Understanding Social-Force Model in Psychological Principles of Collective Behavior

Peng Wang

To well understand crowd behavior, microscopic models have been developed in recent decades, in which an individual's behavioral/psychological status can be modeled and simulated. A well-known model is the social-force model innovated by physical scientists (Helbing and Molnar, 1995; Helbing, Farkas and Vicsek, 2000; Helbing et al., 2002). This model has been widely accepted and mainly used in simulation of crowd evacuation in the past decade. A problem, however, is that the testing results of the model were not explained in consistency with the psychological findings, resulting in misunderstanding of the model by psychologists. This paper will bridge the gap between psychological studies and physical explanation about this model. We reinterpret this physics-based model from a psychological perspective, clarifying that the model is consistent with psychological studies on stress, including time-related stress and interpersonal stress. Based on the conception of stress, we renew the model at both micro-and-macro level, referring to agent-based simulation in a microscopic sense and fluid-based analysis in a macroscopic sense. Existing simulation results such as faster-is-slower effect will be reinterpreted by Yerkes–Dodson law, and herding and grouping effect are further discussed by integrating attraction into the social force. In brief the social-force model exhibits a bridge between the physics laws and psychological principles regarding crowd motion, and this paper will renew and reinterpret the model on the foundations of psychological studies.

I. Introduction

Collective behavior of many individuals is a typical research subject in complex systems, and it is of great theoretical and practical interest. One of the major approaches to study such a system is using computer simulation to investigate how collective behavior on a macroscopic scale emerges from individual interactions. This approach of modeling complex systems is usually called agent-based modeling or agent-based simulation, and it has been widely used in many diverse disciplines, including sociology, economics, molecular biology and so forth.

A well-known model to simulate complex dynamics of pedestrian crowd is called social-force model, and it was initially introduced by physical scientists (Helbing and Molnar, 1995; Helbing, Farkas and Vicsek, 2000; Helbing et al., 2002). Very interestingly, certain psychological factors are abstracted in model parameters such as "desired velocity" and "social force." To some extent, these concepts are not physical entities because they describe people's opinions or certain characteristics of human mind. Thus, the model is not typically within the scope of physics study, but in an interdisciplinary domain, and an important question is whether such innovation is consistent with psychological theories or findings.

On one side, the social-force model has been widely used in simulation of crowd evacuation in the past two decades. Many leading simulators use or extend the social-force model as the pedestrian model. One the other side, some controversial issues were raised for the model. For example, the social-force does not obey the Newton 3rd law and this is questionable in physics study. The controversial issues seem difficult to be explained by the model itself, especially from the perspective of physics. Thus, in this contribution we will try to provide a new perspective to understand the model, and the key idea is borrowed from psychological study of stress. The model will be reexplained and renewed in several aspects, referring to both of self-driven force and social force. In order to justify our explanation about stress, we will reiterates the faster-is-slower phenomenon by

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Yerkes-Dodson law, and the faster-is-slower effect actually shows a doorway scenario where excess of stress impairs human performance in the collective sense. The detailed discussion will be presented in Section 3.

The psychological concept of stress contributes to investigating a pedestrian's response and adaption to the environment, and the social-force model is hereby extended to characterize the interplay between individuals and their surroundings. In this paper we mainly study how an individual interacts with the surrounding individuals, and herding effect and group dynamics will be discussed in detail in Section 4. Very importantly, we will introduce a new form of cohesive force which is used to form groups in pedestrian motion. The cohesive force is an extension of the traditional social-force, and it characterizes the social relationship among individuals.

In Section 5 we will aggregate the Newtonian-like individual model into the macroscopic level and a fluid-based analysis of crowd flow is presented in detail. The micro-and-macro link between the social-force model and fluid dynamics is thus explored. The fluid-based analysis continues with the key concept of stress, and it provides an energy viewpoint to describes the self-driven characteristics of crowd dynamics: the psychological drive arises as a potential form of energy, and it will be transformed to energy in kinetic form or static form, resulting in acceleration or compression of the crowd. Based on such fluid-based analysis the doorway scenario is further revisited.

In brief, the collective behavior of crowd has long been investigated in psychology and social science, and there were a great many valuable theories and findings. As the leading model to study complex dynamics of pedestrian crowd, the social force model should be inherently consistent with psychological theories, and this paper will try to establish a bridge between two research areas, and cast a new light on understanding of the social-force model.

