优能教育

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优能教育

2020 Semester 1

QBUS 2310 期末复习 知识点

TUTOR: Joy

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课程安排

每周知识点复习（共4节）：

~~Week1-3：week4~~

~~Week4-6：week7~~

~~Week8-10：week11~~

~~Week11-13：week13~~

讲解内容：知识点+题型练习+部分tutorial题目讲解

ASM题目练习及讲解：

- ~~ASM1+2：往年2套ASM2题目练手讲解+今年ASM题目讲解提示~~

~~Online test题目讲解~~

考试复习：考试前1-2周内

- ~~期中考试：往年期中考试复习题+期中考试题~~
- ~~期中考试2：期中考试1中部分未讲解题目解析，期中复习题目问题汇总~~
- 期末考试：~~往年复习题+题型复习及练习，~~基础知识复习

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Wk	Topic
Week 01	Management science: introduction
Week 02	Linear programming: model formulation and graphical solution
Week 03	Linear programming: computer solution and sensitivity analysis
Week 04	Linear programming: modelling examples
Week 05	Integer programming
Week 06	1. Transportation; 2. Transshipment; 3. Assignment problems
Week 08	Network flow models
Week 09	Project management
Week 10	Non-linear programming
Week 11	Queuing analysis
Week 12	Simulation
Week 13	Revision

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Week1-3回顾:

解题框架及步骤

- Assumptions
- Define decision variables
- Objective functions
- Constraints
- Optimal solution
- conclusion

定义

- Variable: 定义、i的取值范围
- Objective: parameters、max/min
- Model: constraints、non-negative、integer

Excel\python

- Excel solver: 三要素
- Sensitive report: allowance increase &decrease、reduced cost、shadow price
- Python: variable 角标从0开始

直角坐标系画图解题

- 只能用于解两个未知数
- Constraints限制未知数范围: feasible region
- Objective function: slope 不变上下移动找点/固定点旋转找slope

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Week4-6:

- Cash flow: cash flow excel计算，NPV 概念
- Integer problem
- Binary variables & Logical Conditions
- Fixed charge
- Contract Award
- The Transshipment Model
- The Assignment Model

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The Transportation Model Characteristics：

- 产品以尽可能低的成本从多个来源运输到多个目的地
- 每个货源都能提供固定数量的产品，每个目的地对商品的需求都固定产品
- 线性规划模型对每个来源的供应和每个目的地的需求都有约束
- 在供给等于需求的平衡运输模型中，所有约束都是平等的
- 在供应不等于需求的不平衡模型中，约束包含不平等

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The Assignment Model Characteristics:

Special form of linear programming model similar to the transportation model.

Supply at each source and demand at each destination limited to one unit.

In a balanced model supply equals demand.
In an unbalanced model supply does not equal demand.

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Week8-10知识点

Week8

Network flow models：

- The shortest path problem
- The minimal spanning tree
- The maximal flow problem
- Minimum cost network flow models

Text: chapter 7

Week9

- Goal programming
- Multi-objective LP

Text: chapter 9

Week10

Project management

- The Elements of Project Management
- CPM/PERT Networks Project Crashing and Time-Cost Trade-Off
- Formulating the CPM/PERT Network as a Linear Programming Model
- Probabilistic Activity Times
- Time-Cost Trade-Off with Linear Programming

Text: chapter 8

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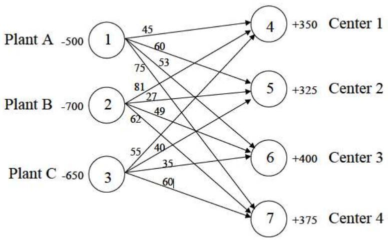
Week 8

Network flow models：

- The shortest path problem
- The minimal spanning tree
- The maximal flow problem
- Minimum cost network flow models

Text: chapter 7



This is a special case of a transshipment problem where:

- There is one supply node with a supply of -1
- There is one demand node with a demand of +1
- All other nodes have supply/demand of +0

