In this document we limit our analysis to one-dimension scenario. Given N pedestrians distributed uniformly in a corridor with closed boundary conditions and neglect the effects of walls on pedestrians (See Figure 7.1).

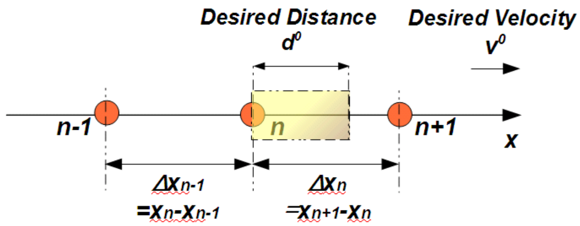


Figure 7.1

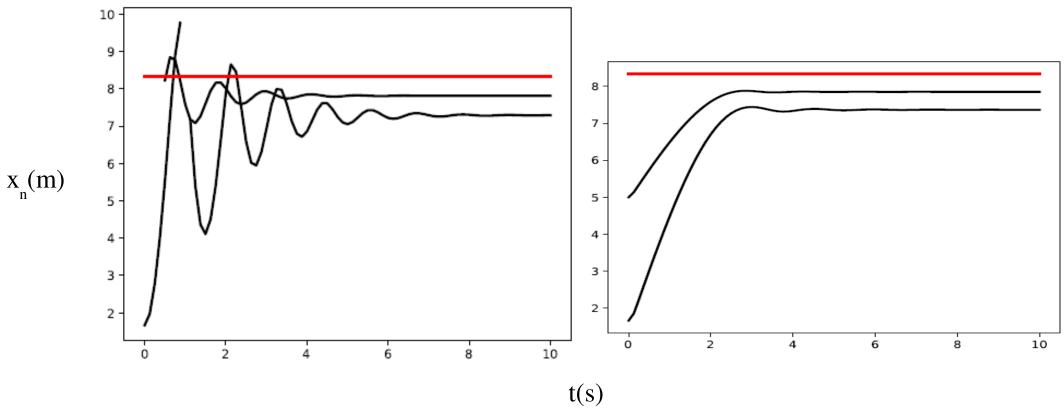
In general, pedestrians are modeled as simple geometric objects of constant size. As shown in Figure 7.1, the physical size of pedestrians is omitted for simplicity and the desired interpersonal distance d_0 is highlighted. Moreover, for interactions among N pedestrians, we assume that pedestrians have foresight such that pedestrian n is only influenced by the pedestrian right in front. Assume $\Delta x_n = x_{n+1} - x_n$, and we have the following equation by Newton Second Law.

Table 7.1 Newtonian Dynamics of Oscillating Walkers

	Equation for Newton Second Law: $m_0 \frac{d^2 x_n}{dt^2} = f^{drv} - f^{rep}$
Interaction Force: Group Social Force (Eq. 7.1)	$m_0 \frac{d^2 x_n}{dt^2} = \frac{m_0}{\tau}(v^0 - v) - A(d^0 - \Delta x_n)\exp\left(\frac{d^0 - \Delta x_n}{B}\right)$
Interaction Force: Group Social Force with V_{ij} Component (Eq. 7.2)	$m_0 \frac{d^2 x_n}{dt^2} = \frac{m_0}{\tau}(v^0 - v) - A(d^0 - \Delta x_n)\exp\left(\frac{d^0 - \Delta x_n}{B}\right) - \frac{\beta}{\Delta x_n} max\left(\frac{dx_n}{dt} - \frac{dx_{n+1}}{dt}, 0\right)$ $max(x, 0) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

In the system of N pedestrians, N equations are formulated and it is feasible to simulate the system by a computer program. The simulation results are illustrated in Figure 7.2 where two pedestrians interact with v_0 =1.2m/s and d_0 =0.6m, and oscillation is observed. However, such oscillations is not a justified phenomenon in reality.

The oscillation phenomenon is common in a control system. From the perspective of control theory, there are a number of methods to offset the oscillation (e.g., PID controller). A widely-used method is adding a derivative term, namely, adding the relative velocity $v_{ij}=v_i- v_j$. In one-dimensional space as introduced above v_{ij} is simplified by $-d\Delta x_n/dt = dx_n/dt - dx_{n+1}/dt$. This term is a best estimate of the future trend of the error based on its current rate of change. It is sometimes called "anticipatory control," which effectively reduce the oscillation when two agents gets closely enough. By using this method a pedestrian will get close to another without oscillation. This derivative term has been applied in several other pedestrian models such as the generalized centrifugal force model (GCFM). Thus, by tuning a force component that is a function of relative velocity v_{ij}, the oscillation phenomenon will be significantly mitigated.



v^0 =1.2 m/s d^0=0.6m τ=0.5s
A_{ij} /m_0 =20 N B_{ij} =3.6m
Group Social Force (Eq. 7.1)

v^0 =1.2 m/s d^0=0.6m τ=0.5s
A_{ij} /m_0 =20 N B_{ij} =3.6m **Beta /m_0** =3.0
Group Social Force with **V**ij Component (Eq. 7.2)

Figure 7.2 Two agents are simulated in 1D space.

As above we discuss the oscillating walkers by control theory and highlight the function of v_{ij} to offset oscillation. There is another perspective of pedestrian modeling. In real-world scenario people usually have a target interpersonal distance and get there without oscillation. In contrast, the oscillation phenomenon implies there may be something wrong with the model or assumption. A major problem is that v_0 and d_0 are assumed to be constant in the above equations. As mentioned before v_0 and d_0 are not physical entities and they actually reflect people's opinions. If v_0 and d_0 are not constant, they may change or oscillates instead of the physical velocity v and distance d=Δx_n.

