QBUS2820 Assignment 1

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Task A

Introduction

Although the NBA is known for being a sport league across globe, it is a vast economic entity as well. Undoubtedly, it has been a major impact in the past decades, and it does not seem to be slowing down anytime soon. Beside the League's branding, its commercial success is contributed by the players at large as they make the trends on social media and attract costumers to buy their products in a constance. However, the most important attribute of a player is none other than his performance on the court. Performance is what NBA players thrive for as it decides their salary level. How much salary a player is worth can be a hard estimation to the teams because the performance of athlete fluctuates. Furthermore, the salary cap of the League as a whole, too, fluctuate every year. Fortunately, the League records players' data in various categories which include field goal attempted, field goal percentage, offensive and defensive ratings, etc. With this data, machine learning techniques can be applied to develop reliable models to predict salary so that to help the NBA better performing their business manner such as human resource management, financial management and marketing strategies.

This project aims to develop predictive models of salary for NBA basketball players. Three types of techniques including k-nearest neignbour regression, linear regression and lasso regression are involved. Predictive models are developed by changing corresponding hyperparameters and features involved, with 5-fold cross validation being applied to assess root mean squared errors of these models. The optimal model is then selected by the lowest validation error for each technique. With the best predictive performance, a lasso regression model having the lowest test set root mean squared error is found to be best-suited the NBA data.

Data processing and exploratory data analysis

Two datasets `NBA_train` and `NBA_test` are analysed in this project. The data is collected by NBA, with the corresponding raw data and metadata being publicly accessible on the NBA websites.

There are 2 categorical variables and 19 numeric variables regarding players' personal information and game performance, with an additional unique ID of each record in the datasets. The numeric variables includes salary, age, number of games played , number of minutes played, personal efficiency rate , true shooting percentage, offensive rebounds , defensive rebounds , turnover percentage , assists , steals, blocks , turnover percentage , usage percentage , offensive rating, defensive rating and win shares while the categorical variables are the position and the team a player in. The corresponding variable names can be found the Table 2

The `NBA_train` dataset is used for training and validating predictive models in this project while the `NBA_test` dataset is used for testing selected models. Therefore the exploratory data analysis is conducted based on the `NBA_train` dataset.

Figure 1 illustrate that win share, defensive win share, offensive win share, number of minutes played and personal efficiency rate show linear relationships with salary, with win share having the strongest linear relationship with salary at a correlation coefficient of 0.68. It also provides evidences of linearity between offensive win share, defensive win share and win share. Although other variables show mild linear relationship with salary, there can be other linkage between salary and these variables. Thus variables showing no linearity with salary can still be potentially informative and should be left for further feature selection when developing predictive models.

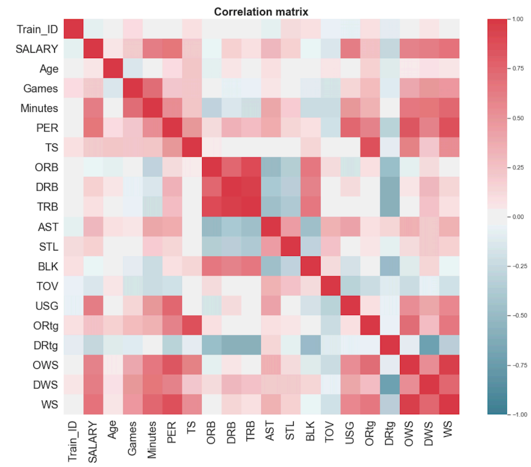
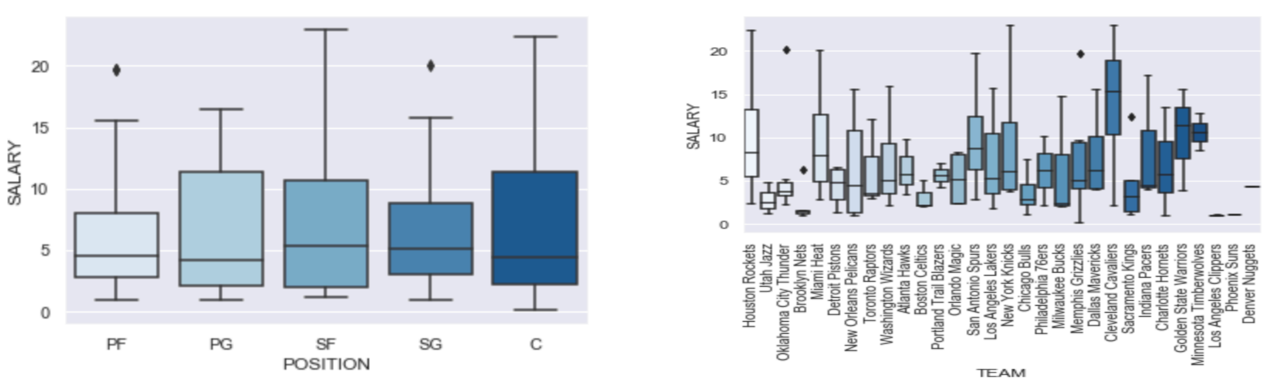