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Week 9

- Goal programming
- Multi-objective LP

Text: chapter 9

Soft constraint

$$X_i - d_i^+ + d_i^- = C$$

$$X_i = C + d_i^+ - d_i^-$$

Where $d^+, d^- \geq 0$

$$MIN \sum_i (d_i^+ + d_i^-)$$

$$MIN \sum_i \frac{1}{t_i}(d_i^+ + d_i^-)$$

$$MIN \sum_i (w_i^+ d_i^+ + w_i^- d_i^-)$$

$MIN: Q$

$d_1^- \leq Q$

$d_1^+ \leq Q$

Ect....

$$w_1\left(\frac{actual - target}{target}\right) \leq Q\ (minimise\ 问题)$$

$$w_1\left(\frac{target - actual}{target}\right) \leq Q(maximum\ 问题)$$

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Week 10

$$Slack = LST_i - EST_i \ or\ LFT_i - EFT_i$$

Project management
- The Elements of Project Management
- CPM/PERT Networks Project Crashing and Time-Cost Trade-Off
- Formulating the CPM/PERT Network as a Linear Programming Model
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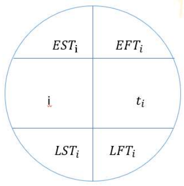
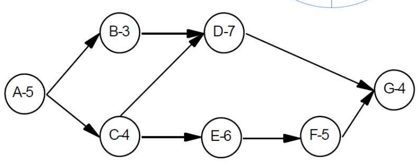
Text: chapter 8

$$t_i = \frac{a_i + 4m_i + b_i}{6}$$

$$v_i = \frac{(b_i - a_i)^2}{36}$$





AON networks should have unique start and finish points.

Minimize the completion time of the last activity (activity M): MIN: $T_M + t_M - C_M$

For each arc in the project network from activity i to activity j, we need a constraint of the form: $T_j >= T_i + t_i - C_i$

$C_i <=$ allowable crash days for activity i

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Week11-13知识点

Week11
Queuing Analysis :

Arrival process
Services process
M/M/1
M/M/s
M/G/1
M/D/1

Text: chapter 13

Week12
Non-Linear Programming:

GRG (Generalized Reduced Gradient)
EOQ (Economic Order Quantity)
Location problem
Non-linear network flow problem
Optimal portfolio
Sensitivity analysis
Lagrange multiple
KKT(Karush-Kuhn-Tucker)

Text: chapter 10

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The Arrival Process

Arrival rate - the manner in which customers arrive at the system for service.

Arrivals are often described by a Poisson random variable:

$$\mathrm{P}(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

λ is arrival rate

The Service Process

Service time - the amount of time a customer spends receiving service (not including time in the queue).

Service times are often described by an Exponential random variable:

$$\mathrm{P}(t_1 \le T \le t_2) = \int_{t_1}^{t_2} \mu e^{-\mu x} dx = e^{-\mu t_1} - e^{-\mu t_2}, for\ t_1 \le t_2$$

μ is service rate

$\frac{1}{\mu}$ is average service time

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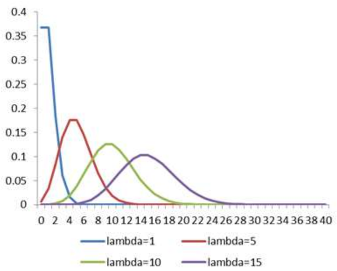
Poisson distribution

