤1、2和3，直到不再有具有可用流量的路径。

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Week 9

- Goal programming
- Multi-objective LP

Text: chapter 9

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Most of the optimization problems considered to this point have had a single objective. 多目标方程

Often, more than one objective can be identified for a given problem.
- Maximize Return or Minimize Risk
- Maximize Profit or Minimize Pollution

These objectives often conflict with one another.

Soft constraint

$$MIN \sum_i (d_i^+ + d_i^-)$$

$$X_i - d_i^+ + d_i^- = C$$

$$X_i = C + d_i^+ - d_i^-$$

Where $d^+, d^- \geq 0$

$$MIN \sum_i \frac{1}{t_i}(d_i^+ + d_i^-)$$

$$MIN \sum_i (w_i^+ d_i^+ + w_i^- d_i^-)$$

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The MiniMax Objective

$MIN: Q$

$d_1^- \leq Q$

$d_1^+ \leq Q$

Ect....

1. Identify the decision variables in the problem.
2. Identify any hard constraints in the problem and formulate them in the usual way.
3. State the goals of the problem along with their target values.
4. Create constraints using the decision variables that would achieve the goals exactly.
5. Transform the above constraints into goal constraints by including deviational variables.
6. Determine which deviational variables represent undesirable deviations from the goals. 确定哪些偏差变量代表了与目标的不合需要的偏差
7. Formulate an objective that penalizes the undesirable deviations. 惩罚项
8. Identify appropriate weights for the objective.
9. Solve the problem.
10. Inspect the solution to the problem. If the solution is unacceptable, return to step 8 and revise the weights as needed.

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Multiple Objective Linear Programming (MOLP) 多目标线性方程

An MOLP problem is an LP problem with more than one objective function.
- MOLP problems can be viewed as special types of GP problems where we must also determine target values for each goal or objective.
- Analyzing these problems effectively also requires that we use the MiniMax objective described earlier.

Solutions obtained using the MiniMax objective are Pareto Optimal.
- Deviational variables and the MiniMax objective are also useful in a variety of situations not involving MOLP or GP.
- For minimization objectives the percentage deviation is: (actual -target)/target
- For maximization objectives the percentage deviation is: (target -actual)/target
- If a target value is zero, use the weighted deviations rather than weighted % deviations.

$$w_1\left(\frac{actual - target}{target}\right) \leq Q\ (minimise\ 问题)$$

$$w_1\left(\frac{target - actual}{target}\right) \leq Q(maximum\ 问题)$$

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Week 10

Project management
- The Elements of Project Management
- CPM/PERT Networks Project Crashing and Time-Cost Trade-Off
- Formulating the CPM/PERT Network as a Linear Programming Model
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Text: chapter 8

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Network representation is useful for project analysis.
Networks show how project activities are organized and are used to determine time duration of projects.

Network techniques used are:

- CPM (Critical Path Method)
- PERT (Project Evaluation and Review Technique)

Management is generally perceived as concerned with planning, organizing, and control of an ongoing process or activity.
Project Management is concerned with control of an activity for a relatively short period of time after which management effort ends.

Primary elements of Project Management to be discussed:

- Project Planning
- Project Team
- Project Control

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Projects can be simple (planning a company picnic) or complex (planning a space shuttle launch).
Successfully completing a project requires:

- Knowledge of the tasks involved
- Accurate estimates of time and resources required
- Knowledge of physical and logical relations between the various tasks
- Project management techniques
- Critical Path Method (CPM)
- Program Evaluation and Review Technique (PERT)

Spreadsheets can be used to manage projects, but dedicated project management software is often more effective.

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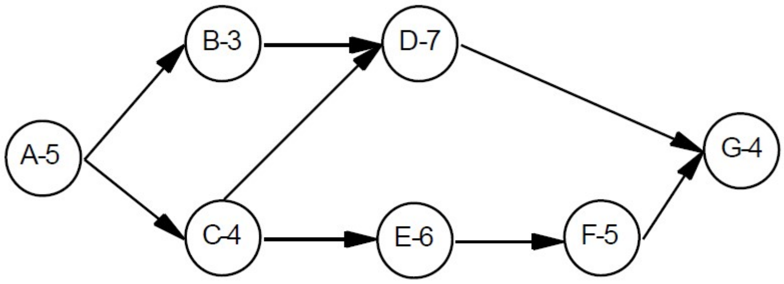
An Activity-On-Node (AON) Network

An Activity-on-Arc (AOA) Network



AON networks should have unique start and finish points.