Fig. 1: Correlations between numeric variables based on correlation coefficients.

Moreover, colinearity is observed between several variables. The number of win shares which is linearly related to salary is also found to be correlated with number of offensive win shares and number of defensive win shares, while total rebound is linearly related to both offensive rebound and defensive rebound.

The relationships between salary and the six relative variables as well as the distribution of numeric variables are further visualized by a scatter plot matrix. In Figure 6, the linearity between numeric variables and salary shown is in line with the correlation matrix (Figure 1). Moreover, salary, win share, defensive win share and offensive win share are significantly right-skewed while usage percentage and personal efficiency rate are slightly right-skewed. In additions, the distributions demonstrates a small variance of number of minutes played.

To analyse the categorical variables, box plots (Figure 2) are generated to visualize the distribution of salary. Figure 2a describes how salary varies for players in different positions. Although the median salaries are similar at $4-6 millions for the five positions, variances of salary are slightly different. Salaries for players in the center and small forward position vary significantly without any outliers whereas variances of salary for both power forward and shooting guard are smaller with an outlier. Nevertheless, the distributions of salary for players in different positions are similar.



(a) Salaries for players in different positions.

(b) Salaries for players in different teams.

Fig. 2: Box plots of salaries for players in different teams and positions.

Outlining in Figure 2b, salary varies across different teams, which is reasonable in a business entity. There are many basketball teams within the NBA, resulting in small sample sizes of salary in each group. Some groups, such as Los Angeles Clippers, Phoenix Surs and Denver Nuggets, have information of only one player being recorded in this dataset. Furthermore, the team variable has no intrinsic order, and the teams in any unseen data can contain new teams. This makes the team variable less informative.

Feature engineering

To discover any missing values involved in the datasets, bar charts of missingness are generated to visualize missingness. As shown in Figure 5, both `NBA_train` and `NBA_test` are complete without any missing values. Therefore no data removal or data imputation is performed.

According to the results of exploratory data analysis, the record ID, position a player in and team played are uninformative for predicting salary. Therefore these three variables are discarded whereas the salary is extracted from the `NBA_train` and `NBA_test` datasets to be the response. There are 19 numeric features including age, number of games played , number of minutes played, personal efficiency rate , true shooting percentage, offensive rebounds , defensive rebounds , turnover percentage , assists , steals, blocks , turnover percentage , usage percentage , offensive rating, defensive rating, win shares and team the player in, engaging in predictive model development.

Methodology of K-nearest neighbour regression models

The K-nearest neighbour regression models are trained by one of the 19 numeric features, with different values of K ranging from 1 to 50. Totally 950 models are developed by changing the feature and the value of k. 5-fold cross validation is applied to assess negative mean square errors of models, followed by transferring negative mean square errors to root mean square errors. The model with the smallest root mean square error is selected as the optimal model.

The model trained by the number of win shares and a k of 19, which has a validation error of 4.2615 ($ Millions), is chosen to be the optimal K-nearest neighbour regression model. With this model, 19 neighbours are considered to examine the value of salary with specific number of win shares. Figure 3 visualizes this k-nearest neighbour regression model with observed data points of salary against number of win shares. Generally, the salary is predicted to increase as number of win shares increases, which is logical with the economic concern of the NBA. The more games a player win, the more valuable he is in the basketball team and thus the play deserves a higher salary. The model predicts salary to remain stable when 10 win shares is reached, which is not in line with observed data points. This may result from the small number of data points with number of win shares greater than 10, resulting in insufficient neigbours for k-nearest neighbour model training.