在实际事例中，当一个随机事件，以固定的平均瞬时速率λ（或称密度）随机且独立地出现时，那么这个事件在单位时间（面积或体积）内出现的次数或个数就近似地服从泊松分布***P*(λ)**。

$$\mathrm{P}(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

λ is arrival rate



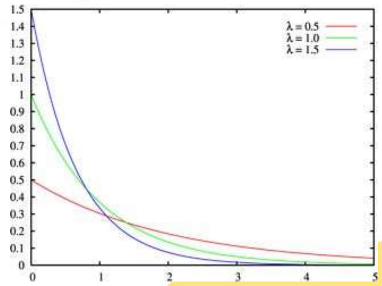
μ is service rate

$\frac{1}{\mu}$ is average service time

Exponential distribution

在概率理论和统计学中，指数分布（也称为负指数分布）是描述泊松过程中的事件之间的时间的概率分布，即事件以恒定平均速率连续且独立地发生的过程。

$$\mathrm{P}(t_1 \le T \le t_2) = \int_{t_1}^{t_2} \mu e^{-\mu x} dx = e^{-\mu t_1} - e^{-\mu t_2}, for\ t_1 \le t_2$$



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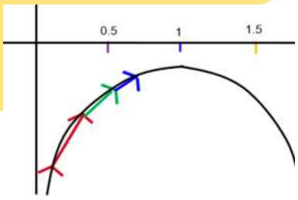
WEEK12



Non-Linear Programming:

GRG (Generalized Reduced Gradient)
EOQ (Economic Order Quantity)
Location problem
Non-linear network flow problem
Optimal portfolio
Sensitivity analysis
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Text: chapter 10

$$Total\ Annual\ Cost = DC + \frac{D}{Q}S + \frac{Q}{2}Ci$$

$$Q^* = \sqrt{\frac{2DS}{Ci}}$$

$$Distance = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$$

MAX: $(1-P_{12}Y_{12})(1-P_{13}Y_{13})(1-P_{14}Y_{14})(1-P_{24}Y_{24})\ldots(1-P_{9,10}Y_{9,10})$

$$\text{MIN: } \sum_{i=1}^{n} \sigma_i^2 p_i^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sigma_{ij} p_i p_j$$

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Problem 1:

Snookers Restaurant is open from 8am to 10pm daily. Besides the hours they are open for business, workers are needed an hour before opening and an hour after closing for setup and clean up duties. The restaurant operates with both full-time and part-time workers on the following shifts:

Shift No.	Shift Time	Daily Pay Rate
1	7am - 11am	\$32
2	7am - 3pm	\$80
3	11am - 3pm	\$32
4	11am - 7pm	\$80
5	3pm - 7pm	\$32
6	3pm - 11pm	\$80
7	7pm - 11pm	\$32

The following numbers of workers are needed during each of the indicated time blocks:

Hours	No. Workers Needed
7am - 11am	11
11am - 1pm	24
1pm - 3pm	16
3pm - 5pm	10
5pm - 7pm	22
7pm - 9pm	17
9pm - 11pm	6

At least one full time worker must be available during the hour before opening and after closing. Additionally, at least 30% of the employees should be full time workers during the busy periods (11am - 1pm and 5pm - 7pm)

1. Formulate an ILP for this problem with the objective of minimizing total daily labor costs. Implement using Excel and Python.
2. What is the optimal solution?

Solution:

1. Let x_i be the number of workers allocated to shift i where $i = 1, \ldots, 7$. The problem can be formulated as:

$$
\begin{aligned}
\min.\ & 32x_1 + 80x_2 + 32x_3 + 80x_4 + 32x_5 + 80x_6 + 32x_7 && (1)\\
\text{s.t. } & x_1 + x_2 \geq 11 && (2)\\
& x_2 + x_3 + x_4 \geq 24 && (3)\\
& x_2 + x_3 + x_4 \geq 16 && (4)\\
& x_4 + x_5 + x_6 \geq 10 && (5)\\
& x_4 + x_5 + x_6 \geq 22 && (6)\\
& x_6 + x_7 \geq 17 && (7)\\
& x_6 + x_7 \geq 6 && (8)\\
& x_2 + x_4 \geq 0.3(x_2 + x_3 + x_4) && (9)\\
& x_4 + x_6 \geq 0.3(x_4 + x_5 + x_6) && (10)\\
& x_2 \geq 1 && (11)\\
& x_6 \geq 1 && (12)\\
& x_i \geq 0, \text{ for } i = 1, \ldots, 7 && (13)\\
& x_i \text{ integer for } i = 1, \ldots, 7 && (14)
\end{aligned}
$$

The objective (1) minimises the total cost of hiring workers. Constraints (2) - (8) ensure that there are sufficient workers during each period. Constraints 9 and 10 ensure that at least 30% of the workers during the busy periods are full-time workers. Constraints 11 and 12 ensure that at least one full-time worker is available before opening and after closing, and constraints 13 and 1 are the non-negativity and integer constraints.
See Canvas for excel and python solutions.

