SID: 480110301

Question 1

1. Formulate a MILP model to solve the problem.

Let X_{1i} be the number of trucks of the model i the company should operate, where $i = 1, 2, 3, 4$.

Let X_{2i} be the amount of product delivered by subcontract in range i, where $i = 1, 2, 3$ indicating the range of less than 25k, 25k to 50k and greater than 50k.

Let Y_{1i} be the binary variable indicating whether the amount of product delivered by subcontract reaches range i, where $i = 1, 2, 3$ representing the range of less than 25k, 25k to 50k and greater than 50k. $Y_{1i} = 1$ when the amount of product delivered by subcontract reaches the range, otherwise $Y_{1i} = 0$.

Let X_{3i} be the binary variables where $i = 1, 2$, indicating the amount of excessive petrol and the amount of additional petrol required.

Let Y_{2i} be the binary variables where $i = 1, 2$, indicating whether the amount of petrol used is greater than and less than 50K litres. $Y_{21} = 1$ when the amount of petrol exceeds while $Y_{22} = 1$ when an additional amount of petrol is needed.

Let X_{41} be the amount of additional drivers needed.

Let Y_{31} be the binary variables indicating whether additional drivers are required. If additional drivers are required, $Y_{31} = 1$.

Constraints:

1. $X_{21} + X_{22} + X_{23} = 280000 - 11000X_{11} - 9000X_{12} - 7000X_{13} - 6000X_{14}$ (sub-contracted product)
2. $X_{21} \leq 25000Y_{11}$ (upper limit for \$1 / liter)
3. $X_{22} \leq 25000Y_{12}$ (upper limit for \$1.3 / liter)
4. $X_{23} \geq Y_{13}$ (upper limit for \$1.6 / liter)
5. $Y_{11} - Y_{12} \geq 0$
6. $Y_{12} - Y_{13} \geq 0$
7. $2000X_{11} + 1700X_{12} + 1200X_{13} + 1100X_{14} \leq 60000$(total petrol available)
8. $50000 - 2000X_{11} - 1700X_{12} - 1200X_{13} - 1100X_{14} \leq 10000Y_{21}$(excessive

petrol available)

9. $2000X_{11} + 1700X_{12} + 1200X_{13} + 1100X_{14} - 50000 \leq 10000Y_{22}$ (additional petrol available)
10. $X_{31} = (50000 - 2000X_{11} - 1700X_{12} - 1200X_{13} - 1100X_{14}) \times Y_{21}$(additional petrol required)
11. $X_{32} = (2000X_{11} + 1700X_{12} + 1200X_{13} + 1100X_{14} - 50000) \times Y_{22}$ (lack of petrol)
12. $Y_{21} + Y_{22} \leq 1$ (lack of petrol and exceed of petrol cannot occur simultaneously)
13. $2X_{11} + 2X_{12} + X_{13} + X_{14} \leq 45$(total drivers available)
14. $2X_{11} + 2X_{12} + X_{13} + X_{14} - 35 \leq 10Y_{31}$(additional drivers available)
15. $X_{41} = Y_{31} \times (2X_{11} + 2X_{12} + X_{13} + X_{14} - 35)$ (additional drivers required)
16. $20X_{11} + 14X_{12} + 12X_{13} + 10X_{14} \leq 400$(maintenance available)
17. X_{ij} is integer for all i, j
18. X_{ij} is non-negative for all i, j

Cost of sub-contract delivery: $X_{21} + 1.3X_{22} + 1.6X_{23}$

Cost of excessive / additional petrol: $-1.1X_{31} + 1.5X_{32}$

Cost of additional drivers: $8000X_{41}$

Total cost = $X_{21} + 1.3X_{22} + 1.6X_{23} - 1.1X_{31} + 1.5X_{32} + 8000X_{41}$

Objective function: $Minimize\ X_{21} + 1.3X_{22} + 1.6X_{23} - 1.1X_{31} + 1.5X_{32} + 8000X_{41}$

2. Solve the problem using Python (refer to Question 1 Appendix).

3. What is the optimal purchasing policy and what is the profit under this policy? Provide a detailed discussion on the company's use of resources and the factors affecting the chose purchasing policy.