II. About the Social-Force Model

Physics is the analysis of nature, conducted to understand how the universe behaves, while psychology is the study of the human mind and behaviors, exploring how human thinks and behaves. In the past decade a pedestrian model involves both of these two subjects in the framework of Newtonian dynamics, and the model is called the social-force model (Helbing and Molnar, 1995; Helbing, Farkas, and Vicsek, 2000; Helbing et al., 2002).

The social-force model presents psychological forces that drive pedestrians to move as well as keep a proper distance with others. In this model an individual's motion is motivated by a self-driven force $\boldsymbol{f}_i^{self}$ and resistances come from surrounding individuals and facilities (e.g., walls). Especially, the model describes the social-psychological tendency of two individuals to keep proper interpersonal distance (as called the social-force) in collective motion, and if people have physical contact with each other, physical forces are also taken into account. Let $\boldsymbol{f}_{ij}$ denote the interaction from individual j to individual i, and $\boldsymbol{f}_{iw}$ denote the force from walls or other facilities to individual i. The change of the instantaneous velocity $\boldsymbol{v}_i(t)$ of individual i is given by the Newton Second Law:

$$m_i \frac{d\,\boldsymbol{v}_i(t)}{dt} = \boldsymbol{f}_i^{self} + \sum_{j(\neq i)} \boldsymbol{f}_{ij} + \sum_{w} \boldsymbol{f}_{iw} + \boldsymbol{\xi}_i \tag{1}$$

where m_i is the mass of individual i, and $\boldsymbol{\xi}_i$ is a small fluctuation force. Furthermore, the self-driven force $\boldsymbol{f}_i^{self}$ is specified by

$$\boldsymbol{f}_i^{self} = m_i \frac{\boldsymbol{v}_i^{\mathbf{0}}(t) - \boldsymbol{v}_i(t)}{\tau_i}\ , \tag{2}$$

This force describes an individual tries to move with a desired velocity $\boldsymbol{v}_i^0(t)$ and expects to adapt the actual velocity $\boldsymbol{v}_i(t)$ to the desired velocity $\boldsymbol{v}_i^0(t)$ within a certain time interval τ_i. In particular, the desired velocity $\boldsymbol{v}_i^0(t)$ is the target velocity existing in one's mind while the actual velocity $\boldsymbol{v}_i(t)$ characterizes the physical speed and direction being achieved in the reality. The gap of $\boldsymbol{v}_i^0(t)$ and $\boldsymbol{v}_i(t)$ implies the difference between the human subjective wish and realistic situation, and it is scaled by a time parameter τ_i to generate the self-driven force. This force motivates one to either accelerate or decelerate, making the realistic velocity $\boldsymbol{v}_i(t)$ approaching towards the desired velocity $\boldsymbol{v}_i^0(t)$. This mathematical description of the self-driven force could be dated back to the Payne-Whitham traffic flow model (Payne, 1971; Whitham, 1974). Sometimes $\boldsymbol{v}_i^0(t)$ is rewritten as $\boldsymbol{v}_i^0(t) = v_i^0(t)\boldsymbol{e}_i^0(t)$, where $v_i^0(t)$ is the desired moving speed and $\boldsymbol{e}_i^0(t)$ is the desired moving direction. In a similar manner, we also have $\boldsymbol{v}_i(t) = v_i(t)\boldsymbol{e}_i(t)$ where $v_i(t)$ and $\boldsymbol{e}_i(t)$ represent the physical moving speed and direction, respectively.

The interaction force of pedestrians consists of the social-force $\boldsymbol{f}_{ij}^{\mathbf{soc}}$ and physical interaction $\boldsymbol{f}_{ij}^{\mathbf{phy}}$. i.e., $\boldsymbol{f}_{ij} = \boldsymbol{f}_{ij}^{soc} + \boldsymbol{f}_{ij}^{phy}$. The social-force $\boldsymbol{f}_{ij}^{\mathbf{soc}}$ characterizes the social-psychological tendency of two pedestrians to stay away from each other, and it is given by

$$\boldsymbol{f}_{ij}^{soc}=A_i\exp\left[\frac{(r_{ij}-d_{ij})}{B_i}\right]\boldsymbol{n}_{ij} \quad \text{or} \quad \boldsymbol{f}_{ij}^{soc}=\left(\lambda_i+(1-\lambda_i)\frac{1+\cos\varphi_{ij}}{2}\right)A_i\exp\left[\frac{(r_{ij}-d_{ij})}{B_i}\right]\boldsymbol{n}_{ij} \tag{3}$$