The idea of varying v_0 and d_0 is not conflicting with the control theory. The control targets are non-physics entities (d_{ij}^0 and v_i^0) and physics entities (d_{ij} and v_i,) are updated to reach the targets. Also, the real situation is that physics entities (d_{ij} and v_i,) are fedback to the human perception so that d_{ij}^0 and v_i^0 are also adjusted in certain conditions. The interplay between the physics entities (d_{ij} and v_i,) and non-physics entities (d_{ij}^0 and v_i^0) forms a close-loop process, and is illustrated in Figure 7.3.

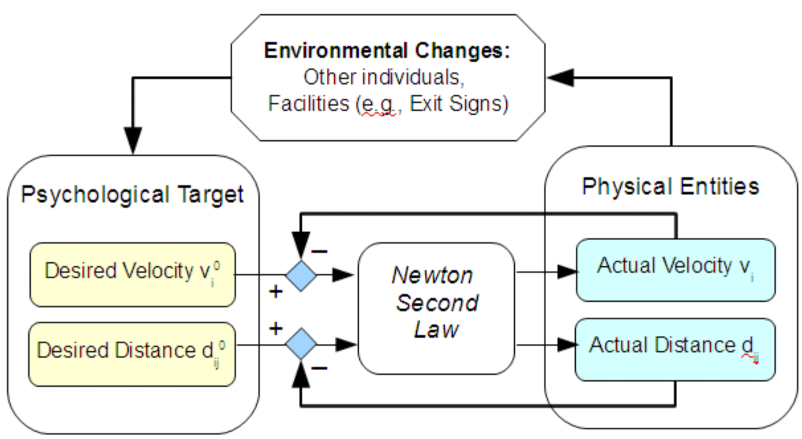


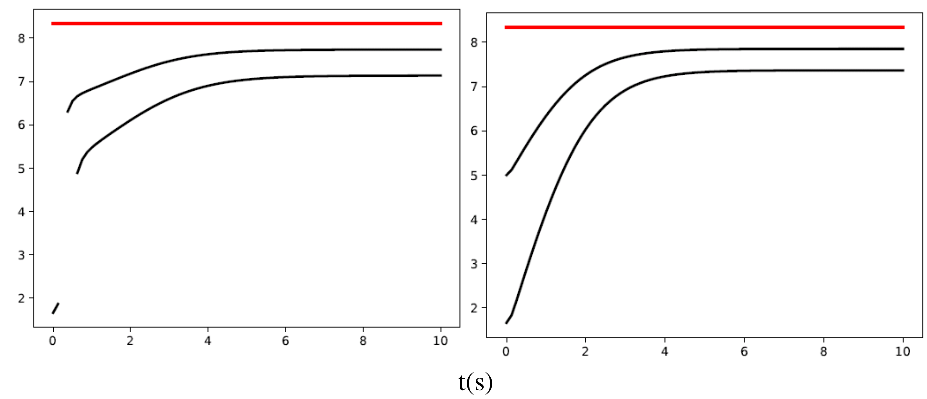
Figure 7.3 Feedback in Close-Loop Process

Based on the above diagram we will present a simplified model where the oscillation is mitigated by decreasing desired velocity v^0 when agents get close to the desired distance sufficiently. In the model $-d\Delta x_n/dt = dx_n/dt - dx_{n+1}/dt$ is integrated as a part of desired velocity v^0, and it also plays an important role of reducing oscillation. This derivative element is the relative velocity of adjacent agents, and it can also be found in other force-based model (Chraibi et. al., 2011)

$$\frac{d^2 x_n}{dt^2} = \frac{(v^0 - v)}{\tau} - \frac{A}{m_0}(d^0 - \Delta x_n)\exp\left(\frac{d^0 - \Delta x_n}{B}\right)$$

$$\left|(d^0 - \Delta x_n)\right| < D: \; v^0 = c_1(d^0 - \Delta x_n) - c_2 \frac{d\,\Delta x_n}{dt} \qquad c_1<0 \text{ and } c_2<0;$$

$$\left|(d^0 - \Delta x_n)\right| \geq D: \; v^0 \text{ and } d^0 \text{ are constant} \tag{7.3}$$



$\boldsymbol{v^0}$ =1.2 m/s $\boldsymbol{d^0}$=0.6m $\boldsymbol{\tau}$=0.5s
$\boldsymbol{A_{ij}}$ **/m₀** =20 N $\boldsymbol{B_{ij}}$ =3.6m
Group Social Force with Dynamic $\boldsymbol{v^0}$ (Eq. 7.3)

$\boldsymbol{v^0}$ =1.2 m/s $\boldsymbol{d^0}$=0.6m $\boldsymbol{\tau}$=0.5s
$\boldsymbol{A_{ij}}$ $\mathbf{/m_0}$ =20 N $\boldsymbol{B_{ij}}$ =3.6m **Beta** $\mathbf{/m_0}$ =16.0
Group Social Force with $\mathbf{V}$ij Component (Eq. 7.2)

Figure 7.4. Group Social Force Model with Varying Desired Velocity

The result of integrating v_{ij} in varying desired velocity (Eq. 7.3) is comparable to that of directly using $\mathbf{v}_{ij}$ Component (Eq. 7.2). The interacting range D in Equation 7.3 determines when the desired velocity becomes a variable. In the above model v^0=0 and d^0=Δx_n is an equilibrium position of the system.

The key idea of our study is modeling pedestrian behavior from human perspective. We will differentiate the pedestrian traffic from vehicle traffic and sufficiently consider human perception and response to surroundings, and this study provides a new angle to understand human behavior in normal and emergency situations.