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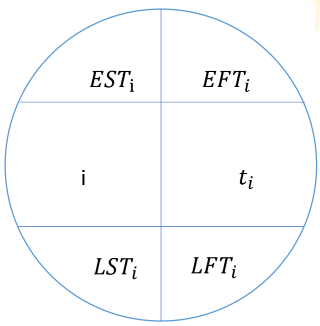
- A Forward Pass through the network determines the earliest times each activity can start and finish.
- A Backward Pass through the network determines the latest times each activity can start and finish without delaying completion of the project.
- The longest path through the network is the "critical path".



t_i= time required to perform activity i
EST_i= earliest possible start time for activity i
EFT_i= earliest possible finish time for activity i
LST_i= latest possible start time for activity i
LFT_i= latest possible finish time for activity i

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The Forward Pass 从前往后

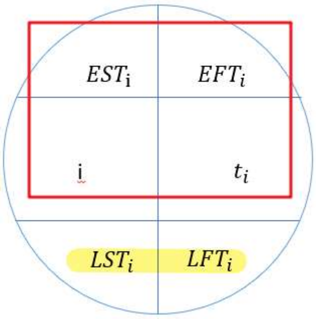
The earliest start time (EST) for the initial activity in a project is "time zero".

- The EST of an activity is equal to the latest (or maximum) early finish time of the activities directly preceding it.
- The EFT of an activity is equal to its EST plus the time required to perform the activity.

The Backward Pass 从后往前

The latest finish time (LFT) for the final activity in a project is equal to its EFT as determined by the forward pass.

- The LFT for any other activity is equal to the earliest (or minimum) LST of the activities directly following (or succeeding) it.
- The LST of an activity is equal to its LFT minus the time required to perform the activity.



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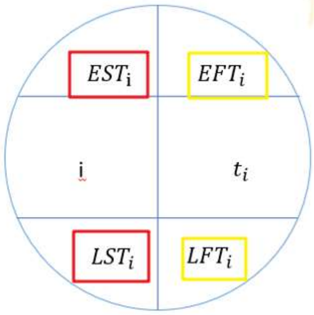
The Critical Path 关键路线 & slack

$$Slack = LST_i - EST_i \text{ or } LFT_i - EFT_i$$

Critical activities have zero slack (懈怠/放松) and cannot be delayed without delaying the completion of the project.

The slack for non-critical activities represents the amount of time by which the start of these activities can be delayed without delaying the completion of the entire project (assuming that all predecessor activities start at their earliest start times).



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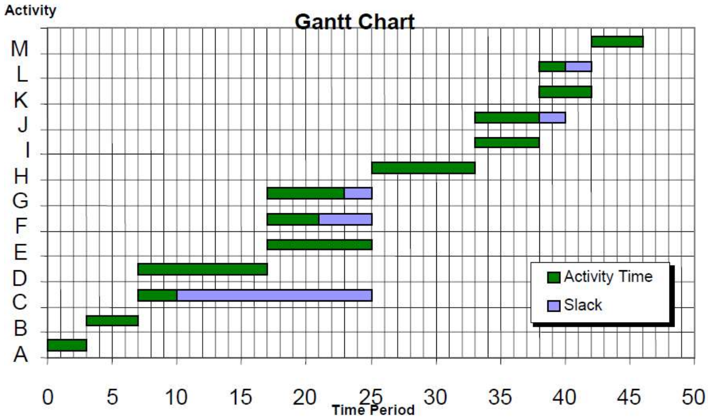
Gantt chart



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Project Crashing 加急

It is often possible to complete activities faster than normal by applying more resources (better equipment, overtime, etc).

This is referred to as "crashing" the project.

We may want to determine the optimal way of crashing a project to:

- complete it more quickly than originally scheduled
- keep it on schedule if critical activities were delayed

T_i = the time at which activity i begins
t_i = the normal activity time of activity i
C_i = the amount of time by which activity i is crashed

Minimize the completion time of the last activity (activity M): MIN: $T_M + t_M - C_M$

For each arc in the project network from activity i to activity j, we need a constraint of the form: $T_j >= T_i + t_i - C_i$

$C_i <=$ allowable crash days for activity i

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PERT: An Overview

CPM assumes all activity times are known with certainty or can be estimated accurately.