where $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are positive constants, which affect the strength and effective range about how two pedestrians are repulsive to each other. The distance of pedestrians i and j is denoted by d_{ij} and the sum of their radii is given by r_{ij}. $\boldsymbol{n}_{ij}$ is the normalized vector which points from pedestrian j to i. The geometric features of two pedestrians are illustrated in Figure 1. In practical simulation, an anisotropic formula of the social-force is widely applied where Equation (3) is scaled by a function of λ_i. The angle φ_{ij} is the angle between the direction of the motion of pedestrian i and the direction to pedestrian j, which is exerting the repulsive force on pedestrian i. If $\lambda_i = 1$, the social force is symmetric and $0 < \lambda_i < 1$ implies that the force is larger in front of a pedestrian than behind. This anisotropic formula assumes that pedestrians move forward, not backward, and thus we can differ the front side from the backside of pedestrians based on their movement. Although the anisotropic formula is widely used in pedestrian modeling, it also brings a controversial issue that the anisotropic formula of social force disobeys Newton's 3rd law.

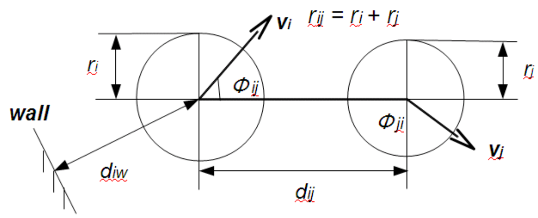


Figure 1. A Schematic View of Two Pedestrians

The physical interaction $\boldsymbol{f}_{ij}^{phy}$ describes the physical interaction when pedestrians have body contact, and it is composed by an elastic force that counteracts body compression and a sliding friction force that impedes relative tangential motion of two pedestrians. Both of them are valid only when $r_{ij}>d_{ij}$. In Helbing, Farkas and Vicsek, 2000 the interaction force is repulsive. The model may also include an attraction force in its original version (Helbing and Molnar, 1995, Helbing et al., 2002 Korhonen, 2017). The interaction of a pedestrian with obstacles like walls is denoted by $\boldsymbol{f}_{iw}$ and is treated analogously, i.e., $\boldsymbol{f}_{iw} = \boldsymbol{f}_{iw}^{soc} + \boldsymbol{f}_{iw}^{phy}$. Here $\boldsymbol{f}_{iw}^{soc}$ is also an exponential term and $\boldsymbol{f}_{ij}^{phy}$ is the physical interaction when pedestrians touch the wall physically.

By simulating many such individuals in collective motion, several scenarios in crowd movement were demonstrated, and one is called the "faster-is-slower" effect. This scenario was observed when a crowd pass a bottleneck doorway, and it shows that increase of desired speed (i.e., $|\boldsymbol{v}_i^0(t)|$) can inversely decrease the collective speed of crowd passing through the doorway. Another paradoxical phenomenon is called "freezing-by-heating," and it studies two groups of people moving oppositely in a narrow passageway, and the simulation shows that increasing the fluctuation force in Equation (1) can also cause blocking in the counter-flow of pedestrian crowd. Other spatio-temporal patterns include herding effect, oscillation of passing directions, lane formation, dynamics at intersections and so forth.

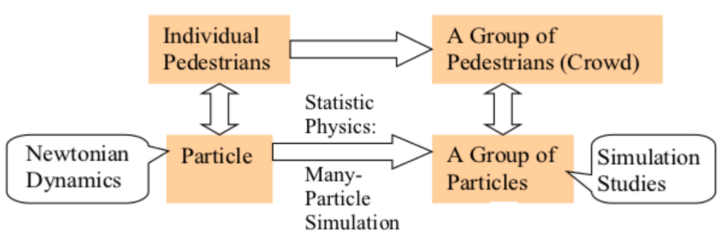


Figure 2. Framework in Helbing, Farkas, and Vicsek, 2000

In the past decade, the social-force model has generated considerable research on evacuation modeling (Helbing and Johansson, 2010), and it has been incorporated into several egress simulators, such as Fire Dynamics Simulator with Evacuation (Korhonen et. al., 2008, Korhonen, 2017), Pedsim, SimWalk, MassMotion (Oasys, 2018), VisWalk and Maces (Pelechano and Badler, 2006). The model has been partly validated based on data sets from real-world experiments. The method of validation involves comparing the simulation of the model with associated observations drawn from video-based analysis (Johansson, Helbing and Shukla, 2007; Johansson et al., 2008).