The optimal purchasing policy is that the shipping company operates 2 trucks of model B and 31 trucks of model C to deliver 245K products. No truck of model A and C is operated. The remaining 4.5K products will be subcontracted to the other shipping company to deliver. This resulted in a minimum cost of $40660 to deliver all of 280K products.

With this purchasing policy, 40600 liters of petrol are required. There are 9400 liters of excessive petrol which can be sold by $1.1 per liter. 35 drivers are required to operate trucks, which is the exact number of drivers with fixed salaries, therefore no additional driver is needed. 400 hours of maintenance are quired, which is exactly the maximum time of maintenance provided.

Question 1 Appendix

```
import gurobi as grb

A = grb.Model('Q1')

subcontract = {1:1,2:1.3,3:1.6}
delivery = {1,2,3,4}

petrol = {1:-1.1,2:1.5}
petrol_amount = {1:-1.1,2:1.5}
driver = {1:8000}

X1 = A.addVars(delivery, obj = 0, vtype = grb.GRB.INTEGER)
X2 = A.addVars(subcontract, obj = subcontract.values(), vtype = grb.GRB.INTEGER)
X3 = A.addVars(petrol, obj = petrol.values())
X4 = A.addVars(driver, obj = driver.values(), vtype = grb.GRB.INTEGER)
Y1 = A.addVars(subcontract, obj =0, vtype = grb.GRB.BINARY)
Y2 = A.addVars(petrol, obj =0, vtype = grb.GRB.BINARY)
Y3 = A.addVars(driver, obj =0, vtype = grb.GRB.BINARY)

A.modelSense = grb.GRB.MINIMIZE

# subcontract constraints
A.addConstr(X2[1]+X2[2]+X2[3]+11000*X1[1]+9000*X1[2]+7000*X1[3]+6000*X1[4]==280000)
A.addConstr(X2[1]<=25000*Y1[1])
A.addConstr(X2[2]<=25000*Y1[2])
A.addConstr(X2[3]>=Y1[3])
A.addConstr(Y1[1]-Y1[2]>=0)
A.addConstr(Y1[2]-Y1[3]>=0)

# petrol constraints
A.addConstr(2000*X1[1]+1700*X1[2]+1200*X1[3]+1100*X1[4]<=60000)
A.addConstr(50000-2000*X1[1]-1700*X1[2]-1200*X1[3]-1100*X1[4]<=10000*Y2[1]) # excess
A.addConstr(2000*X1[1]+1700*X1[2]+1200*X1[3]+1100*X1[4]-50000<=10000*Y2[2]) # additional
A.addConstr(Y2[1]+Y2[2]<=1)
A.addConstr(X3[1] == (50000-2000*X1[1]-1700*X1[2]-1200*X1[3]-1100*X1[4])*Y2[1])
A.addConstr(X3[2] == (2000*X1[1]+1700*X1[2]+1200*X1[3]+1100*X1[4]-50000)*Y2[2])

# Drivers
A.addConstr(2*X1[1]+2*X1[2]+X1[3]+X1[4]<=45)
A.addConstr(2*X1[1]+2*X1[2]+X1[3]+X1[4]-35<=10*Y3[1])
A.addConstr(X4[1]==Y3[1]*(2*X1[1]+2*X1[2]+X1[3]+X1[4]-35))

# Maintainance
A.addConstr(20*X1[1]+14*X1[2]+12*X1[3]+10*X1[4]<=400)

A.optimize()

A.write('Assignment Q1.lp')

model = ["A","B","C","D"]
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```
for i in range(1,5):
    print("The company should operate {} trucks of model {}.".format(round(X1[i].x), model[i-1]))

print("The minimum cost is ${}.".format(round(A.objVal)))

op_subcontract = round(X2[1].x+X2[2].x+X2[3].x)
print("The company should sub-contract the delivery of {} products to other
companies.".format(op_subcontract))

petrol_used = 2000*X1[1].x + 1700*X1[2].x + 1200*X1[3].x + 1100*X1[4].x
print("The amount of petrol required is {} liters.".format(round(petrol_used)))
print("The amount of excessive petrol is {} liters.".format(round(X3[1].x)))
print("The amount of additional petrol required is {} liters.".format(round(X3[2].x)))

driver_required = 2*X1[1].x + 2*X1[2].x + X1[3].x + X1[4].x
print("The number of drivers required:", round(driver_required))
print("The number of additional drivers:", round(X4[1].x))

maintainance_required = 20*X1[1].x+14*X1[2].x+12*X1[3].x+10*X1[4].x
print("The amount of maintainance required:", round(maintainance_required),"hours.")
```