PERT accounts for uncertainty in activity times by using three time estimates:
ai = duration of activity i assuming the most favourable conditions
bi = duration of activity i assuming the least favourable conditions
mi = estimate of the most likely duration of activity i

PERT then estimates expected duration and variance of each activity's duration as:

$$t_i = \frac{a_i + 4m_i + b_i}{6}$$

$$v_i = \frac{(b_i - a_i)^2}{36}$$

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The expected (or mean) time required to complete any path in the network is the sum of the expected times (the t_i) of the activities on the path. 完成网络中任何路径所需的预期（或平均）时间是路径上活动的预期时间（ti）的总和

Assuming the individual activity times in a project are independent of one another, we may also calculate the variance of the completion time for any path as the sum of the variances (the v_i) of the activities on the path. 假设一个项目中的各个活动时间是相互独立的，我们也可以计算任何路径的完成时间的方差，作为该路径上活动的方差之和（vi）

PERT considers the path with the largest expected completion time to be the critical path. PERT将期望完成时间最大的路径作为关键路径

PERT's reasoning may be flawed...

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优缺点

优点 1.是一种有效的事前控制方法。

2.通过对进行时间网络分析可以使各级主管人员熟悉整个工作过程并明确自己负责的项目在整个工作过程中的位置和作用，增强全局观念和对计划的接受程度。

3.通过时间网络分析使主管人员更加明确其工作重点，将注意力集中在可能需要采取纠正措施的关键问题上，使控制工作更加富有有效。

4.是一种计划优化方法。

缺点

并不适用于所有的计划和控制项目，其应用领域具有较严格的限制。适用PERT法的项目必须同时具备以下条件:

1、事前能够对项目的工作过程进行较准确的描述;

2、整个工作过程有条件划分为相对独立的各个活动;

3、能够在事前较准确地估计各个活动所需时间、资源。

4、对事件时间的耗用是基于经验的判断，但是当事件是首次执行，没有经验参考的情况，经过判断得出的时间显然是不科学的。

5、即使事件时间的判断是比较准确的，但是此方法是假设事件时间是基于β分布的，但是现实中是不是真的遵循β分布呢，我们也无从判断。

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题目练习

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A company needs to ship 100 units from Roanoke to Washington at the lowest possible cost. The costs associated with shipping between the cities are:

	To		
From	Lexington	Washington	Charlottesville
Roanoke	50	-	80
Lexington	-	50	40
Charlottesville	-	30	

Draw the network representation of this problem.

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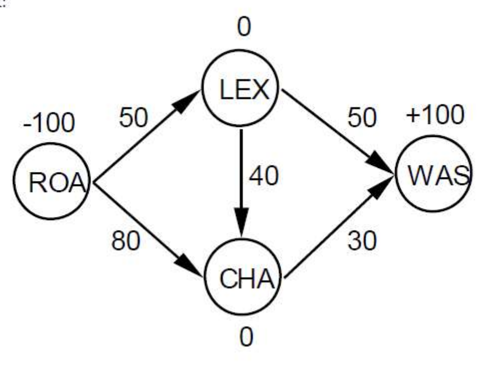
ANSWER:



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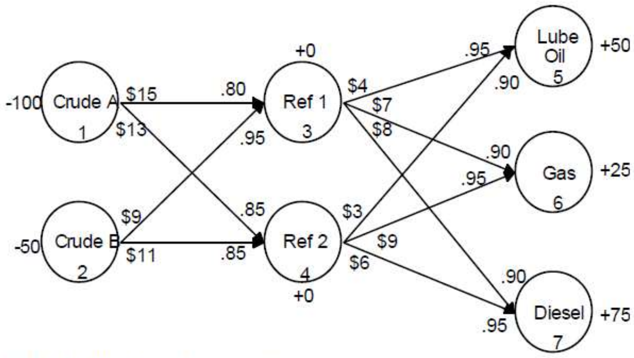
Question 10:

An oil company wants to create lube oil, gasoline and diesel fuel at two refineries. There are two sources of crude oil. The following network representation depicts this problem.



Write out the LP formulation for this problem.

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ANSWER:

MIN $15X_{13} + 13X_{12} + 9X_{23} + 11X_{24} + 4X_{35} + 7X_{36} + 8X_{37} + 3X_{45} + 9X_{46} + 6X_{47}$

Subject to:

$-X_{13} - X_{14} = -100$
$-X_{23} - X_{24} = -50$
$0.80X_{13} + 0.95X_{23} - X_{35} - X_{36} - X_{37} = 0$
$0.85X_{14} + 0.85X_{24} - X_{45} - X_{46} - X_{47} = 0$
$0.95X_{35} + 0.90X_{45} =< 50$
$0.90X_{36} + 0.95X_{46} =< 25$
$0.90X_{37} + 0.95X_{47} = <75$

$X_{ij} \geq 0$

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Question 11:

The Wilson Doors company manufacturers, three styles of doors – exterior, interior and commercial. Each door requires a certain amount of steel and two separate production steps: forming and assembly.