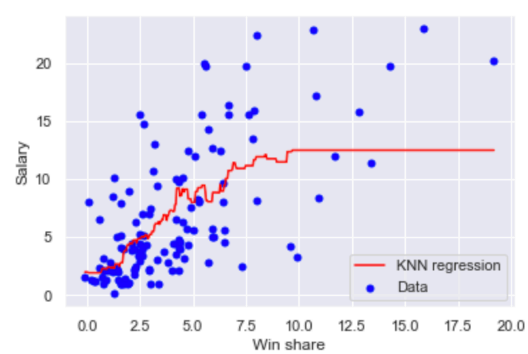


Fig. 3: *Visualizing the K-nearest neighbour model with K = 19 and number of win shares as the feature.*

Methodology of linear regression models

The polynomial regression models are trained by one of the 19 numeric features with a polynomial degree varying between 1 and 10. 190 polynomial regression models are developed by selecting different feature and the polynomial degree of the model.

Multiple linear regression models are also developed with different features. Recursive feature elimination (RFE) which is a feature selection method is applied. It removes weakest features based on corresponding coefficients of linear regression model until the required number of features is reached. The dependencies and colinearity between features are also captured and eliminated by this method. As discussed in the exploratory data analysis, there is colinerity between number of defensive win shares, offensive win shares and win shares, and between total rebound, offensive rebound and defensive rebounds. In order to avoid violating the assumption of multiple linear regression, the number of offensive win share and defensive win shares, offensive rebound and defensive rebound are removed from the feature pool for feature selection, leading to 15 numeric variables remaining in the feature pool. The number of features required for recursive feature elimination varies from 1 to 15, as such simple linear regression models with one feature are also estimated in this approach.

Negative mean square errors of these models are obtained by implementing 5-fold cross validation. Root mean square errors are then calculated to determine the optimal models of polynomial linear regression and multiple linear regression.

The optimal polynomial regression model contains number of win shares as the feature and a polynomial degree of 2. The predictive function of the model is: $Salary = 1.4213 + 1.4339\ WS - 0.0219\ WS^2$, where WS is the number of win shares. As shown in Figure 4, The polynomial regression better fits the observed data compared to the linear regression model as it captures more variance of salary with regard to different number of win shares.

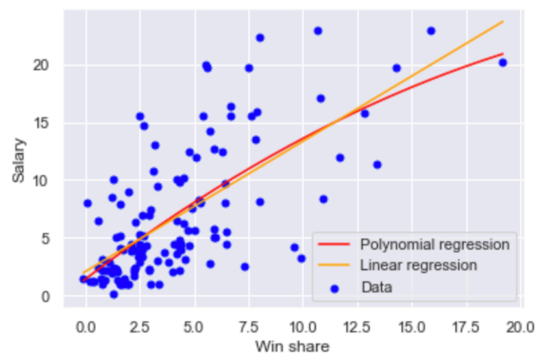


Fig. 4: *Visualizing the polynomial regression model and linear model with number of win share as feature. The polynomial regression model has a polynomial degree of 2.*

The optimal multiple regression model involves 14 numeric features, having the root mean square error of 3.792 ($ Millions). The predictive function is $Salary = 10.53 - 0.1\ Age - 0.16\ Games + 0.0037\ Minutes - 0.25\ PER - 0.098\ TS + 0.2\ TRB - 0.062\ AST + 0.58\ STL - 0.21\ BLK + 0.12\ TOV + 0.51\ USG + 0.11\ ORtg - 0.19 DRtg + 0.49\ WS$, where salary rises with incrase in number of minutes played, total rebounds, number of steals, turnover percentage, usage percentage, offensive rating, win shares, and decrease in age, number of games played, personal efficiency rating, true shooting percentage, assists, blocks, defensive rating.

With a smaller root mean square error of training model, the optimal multiple linear regression model including 14 features are finally selected as the optimal linear regression model.

Methodology of lasso regression models

Least absolute shrinkage and selection operator (Lasso), as an extension of liner regression analysis, conducts both feature selection and regularization. This helps to enhance the predictive performance and interpretability of the model developed.