A criticism about the model, as mentioned before, is that the anisotropic formula of social force disobeys Newton's 3rd law. If the model does not obey Newton 3rd law, it becomes questionable in field of physics studies. Another problem about the social force is the dilemma of choosing proper parameters to avoid both overlapping and oscillations of the moving pedestrians (Chraibi, 2011). Certain scenarios about oscillation of walking behavior are not realistic in the physical world. From the perspective of physics these problems are difficult to explain. However, as mentioned previously, human motion is self-driven and self-adapted, and it is subject to both physical laws and psychological principles. Psychological study will give a new perspective to understand the model and provide us with a new angle to understand the problems. This is why we want to bridge the gap between psychological studies and physical explanation about this model.

III. Psychological Explanation of Self-driven Force and Social-force

This section provides a psychological perspective to understand self-driven force and social-force, and the conception of stress is critically involved. Very importantly we introduce the new concept of desired distance in the social force, which is the counterpart of the desired velocity in the self-driven force, and this innovation is the foundation of our discussion in the next few sections.

3.1 Panic, Stress and Time-Pressure

One problem about the social-force model is that most of the testing results were explained by "panic" behavior of people (Helbing, Farkas and Vicsek, 2000; Helbing et al., 2002; Helbing and Johansson, 2010) while existing egress research clarifies the psychological state of panic occurs relatively rarely in real-world evacuation events (Sime, 1980; Proulx, 1993; Ozel, 2001), and this could cause misunderstanding of the model by social psychologists. Defined psychologically, "panic" means a sudden over-whelming terror which prevents reasoning and logical thinking, and thus results in irrational behavior. Based on Equation (1) and (2), we see that the equations do not imply any irrational behavior aroused by fear, but describe a kind of rational mechanics that govern an individual's motion. Thus, we think that the general use of the term panic is not essential to the social-force model.

By searching in literature of social psychological studies in emergency egress, we think that "stress" is more accurate conceptualizations of the social-force model than "panic." (Sime, 1980; Ozel, 2001). Psychological stress can be understood as the interaction between the environment and the individual (Selye, 1978, Staal, 2004), emphasizing the role of the individual's appraisal of situations in shaping their responses. In Stokes and Kite, 2001, such stress is the result of mismatch between psychological demand and realistic situation, and Equation (2) characterizes the mismatch in terms of velocity: the psychological demand is represented by desired velocity $\boldsymbol{v}_i^0$ while the physical reality is described by the physical velocity $\boldsymbol{v}$. The gap of two variables describes how much stress people are bearing in mind, and thus are motivated into certain behavior in order to make a change in reality. Such behavior is formulated as the self-driven force in Equation (1) and (2).

Furthermore, velocity is a time-related concept in physics and the gap of velocities actually describes a kind of time-related stress, or commonly known as time-pressure. Such a kind of stress is caused by insufficient time when people are dealing with a time-critical situation, and time is the critical resource to complete the task. In sum, although the social-force model is labeled with the term "panic," its mathematical description is not directly related to "panic" in a psychological sense and the self-driven force critically characterizes the psychological concept of stress and time-pressure. This also explains why the model can be well used in simulation of emergency egress because "emergency" implies shortage of time in a process.

3.2 Interpersonal Stress and Social-Force

As above we briefly discuss the time-related stress and explain its relationship to desired velocity. There is another kind of stress originating from social relationship, and it leads to competition or cooperation in crowd movement, and such stress is characterized by the social-force. This subsection will discuss such stress and its relationship with interpersonal distance, and we will introduce a new concept of desired distance in the social force, and it is the counterpart of desired velocity in the self-driven force.

Interpersonal distance refers to a theory of how people use their personal space to interact with surrounding people. In Hall, 1963 the theory was named by proxemics, and it was defined as "the interrelated observations and theories of man's use of space as a specialized elaboration of culture." Proxemics suggests that we surround ourselves with a "bubble" of personal space, which protects us from too much arousal and helps us keep comfortable when we interact with others. People normally feel stressed when their personal space is invaded by others. There are four interpersonal distances mentioned in Hall, 1966: intimate (<0.46m), personal (0.46m to 1.2m), social (1.2m to 3.7m), and public (>3.7m), and each one represents a kind of social relationship between individuals. Here we highlight two issues as below.

(a) The interpersonal distance is object-oriented. For example, we usually keep smaller distance to a friend than to a stranger, and such distance is an indication of familiarity. As named by personal distance (0.46m to 1.2m) in proxemics, this range is widely observed as the distance to interact with our friends or family, and normal conversations take place easily at this range.

(b) The interpersonal distance reflects a kind of social norms, and it is also redefined in different cultures. For example, in a crowded train or elevators, although such physical proximity is psychologically disturbing and uncomfortable, it is accepted as a social norm of modern life. Also, it is also known that the male and female commonly keep larger distance in public place in Muslim culture than other cultures. In brief although proximal distance origins from basic human instincts, it is also widely redefined in different social norms and cultures nowadays.