	EXTERIOR	INTERIOR	COMMERCIAL	AVAILABILITY
Steel (lb/door)	4	3	7	9,000 pounds
Forming (hr/door)	2	4	3	6,000 hours
Assembly (hr/door)	2	3	4	5,200 hours
Selling price/door	\$70	\$110	\$110	

Wilson sets the following goals:
Goal 1: Achieve total sales of at least \$180,000
Goal 2: Achieve exterior doors sales of at least \$70,000
Goal 3: Achieve interior doors sales of at least \$60,000
Goal 4: Achieve commercial doors sales of at least \$35,000

Formulate a GP to minimize the sum of percentage deviations.

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ANSWER:

Let E, I *and* C, be the number of Exterior, Interior and Commercial doors produced. Let d_i^+ *and* d_i^- be the amount by which we overachieve or under achieve goal $i, i = 1,2,3,4$.

Our formulation is:

$$Minimize\ \frac{d_1^+ + d_1^-}{180{,}000} + \frac{d_2^+ + d_2^-}{70{,}000} + \frac{d_3^+ + d_3^-}{60{,}000} + \frac{d_4^+ + d_4^-}{35{,}000}$$

$st.$

$70E + 110I + 110C - d_1^+ + d_1^- = 180{,}000\ (total\ sales\ goal)$
$70E - d_2^+ + d_2^- = 70{,}000\ (exterior\ door\ sales\ goal)$
$110I - d_3^+ + d_3^- = 60{,}000\ (interior\ doors\ sales\ goal)$
$110C - d_4^+ + d_4^- = 35{,}000\ (commercial\ doors\ sales\ goal)$
$4E + 3I + 7C \le 9{,}000\ (steel\ usage\ constraint)$
$2E + 4I + 3C \le 6{,}000\ (forming\ time\ constraint)$
$2E + 3I + 4C \le 5{,}200\ (assembly\ time\ constraint)$
$E, I, C, d_i^+, d_i^- \ge 0$

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Question 12:

How would your formulation change if your goal is to minimize the maximum percentage underachievement with respect to each of the goals in Question 10? (Note that you don't mind overachieving).

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Goal 4: Achieve commercial doors sales of at least $35,000

Formulate a GP to minimize the sum of percentage deviations.

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ANSWER:

We have the same decision variables as in Question 10: let E, I *and* C, be the number of Exterior, Interior and Commercial doors produced. Let d_i^+ *and* d_i^- be the amount by which we overachieve or under achieve goal $i, i = 1,2,3,4$.
We add a "dummy" variable Q

Our formulation is:

$Minimize\ Q$
$st.$

$$Q \geq \frac{d_1^-}{180{,}000}$$
$$Q \geq \frac{d_2^-}{70{,}000}$$
$$Q \geq \frac{d_3^-}{60{,}000}$$
$$Q \geq \frac{d_4^-}{35{,}000}$$
$$70E + 110I + 110C - d_1^+ + d_1^- = 180{,}000\ (total\ sales\ goal)$$
$$70E - d_2^+ + d_2^- = 70{,}000\ (exterior\ door\ sales\ goal)$$
$$110I - d_3^+ + d_3^- = 60{,}000\ (interior\ doors\ sales\ goal)$$
$$110C - d_4^+ + d_4^- = 35{,}000\ (commercial\ doors\ sales\ goal)$$
$$4E + 3I + 7C \leq 9{,}000\ (steel\ usage\ constraint)$$
$$2E + 4I + 3C \leq 6{,}000\ (forming\ time\ constraint)$$
$$2E + 3I + 4C \leq 5{,}200\ (assembly\ time\ constraint)$$
$$E, I, C, d_i^+, d_i^- \geq 0$$

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Question 3:

A research director must pick a subset of research projects to fund over the next five years. He has five candidate projects, not all of which cover the entire five-year period. Although the director has limited funds in each of the next five years, he can carry over unspent research funds into the next year. Additionally, up to $30K can be carried out of the five-year planning period. The following table summarizes the projects and budget available to the research director.

	Project Funds Required (in $000s)					Benefit
Project	1	2	3	4	5	(in $000s)
1	$70	$40	$30	$15	$15	$160
2	$82	$35	$20	$20	$10	$190
3	$55	$10	$10	$5		$125
4	$69	$17	$15	$12	$8	$139
5	$75	$20	$25	$30	$45	$174
Budget	$225K	$60K	$60K	$50K	$50K	

Define the ILP formulation for this capital budgeting problem.