The objective of a lasso regression model is to minimize $\sum_{i=1}^{n}(y_i - \sum_j x_{ij}\beta_j)^2 + \alpha \sum_{j=1}^{p} |\beta_j|$, where α is a tuning parameter which represents how strong the L1 regularization penalize coefficients of the lasso regression model. Changing the value of α influences the number of features eliminated. When $\alpha = 0$, coefficients of features are not Palisades such that no feature is removed. As the value of α increases, L1 penalty gets stronger and therefore more features are eliminated, vice versa. Furthermore, the value of α also affects the bias-variance trade-off. An increase of α leads to increase in bias while a decrease of α results in increase in variance.

Lasso regression model automates feature selection whereas multiple linear regression model require additional feature selection approach to deal with multicollinearity. Moreover, L1 regularization which penalizes the coefficients of linear regression model is performed with lasso regression. In the source data of this project, outliers and multicollinearity exist in several pairs of features. In this case, lasso regression with automated feature selection and regularization could be better-suited for the data compared to regular linear regression.

Lasso regression is a supervised learning technique while K-nearest neighbour regression is an unsupervised learning technique, which means a linear function is pre-defined for lasso regression when K-nearest neighbour regression observes pattern in the data without fix function. As such, lasso regression is more interpretable but less flexible than k-nearest neighbour regression. In this project, one of the goals is to discover dominant factors of salary for NBA players. To achieve this goal, a lasso regression model which can clearly define weights of features is more powerful compared to a KNN regression model.

19 numeric features are fed into lasso regression models with 7 values of alpha: 0.0001, 0.001, 0.01, 0.1, 0.5, 5 and 10. 5-fold cross validation is then applied to assess predictive performance of models. The optimal lasso model is selected based on root mean squared error, the smaller the root mean squared error, the better the lasso model performs.

The optimal value of α is 0.5 for this data. With this value of α, 10 features are selected with the predictive function: $Salary = 25.18 - 0.0044\,Age - 0.18\,Games + 0.0056\,Minutes + 0.1\,PER + 0.036\,DRB - 0.054\,AST + 0.098\,TOV + 0.27\,USG - 0.23\,RDtg + 0.17WS$. This indicates that salary increases with increases in number of minutes played, personal efficiency rate, defensive rebounds, turn over percentage, usage percentage and number of win shares while decreases in age, number of games played, assists and defensive rating result in an increase in salary. The validation root mean squared of the model is 3.5857 ($ Millions), which is the best performance among 7 lasso regression models developed.

Test set performance

Root mean squared error is used to assess the predictive performance of three models selected.

The selected k-nearest neighbour regression model has a test set root mean squared error of 4.2288, representing a standard deviation of unexplained variance of salary at 4.2288 millions dollars.

The selected multiple linear regression model has a test set error at 4.0532, indicating that the standard deviation of the unexplained variance of salary by the multiple linear regression is 4.0532 millions dollars.

The test set error of the optimal lasso regression model is 3.9570, demonstrating a standard deviation of uncaptured variance of salary by the lasso regression at 3.9570 millions dollars.

With the rule of thumb of 4.1 millions dollars, the predictive performances of the multiple linear regression model with 14 features and the lasso regression model with $\alpha = 0.5$ are satisfactory.

As shown in Table 1, a predictive model with a lower validation error tends to have a lower testing error. The lasso regression model with $\alpha = 0.5$ has the lowest testing error, at 3.957 millions dollars. Therefore generally the lasso regression model is the best-suited the data with greatest predictive performance.

Model	Validation error	Test set error
KNN regression	4.2615	4.2288
Multiple linear regression	3.7920	4.0532
Lasso regression	3.5857	3.9570

Table 1: *Summary of training and testing performance of three predictive models. Both validation error and testing errors are estimated by root mean squared error.*

Analysis and conclusions

In conclusion, predictive performance can vary significantly with different features and hyperparameters of machine learning techniques. The optimal multiple linear regression model contains 14 features while the optimal lasso regression model selects

10 features and a α value of 0.5. These two models satisfies the predictive performance rule of thumb which requires root mean squared errors less than 4.1 millions dollars.