Question 2

a. **Does the solution have multiple optimal solutions? Provide a short explanation.**

   Answer: All of the values of allowable increase and allowable decrease are not equal to zero, meaning that the coefficients can be changed in both directions to remain the unique optimal solution. Therefore the solution does not have multiple optimal solutions.

b. **What is the optimal objective function value if the RHS value of the first constraint increases to 60? Provide a short explanation.**

   Answer: The original optimal objective function value can be calculated by summing up the products of objective coefficients and final values in the sensitivity report: $2 \times 5 + 0 \times 4 + 8 \times 7 + 3 \times 6 = 84$. The shadow price of the first constraint is 1.4375, which means that the objective value will increase by 1.4375 for 1 unit increase of RHS of the first constraint. There is a 20-unit increase when the RHS value increases from 40 to 60. Therefore the objective value will increase by $20 \times 1.4375 = 28.75$ units. The optimal objective function value will be 112.75 ($84 + 28.75 = 112.75$) after increasing the RHS value of the first constraint to 60.

c. **What is the optimal objective function value if the RHS value of the second constraint decreases to 40? Provide a short explanation.**

   Answer: The original optimal objective function value is 84 as calculated above. The shadow price of the second constraint is 0.3375, which means that the objective function value will decrease by 0.3375 for 1 unit decrease of the RHS value of the second constraint. There is a 7-unit decrease when the RHS value decreases from 47 to 40. Therefore the objective function value will decrease by $7 \times 0.3375 = 2.3625$ units. The optimal objective function value will be 81.6375 ($84 - 2.3625 = 81.6375$) after decreasing the RHS value of the second constraint to 40.

d. **What is the optimal objective function value if the RHS value of the third constraint increases to 50?**

   Answer: The original optimal objective function value is 84 as calculated above. The shadow price of the third constraint is 0.2875, which means that the objective function value will increase by 0.2875 for 1 unit increase of the

RHS value of the third constraint. There is a 13-unit increase when the RHS value decreases from 37 to 50. Therefore the objective function value will decrease by $13 \times 0.2875 = 3.7375$ units. The optimal objective function value will be 87.7375 ($84 + 3.7375 = 87.7375$) after increasing the RHS value of the third constraint to 50.

e. **Assume that new technology has been introduced to our production system and that producing product 2 (X2) now requires the use of 2 units less of resource 1 (Constraint 1) and 1 unit less of resource 3 (Constraint 3) compared to current practice (the use of resource 2 has not changed). Would the optimal solution be different after this new technology is introduced?**

   Answer: Two coefficients are changing simultaneously therefore we apply the 100% Rule. The percentages of decrease can be calculated by the number of units divided by the allowable decrease of the corresponding constraints, which are $2 \div 25.6 = 3.9$% and $1 \div 8 = 12.5$%. The sum of decrease percentage is 12.5%+3.9%=16.4%, which is smaller than 100%. Therefore the optimal solution remains the same after this new technology is introduced.