2. The optimal solution is: $x^* = [9, 2, 3, 16, 6, 15, 1, 16]$ with the total cost minimised at \$ 2,512.

Problem 3:

You have been asked to help the chief of the Sydney Fire Brigade in determining the location of fire stations throughout the city. To make your task easier, the city has been divided into 7 regions. The travelling time (in minutes) between the different regions is given in the following table. (Note: travel times between a region and itself are zero. The table is symmetric, and only the upper triangle has been provided).

Driving times between regions

	Region 1	Region 2	Region 3	Region 4	Region 5	Region 6	Region 7
Region 1	0	6	11	8	14	12	19
Region 2	-	0	5	18	8	6	20
Region 3	-	-	0	12	11	5	4
Region 4	-	-	-	0	9	6	15
Region 5	-	-	-	-	0	12	6
Region 6	-	-	-	-	-	0	13
Region 7	-	-	-	-	-	-	0

Under the condition that there must be a station within a 10 minute driving time of any region, determine the minimum number of fire stations required and their locations.

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Solution: Let $x_i \in \{0,1\}$ be a binary variable representing whether or not a fire station is built in region i, where $i = 1, \ldots, 7$. The ILP formulation is as follows:

$$\min . \sum_{i=1}^{7} x_i \quad (24)$$

$$\text{s.t. } x_1 + x_2 + x_4 \geq 1 \quad (25)$$

$$x_1 + x_2 + x_3 + x_5 + x_6 \geq 1 \quad (26)$$

$$x_2 + x_3 + x_6 + x_7 \geq 1 \quad (27)$$

$$x_1 + x_4 + x_5 + x_6 \geq 1 \quad (28)$$

$$x_2 + x_4 + x_5 + x_7 \geq 1 \quad (29)$$

$$x_2 + x_3 + x_4 + x_6 \geq 1 \quad (30)$$

$$x_3 + x_5 + x_7 \geq 1 \quad (31)$$

$$x_i \in \{0,1\}, i = 1, \ldots, 7 \quad (32)$$

Where the objective (24) minimises the total number of fire stations built and constraints (25) - (31) ensure that regions 1 - 7 are served by at least one station respectively.

The optimal solution is to build fire stations in regions 2 and 5 ($x_2 = x_5 = 1$).

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Problem 3:

The Ortega Food Company needs to ship 100 cases of hot tamales from its warehouse in San Diego to a distributor in New York City at minimum cost. The costs associated with shipping 100 cases between various cities are:

From	To					
	Los Angeles	Denver	St. Louis	Memphis	Chicago	New York
San Diego	5	13	-	45	-	105
Los Angeles	-	27	19	50	-	95
Denver	-	-	14	30	32	-
St. Louis	-	14	-	35	24	-
Memphis	-	-	35	-	18	25
Chicago	-	-	24	18	-	17

1. Draw the network representation of this problem.
2. Write out the LP formulation of this problem.
3. Solve the problem using Solver. Interpret your solution.
4. Solve the problem in Python. Try using a data structure of a 7 by 7 distance matrix. If there is no path between cities, put it down as a very large number.