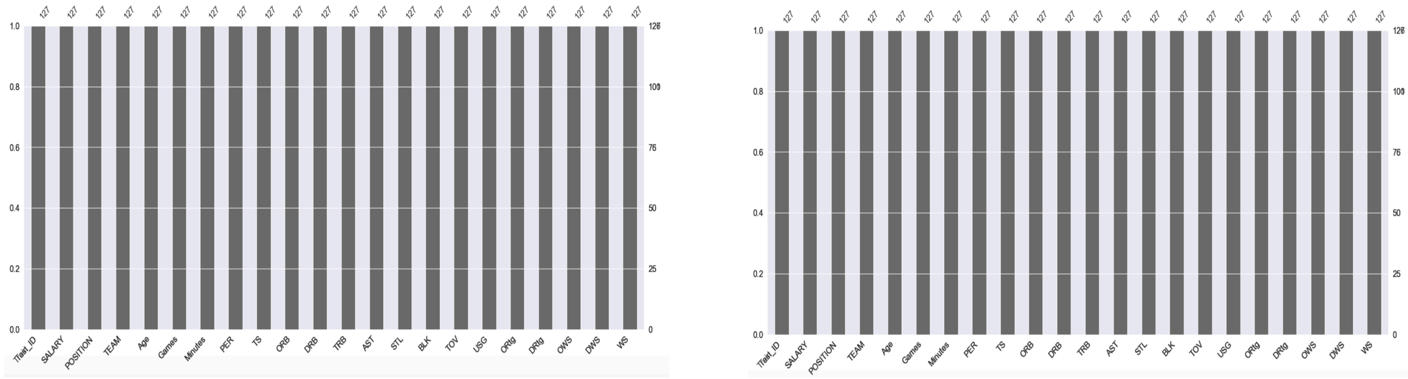
Although the 19-nearest neighbour regression model with win shares as feature is the optimal knn model, it is not satisfactory with a relatively high test set error compared to other techniques. With regards to previous discussion, a limitation of the optimal k-nearest neighbour model is that it does not accurately predict salary for a large number of win shares because of the outliers. This can also be the main reason of the undesired predictive performance. A potential solution to this issue is to choose a smaller values of k so that the salary is estimated based on fewer neibours. However, this could worsen predictive performance of the whole model as root mean squared error increases. More attention should be paid on this trade-off in further study. Another solution could be scaling the salary with logarithm to deal with outliers. Moreover, a problem of whether and how the feature selection should be perform for k-nearest neighbour is yet to be discussed.

Beside the k-knerest neighbour regression, more unsupervised learning techniques should be evaluated in further study, so that any differences of predictive performance between supervised and unsupervised learning can be discovered.

Appendix

Variables	Description	Data type
ID	Unique identification number of the record	Numeric
SALARY	Salary for the NBA player	Numeric
POSITION	Position played	Categorical
TEAM	Team the player in	Categorical
Age	Age of the player	Numeric
Games	Number of games played	Numeric
Minutes	Number of minutes played	Numeric
PER	Personal efficiency rate	Numeric
TS	True shooting percentage	Numeric
ORB	Offensive rebounds	Numeric
DRB	Defensive rebounds	Numeric
TRB	Total rebounds	Numeric
AST	Number of assists	Numeric
STL	Number of steals	Numeric
BLK	Number of blocks	Numeric
TOV	Turnover percentage	Numeric
USG	Usage percentage	Numeric
ORtg	Offensive rating	Numeric
DRtg	Defensive rating	Numeric
OWS	Offensive win shares	Numeric
DWS	Defensive win shares	Numeric
WS	Win shares	Numeric

