

Management science process: **observation** (identify problems) - **problem definition** - **model construction** (develop function: DV, objective, constraints) - solution - **implementation** (use of model)

Decision variables: mathematical symbols representing levels of activity of a firm

Object function: a linear mathematical relationship describing an objective, the function is maximized or minimized

Constrains: requirements/restrictions

Parameters: numerical coefficients and constants used in objective function and constraints

A **feasible solution** does not violate any constraints while **infeasible solution** violates at least one of the constraints

Slack variable / Surplus variable:

- is added to a \leq / \geq constraint to convert it to an equation ($=$).
- Represents unused / excess resource
- No contribution to objective function value.

Special types of problems: multiple optimal solutions, infeasible solutions, unbounded solutions.

Characteristics of linear programming:

- **Decision variable:** A decision amongst alternative courses of action
- **Objective function:** expresses a goal that the decision maker wants to achieve
- **Constrains:** limits the extent of achievement of the objective
- Objective and constraints must be definable by **linear** mathematical function.

Properties of linear programming model:

- **Proportionality:** the slope of objective function and constraint equations is constant
- **Additivity:** terms in objective function and constraints must be additive
- **Divisibility:** decision variables can take on any fractional value and are therefore continuous.
- **Certainty:** value of all parameters are assumed to be known with certainty.

LP model in Solver:

- **Target cell:** the cell representing the objective function
- **Changing cell:** cells representing decision variables
- **Constraint cell:** cells representing the LHS formulas on the constraints

Goals for spreadsheet design:

- Communication: the primary business purpose is communicating information to managers.
- Reliability: the output generated should be correct and consistent.
- Auditability: a manager should be able to retrace the steps followed to generate the different outputs of the model.
- Modifiability: should be easy to change or enhance in order to meet dynamic user requirements.

Sensitivity report:

- **Allowable increase / decrease:** the coefficients of the original objective function can be increased / decreased by _____ without changing the optimal solution.
 - **allowable increase / decrease = 0**
 - For **objective function:** **Alternative** optimal solution exists
 - For **constraints:** LP problem degenerate
 - Cannot detect alternate optimal solutions
 - Reduced cost may not be unique; the coefficients in objective must change by at least as much as reduced costs before the optimal solution changes.
 - Allowable increase/decrease still hold for objective
 - Shadow price may not be unique.
- **Reduced cost:** = unit profit - unit cost
 - Interpretation: the objective coefficients (unit profits) can be increased or decreased by _____ before the optimal solution changes.
- **Shadow price:** the objective values will increase / decrease by shadow price for every unit change of RHS value.
 - **Shadow price** is **only valid** within the corresponding range of **allowable increase / decrease**
 - Shadow price for **nonbinding** constraints are **0**.
 - Changing a RHS value for a **binding constraint** will **change** the feasible

region and the **optimal solution**.

- **calculate profit:** a marginal profit of \$320 and requires: 1 pump (shadow price = \$200), 8 labor (sp = \$16.67), 13 tubing (sp=\$0). Would it be profitable to produce any? Ans: $320 - 200 \cdot 1 - 16.67 \cdot 8 - 0 \cdot 13 = -13.33 = \text{No!}$
- Calculate **max. resource required** so that still **profitable**: unit profit - cost of resources ≥ 0

Simultaneous changes in coefficients in objective function & constraints:

- all changes within the range of allowable increase/decrease (all **reduced cost** $\neq 0$): Solution remains optimal
- **100% rule:** if at least change is outside the range of allowable (variable has reduced cost = 0): $\sum \frac{\text{changes}}{\text{allowable increase / decrease}}$
 - If sum ≤ 1 (100%), **solution remains optimal**; if sum > 1 , solution might be optimal but not guaranteed.

Bound: optimal solution to an LP relaxation of an integer LP problem gives us a bound on the optimal objective value (for max / min, the optimal relaxed objective value is an upper / lower bound on the optimal integer value)

Branch-and-Bound can solve any ILP, it requires the solution of a series of LP problems termed "candidate problems"

- Stopping rules: stop once an integer solution is found that is within some % of the global optimal solution

Transportation model:

- Has constraints for supply at each source and demand at each destination
- All constraints are equalities where supply equals demand
- Constraints contain inequalities where supply does not equal demand.

Transshipment model (for minimum cost)

- Total supply $>$ total demand: inflow-outflow \geq supply / demand (in each node)
- Total supply $<$ total demand: inflow-outflow \leq supply/demand
- Total supply = total demand: inflow-outflow = supply/demand

Assignment model:

- Supply at each source and demand at each destination is 1 unit.

- Supply = demand in a balanced model
- Supply \neq demand in an unbalanced model

The shortest path problem

- One supply node with a supply of -1
- One demand node with a demand of +1
- All other nodes have supply/demand of 0