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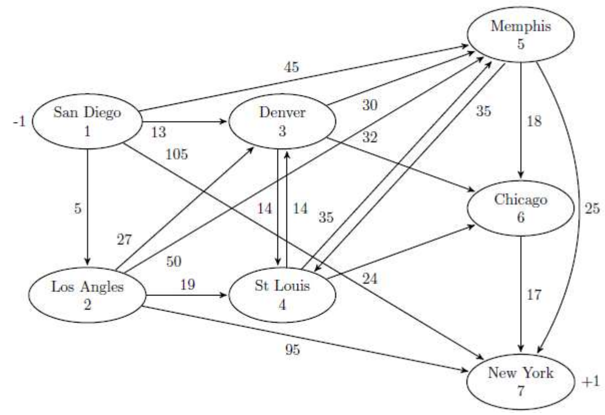

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2. Let x_{ij} be the amount of tamales (in 100 cases) shipped from i to j, for $j = 1, 2, \dots, 7$, where i, j represent corresponding cities in nodes above.

$$
\begin{aligned}
\min.\ & 5x_{12} + 13x_{13} + 45x_{15} + 105x_{17} + 27x_{23} + 19x_{24} + 50x_{25} + 95x_{27} + 14x_{34} \\
& + 30x_{35} + 32x_{36} + 14x_{43} + 35x_{45} + 24x_{46} + 35x_{54} + 18x_{56} + 25x_{57} + 24x_{64} \quad (1) \\
& + 18x_{65} + 17x_{67} \\
\text{s.t.}\ & -x_{12} - x_{13} - x_{15} - x_{17} = -1 \quad (2) \\
& x_{12} - x_{23} - x_{24} - x_{25} - x_{27} = 0 \quad (3) \\
& x_{13} + x_{23} + x_{43} - x_{34} - x_{35} - x_{36} = 0 \quad (4) \\
& x_{24} + x_{34} + x_{54} + x_{64} - x_{43} - x_{45} - x_{46} = 0 \quad (5) \\
& x_{15} + x_{25} + x_{35} + x_{45} + x_{65} - x_{54} - x_{56} - x_{57} = 0 \quad (6) \\
& x_{36} + x_{46} + x_{56} - x_{64} - x_{65} - x_{67} = 0 \quad (7) \\
& x_{17} - x_{27} + x_{57} + x_{67} = 1 \quad (8) \\
& x_{ij} \geq 0,\ \forall i, j \quad (9)
\end{aligned}
$$

The objective (1) minimises the total cost. Constraints (2) - (8) are the flow conservation constraints and constraint (9) is the non-negative constraint.

3. The optimal solution is: $x_{13} = x_{36} + x_{67} = 1$ and all other variables are equal to zero. This policy results in a minimised total cost of \$62. See Canvas for Excel and Python solutions.

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Problem 2:

The Royal Seas Company runs a three-night cruise to the Carribean from Port Canaveral. The company wants to run TV ads promoting its cruises to high-income men, high-income women, and retirees. The company has decided to consider airing ads during prime time, afternoon soap operas and during the evening news. The number of exposures (in millions) expected to be generated by each type of ad in each of the company's target audiences is summarized in the following table.

	Prime Time	Soap Operas	Evening News
High Income Men	6	3	6
High Income Women	3	4	4
Retirees	4	7	3

Ads during prime time, the afternoon soaps, and the news hour cost \$120,000, \$85,000 and \$100,000 each respectively. Royal Seas wants to achieve the following goals:

- Goal 1: To spend approximately \$900,000 of TV advertising
- Goal 2: To generate aproximately 45 million exposures among high-income men
- Goal 3: To generate approximately 60 million exposures among high-income women
- Goal 4: To generate approximately 50 million exposures among retirees

Solve the following:

1. Formulate a GP model for this problem. Assume overachievement of the first goal is equally as undesirable as underachievement of the remaining goals on a percentage deviation basis
2. Implement your model in a spreadsheet and solve it
3. What is the optimal solution?
4. What solution allows the company to spend as close to \$900,000 as possible without exceeding this amount?