Table 2: *Table of variables.*



(a) *Missingness bar chart of the 'NBA train' dataset.*

(b) *Missingness bar chart of the 'NBA test' dataset.*

Fig. 5: *Visualizing the missingness of two datasets used.*

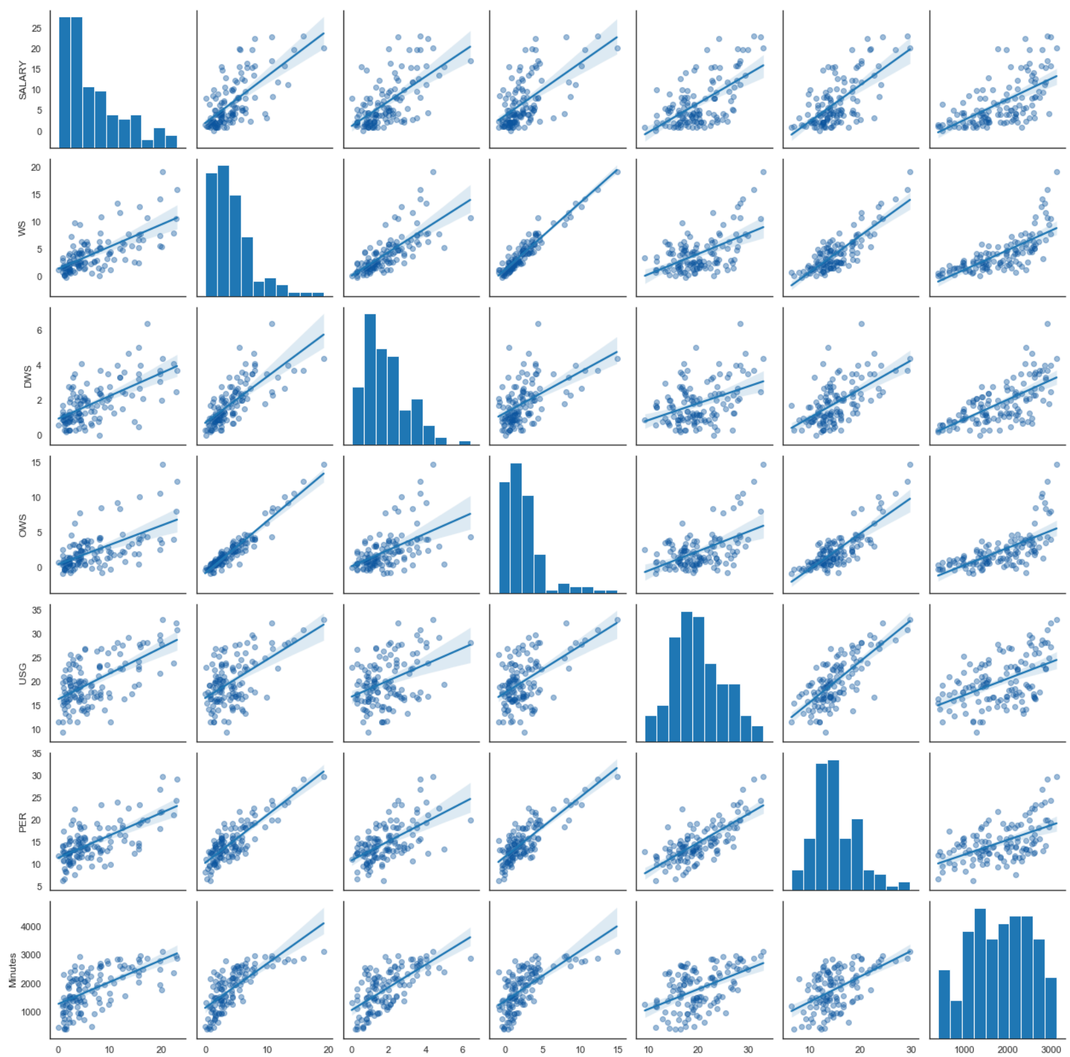


Fig. 6: *Distribution of numeric variables in 'NBA train' dataset.*

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Task B

Exploratory data analysis

There are 14 numeric variables in the `Boston housing` dataset. As shown in Figure 7, the dataset is 100% complete without missing values.

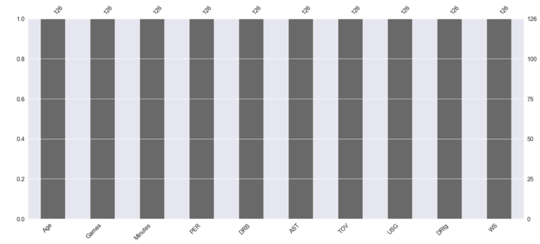

Fig. 7: Bar chart of missingness for ‘Boston housing‘ dataset.

Based on the correlation coefficients shown in Figure 8 and Table 4, “lstat”, “rm” and “ptrtio” have relatively strong linear relationship with “medv” while “chas” and “dis” is mildly linearly related to “medv”. To reduce the number of features, features with an absolute value of correlation coefficients with “medv” (refer to Table 4) at less than 0.4 are discarded. 6 features which are “nox”, “tax”, “indus”, “ptratio”, “rm” and “lstat” remain for further inspection.