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ANSWER:

Let $X_i = 0$ if project i not selected, 1 if project i selected for i=1, 2, 3, 4, 5
Let C_j = amount carried out of year j, j =1, 2, 3, 4, 5

MAX $160X_1 + 190X_2 + 125X_3 + 139X_4 + 174X_5$
Subject to:

$70X_1 + 82X_2 + 55X_3 + 69X_4 + 75X_5 + C_1 = 225$
$40X_1 + 35X_2 + 10X_3 + 17X_4 + 20X_5 + C_2 = 60 + C_1$
$30X_1 + 20X_2 + 10X_3 + 15X_4 + 25X_5 + C_3 = 60 + C_2$
$15X_1 + 20X_2 + 5X_3 + 12X_4 + 30X_5 + C_4 = 50 + C_3$
$15X_1 + 10X_2 + 8X_4 + 45X_5 + C_5 = 50 + C_4$
$C_5 \leq 30$
X_i binary, $C_1, C_2, C_3, C_4, C_5 \geq 0$

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Question 7:

A company needs to manage a project which consists of the following set of activities:

Activity	Days Required	Predecessor Activities
A	5	–
B	3	A
C	4	A
D	7	B, C
E	6	C
F	5	E
G	4	D, F

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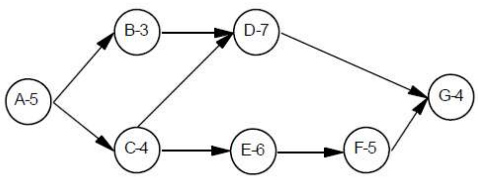
a. Draw the CPM network for this problem. Use AON notation.

ANSWER:



b. Manually determine the earliest and latest start and finish times for each activity and the amount of slack for each activity. What is the critical path for the project and how long should it take to complete it?

ANSWER:

Activity	Time	EST	EFT	LST	LFT	Slack
A	5	0	5	0	5	0
B	3	5	8	10	13	5
C	4	5	9	5	9	0
D	7	9	16	13	20	4
E	6	9	15	9	15	0
F	5	15	20	15	20	0
G	4	20	24	20	24	0

Critical path = A-C-E-F-G

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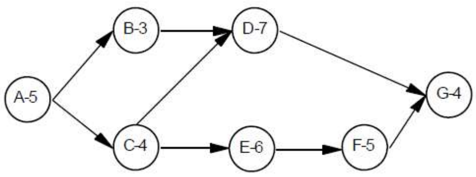
a. Draw the CPM network for this problem. Use AON notation.

ANSWER:



c. Identify each path through the network and its expected length.

ANSWER:

A-B-D-G = 19 DAYS
A-C-E-F-G = 24 DAYS
A-C-D-F = 20 DAYS

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Question 8:

Consider the following Project:

Activity	Immediate predecessor	Normal time	Crash time	Cost per crashing 1 time unit
A	-	6	3	300
B	-	8	6	500
C	-	5	4	100
D	A	4	3	250
E	B,C	4	3	600
F	B,D	3	3	500
G	B	7	5	300
H	B,D	7	5	200

- Find the critical path and the time required to complete the project.
- Can the project be crashed so it is completed by time 12? If so, how much will it cost?

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ANSWER:

Activity	Immediate predecessor	time	ES	EF	LS	LF	SL
A	-	6	0	6	0	6	**0**
B	-	8	0	8	2	10	2
C	-	5	0	5	8	13	8
D	A	4	6	10	6	10	**0**
E	B,C	4	8	12	13	17	5
F	B,D	3	10	13	14	17	4
G	B	7	8	15	10	17	2
H	B,D	7	10	17	10	17	**0**

The earliest the project can be completed is time 17.
The critical path is A-D-H.
The slacks are given in the rightmost column of the table.

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Shortening the project to 12 time units:

Path	Length	2H (400)	D&G (550)	A&G (600)	A&B (800)
ADF	13	13	12	11	10
ADH	**17**	**15**	**14**	**13**	**12**
BE	12	12	12	12	11
BF	11	11	11	11	10
BG	15	**15**	**14**	**13**	**12**
BH	15	13	13	**13**	**12**
CE	9	9	9	9	9

The optimal crashing policy is 2A, 1B, 1D, 2G and 2H. We obtain 3 critical paths, each of

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