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Solution:

1. Let x_i be the number of ads to air during period i for $i \in \{1,2,3\}$, where 1, 2, 3 stand for prime time, soap operas and evening news respectively.

$$\min.\ \frac{d_1^+}{900} + \frac{d_2^-}{45} + \frac{d_3^-}{60} + \frac{d_4^-}{50} \quad (1)$$
$$\text{s.t. } 120x_1 + 85x_2 + 100x_3 + d_1^- - d_1^+ = 900 \quad (2)$$
$$6x_1 + 3x_2 + 6x_3 + d_2^- - d_2^+ = 45 \quad (3)$$
$$3x_1 + 4x_2 + 6x_3 + d_3^- - d_3^+ = 60 \quad (4)$$
$$4x_1 + 7x_2 + 3x_3 + d_4^- - d_4^+ = 50 \quad (5)$$
$$x_i \geq 0,\ x_i \text{ is an integer}, \forall i \quad (6)$$
$$d_j^-, d_j^+ \geq 0,\ \forall j \quad (7)$$

The objective (1) minimises the total undesired deviations. Constraints (2) - (5) are the goals. Constraint (6) and (7) are the integer and non-negative constraints.

2. The optimal solution is $x_1 = 0,\ x_2 = 5,\ x_3 = 5$.

3.

$$\min.\ d_1^- \quad (1)$$
$$\text{s.t. } 120x_1 + 85x_2 + 100x_3 + d_1^- = 900 \quad (2)$$
$$x_i \geq 0,\ x_i \text{ is an integer}, \forall i \quad (3)$$
$$d_1^- \geq 0 \quad (4)$$

There are multiple optimal solutions. An optimal solution should have $d_1^- = 0$, i.e. all $900,000 budget has been spent.

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Problem 2:

You are given the following project.

Activity	Time	Predecessor Activities
A	3	–
B	4	A
C	4	A
D	3	A
E	3	D
F	4	B
G	6	B
H	5	F, C, E
I	6	G, H
J	4	F, C, E
K	2	D
L	6	I, J, K

1. Draw the network which corresponds to this project.
2. Find the earliest and latest start and finish times, and the slack for each activity.
3. What is the critical path? What is the earliest the project can be completed?
4. The following data summarize the per-day cost of crashing the activities in the project.

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Activity	Crashing cost/day ($)	Maximum crash days*
A	50	2
B	60	3
C	57	1
D	45	2
E	25	2
F	30	2
G	65	5
H	55	2
I	28	4
J	33	3
K	40	1
L	37	2

*The maximum number of days by which a given activity can be shortened.

Determine the least costly way of crashing the project if it must be completed within 20 day.

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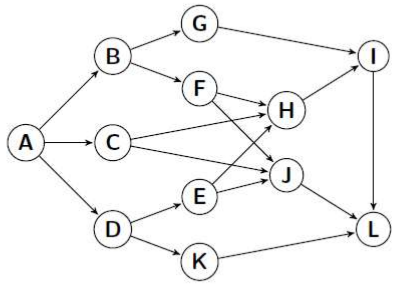
Solution:

1.



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2.

Activity	Time	EST	EFT	LST	LFT	Slack	Critical
A	3	0	3	0	3	0	y
B	4	3	7	3	7	0	y
C	4	3	7	7	11	4	n
D	3	3	6	5	8	2	n
E	3	6	9	8	11	2	n
F	4	7	11	7	11	0	y
G	6	7	13	10	16	3	n
H	5	11	16	11	16	0	y
I	6	16	22	16	22	0	y
J	4	11	15	18	22	7	n
K	2	6	8	20	22	14	n
L	6	22	28	22	28	0	y

3. The critical path is $A \to B \to F \to H \to I \to L$. We require at least 3+4+4+5+6+6 = 28 days to complete the project.