Figure 9 further visualize the distribution of 6 features which show linear relationship with “medv”. The linear relationships are clearly shown between “rm”, “lstat” and ’medv“, which indicates multicollinearity between the three variables. As such, only one of”rm" and “lastat” can be involved in the linear regression model. “nox” and “indus” are also visualized to have mediate linear relationship with “medv” whereas the pattern of linearity of “ptratio” and “tax” is unclear.

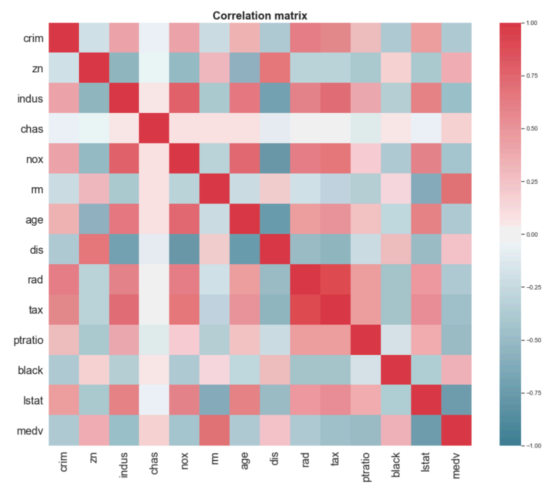

Fig. 8: Correlations between numeric variables in ‘Boston Housing‘ dataset.

Furthermore, Figure 9 shows that “medv”, “lstat” and “nox” are right-skewed while “ptratio” is left-skewed, and “rm” follows a normal distribution. “indus” and “tax” follow unknown distribution with large amount of outliers.

To select 3 features from 6 feature, the correlation coefficient and distribution of the data play is referred. As “lstat” and “rm” show no clear linear relationship with other variables except “medv”, “lstat” is preferred as it has a greater correlation coefficient with the response “medv”. Besides, “nox” and “indus” are also selected as they have higher correlation coefficient and follow a clear distribution compared to “ptratio” and “tax”. As such, by considering multicolinearity and the strength of linear relationships with “medv”, “lstat”, “nox” and “indus” are selected to engage in the linear regression model.

Gradient ascent algorithm

To develop the gradient ascent algorithm for maximum likelihood estimation, the log-likelihood is calculated by $L = -\frac{1}{2N}\sum_{i=1}^{N}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_i x_{ij})^2$. The reason why log-likelihood rather than likelihood is used is that the logarithm transfers multiplication to sum in the function so that it is easier to calculate differentiation. The gradients of parameters are then calculated by the matrix representation $\frac{\triangle L(\beta)}{\triangle \beta} = \frac{1}{N}X^T(y - f(X))$. The gradient ascent approach is initialized by a point of $\beta = [0,...,0]^T$. In each iteration, β is updated to $\beta + \alpha\frac{\triangle L(\beta)}{\triangle \beta}$, where α is the learning rate controlling the step of increase in β for each iteration, until convergence (i.e. the gradient has few changes).

With the same number of iteration, different values of learning can influences the predictive performance of models. In this project, performance of gradient ascent models with learning rates of 0.001, 0.01, 0.1 and 1 is estimated while the number of iteration is set to be 500 which is large enough for likelihood to converge. Leave one out cross validation is applied, with mean squared error being the cross validation score to assess the performance of models.

As shown in Table 3, the optimal learning rate is 0.001, which results in the smallest mean squared error of the linear regression model at 0.4447. This indicates that the linear regression model applying gradient ascent with $\alpha = 0.001$ for maximum likelihood estimation has a standard deviation of unexplained variance of "medv" at 0.4447.

Learning rate	CV score (MSE)
0.001	0.4447
0.01	0.4485
0.1	0.4485
1	0.04485

Table 3: *CV scores of gradient ascent models*

Appendix

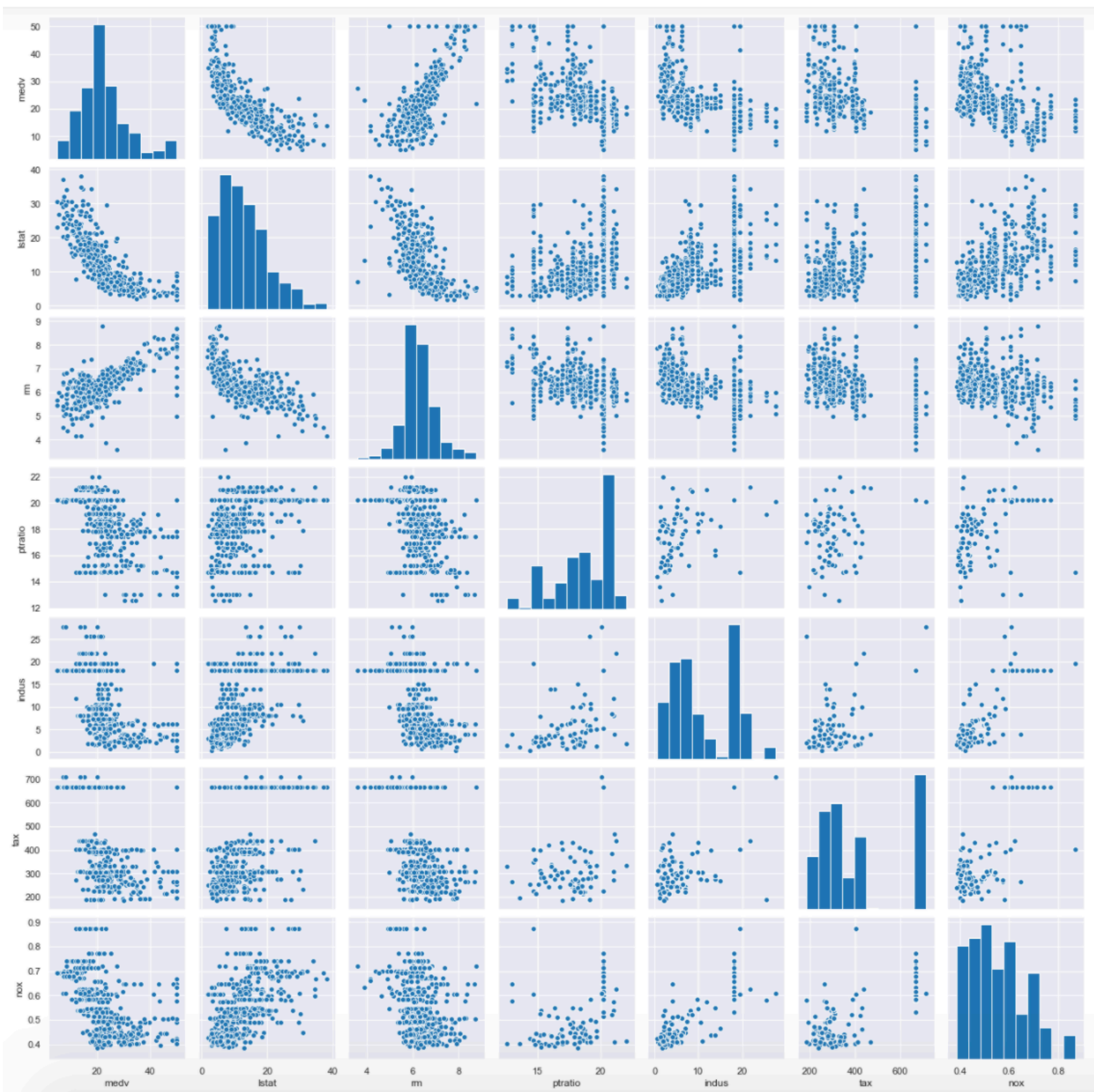

Fig. 9: *Distribution of numeric variables in 'Boston housing' dataset.*

Variables	Correlation with "medv"	Absolute coefficient
chas	0.1753	0.1753
dis	0.2499	0.2499
black	0.3335	0.3335
zn	0.3604	0.3604
age	-0.3770	0.3770
rad	-0.3816	0.3816
crim	-0.3883	0.3883
nox	-0.4273	0.4273
tax	-0.4685	0.4685
indus	-0.4837	0.4837
ptratio	-0.5078	0.5078
rm	0.6954	0.6954
lstat	-0.7377	0.7377

Table 4: *Correlation coefficients between "medv" and other variable in 'Boston housing' dataset.*

References

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