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Let S_i, F_i and C_i be the variables representing the start time, finish time and crash days of activity i, for $i = A, B, \ldots, L$.

$$\begin{aligned} \min.\ & 50C_A + 60C_B + 57C_C + 45C_D + 25C_E + 30C_F + 65C_G + 55C_H + 28C_I \\ & + 33C_J + 40C_K + 37C_L \end{aligned} \tag{1}$$

$$F_A = S_A + 3 - C_A \tag{2}$$
$$F_B = S_B + 4 - C_B \tag{3}$$
$$F_C = S_C + 4 - C_C \tag{4}$$
$$F_D = S_D + 3 - C_D \tag{5}$$
$$F_E = S_E + 3 - C_E \tag{6}$$
$$F_F = S_F + 4 - C_F \tag{7}$$
$$F_G = S_G + 6 - C_G \tag{8}$$
$$F_H = S_H + 5 - C_H \tag{9}$$
$$F_I = S_I + 6 - C_I \tag{10}$$
$$F_J = S_J + 4 - C_J \tag{11}$$
$$F_K = S_K + 2 - C_K \tag{12}$$
$$F_L = S_L + 6 - C_L \tag{13}$$

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$S_B \geq F_A$ (14)
$S_C \geq F_A$ (15)
$S_D \geq F_A$ (16)
$S_G \geq F_B$ (17)
$S_F \geq F_B$ (18)
$S_H \geq F_C$ (19)
$S_J \geq F_C$ (20)
$S_E \geq F_D$ (21)
$S_K \geq F_D$ (22)
$S_H \geq F_E$ (23)
$S_J \geq F_E$ (24)
$S_H \geq F_F$ (25)
$S_J \geq F_F$ (26)
$S_I \geq F_G$ (27)
$S_I \geq F_H$ (28)
$S_L \geq F_I$ (29)
$S_L \geq F_J$ (30)
$S_L \geq F_K$ (31)

$C_A \leq 2$ (32)
$C_B \leq 3$ (33)
$C_C \leq 1$ (34)
$C_D \leq 2$ (35)
$C_E \leq 2$ (36)
$C_F \leq 2$ (37)
$C_G \leq 5$ (38)
$C_H \leq 2$ (39)
$C_I \leq 4$ (40)
$C_J \leq 3$ (41)
$C_K \leq 1$ (42)
$C_L \leq 2$ (43)
$F_L \leq 20$ (44)
$S, F_i, C_i \geq 0, \ for\ i = A, B \ldots, F$ (45)

The objective (1) minimises the total crashing cost. Constraints (2) - (13) ensure that the activity will be finished after the required time minus the crash days. Constraints (14) - (31) ensure that the succeeding activities will start after the preceding activities. Constraints (32) - (43) are the maximum crash days constraints. Constraint (44) ensures the project will be completed within 20 days. Constraint (45) is the non-negativity constraint.

The optimal solution is $C_F = 2,\ C_I = 4,\ C_L = 2,\ C_A = C_B = C_C = C_D = C_E = C_G = C_H = C_J = C_K = 0$, the minimised crashing cost is \$246.

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Problem 5:

Customers checking out at Food Tiger arrive in a single-line queue served by two cashiers at a rate of eight per hour according to a Poisson distribution. Each cashier processes customers at a rate of eight per hour according to an exponential distribution.

1. If, on average, customers spend 30 minutes shopping before getting in the check out line, what is the average time a customer spends in the store?
2. What is the average number of customers waiting for service in the check out line?
3. What is the probability that a customer must wait?
4. What assumption did you make to answer this question?

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Solution: We can apply an M/M/s model with arrival rate $\lambda = 8$, service rate $\mu = 8$ and number of servers $s = c = 2$.

1. The average time a customer spends in the queueing system (waiting and being served) is

$$\begin{aligned} W &= \frac{L}{\lambda} \\ &= 0.1667 \text{ hours} = 10 \text{ minutes} \end{aligned}$$

   So the average time a customer spends in the store would be $30 + 10 = 40$ minutes.

2. $$\begin{aligned} L_q &= L - \frac{\lambda}{\mu} \\ &= 0.0333 \end{aligned}$$

3. $$\begin{aligned} P_w &= \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu - \lambda} P_0 \\ &= 0.3333 \end{aligned}$$

4. No balking or reneging. 

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