Proxemics implies that when the interpersonal distance is smaller than the desired, people feel stressed. Repulsion comes into being in this situation, and repulsion increases when the distance further decreases. This theory justifies the assumption of repulsive social-force in Equation (3). However, the repulsion is not related to physical size of two people (i.e., r_{ij}), but the social relationship, culture and occasions. Comparing social force with self-driven force, we suggest that there should be a subjective concept of desired distance d_{ij}^0 in the social force, and it replaces r_{ij} in Equation (3). Here d_{ij}^0 is the target distance that individual i expects to maintain with individual j. This distance describes the desired interpersonal distance when people interact, and it is a function of the social relationship of individual i and j as well as the culture and social occasions. If we keep using the exponential form in Equation (3), the social force is rewritten as

$$\boldsymbol{f}_{ij}^{soc} = A_i \exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_i}\right]\boldsymbol{n}_{ij} \quad \text{or} \quad \boldsymbol{f}_{ij}^{soc} = \left(\lambda_i + (1-\lambda_i)\frac{1+\cos\varphi_{ij}}{2}\right) A_i \exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_i}\right]\boldsymbol{n}_{ij} \tag{4}$$

Similar to desired velocity $\boldsymbol{v}_i^0$, the desired distance d_{ij}^0 is the target distance in one's mind, specifying the distance that one expects to adapt oneself with others. The physical distance d_{ij} is the distance achieved in the reality. The gap of d_{ij}^0 and d_{ij} implies the difference between the subjective wish in one's mind and objective feature in the reality. Similar to $\boldsymbol{v}_i^0 - \boldsymbol{v}_i$, as an indication of time-related stress concerning emergencies, $d_{ij}^0 - d_{ij}$ is an indication of interpersonal stress related to the social composition of crowd. Such stress depends on the intrinsic social characteristics of the crowd, not directly related to the emergency situation. Here A_i and B_i are parameters as introduced before, and $\boldsymbol{n}_{ij}$ is the normalized vector which points from pedestrian j to i. In a similar manner, an anisotropic formula of the social-force is also modified in Equation (4). The social force also functions in a feedback manner to make the realistic distance d_{ij} approaching towards the desired distance d_{ij}^0. A difference is that $\boldsymbol{v}_i^0$ and $\boldsymbol{v}_i$ are vectors while d_{ij}^0 and d_{ij} are scalars.

Although a major difference exists between the concepts of r_{ij} and d_{ij}^0, most of the simulation results in Helbing, Farkas and Vicsek, 2000 still stand. In fact, the coding framework of social-force model is not affected when r_{ij} is replace by d_{ij}^0. When realizing the model in computer programs, r_{ij} and d_{ij}^0 are exactly at the same position in coding work, and we can simply have $d_{ij}^0 = r_{ij} \cdot c_{ij}$ to extend the traditional social force to the new force, and $c_{ij}>1$ is a scale factor.

In a psychological sense d_{ij}^0 and $\boldsymbol{v}_i^0$ are both subjective concepts which exist in people's mind, and they characterize how an individual intends to interact with others and environment. As a result, the social-force given by Equation (4) and the self-driven force are both subjective forces which are generated involving one's mental activities and opinions. In a physics sense the subjective forces are generated by the foot-floor friction, which exactly obey physics laws. The social-force model thus exhibits a bridge between the physics laws and psychological principles regarding crowd motion.

3.3 Faster-is-Slower Effect and Yerkes–Dodson law

In this subsection we would like to further justify the social-force model by the concept of stress, and we will investigate a typical scenario of crowd evacuation. This scenario was named by "faster-is-slower" effect in Helbing, Farkas and Vicsek, 2000, and it refers to egress performance when a large number of individuals pass through a narrow doorway. The simulation result shows that the egress time may inversely increase if the average desired velocity keeps increasing. In other words, egress performance may degenerates if the crowd desire moving too fast to escape. We will explain the simulation result from the psychological perspective of stress and time-pressure. In particular, this scenario reiterates an existing psychological knowledge: moderate stress improves human performance (i.e., speeding up crowd motion); while excessive stress impairs their performance (i.e., disorders and jamming), and this theorem is widely known as Yerkes–Dodson law in psychological study (Yerkes and Dodson, 1908; Teigen, 1994; Wikipedia, 2016). In addition, this subsection will keep using psychological theory to understand the simulation result of the social force model, and if readers are not quite interested in the doorway scenario, you can omit this section and move on to the next section without any problems.

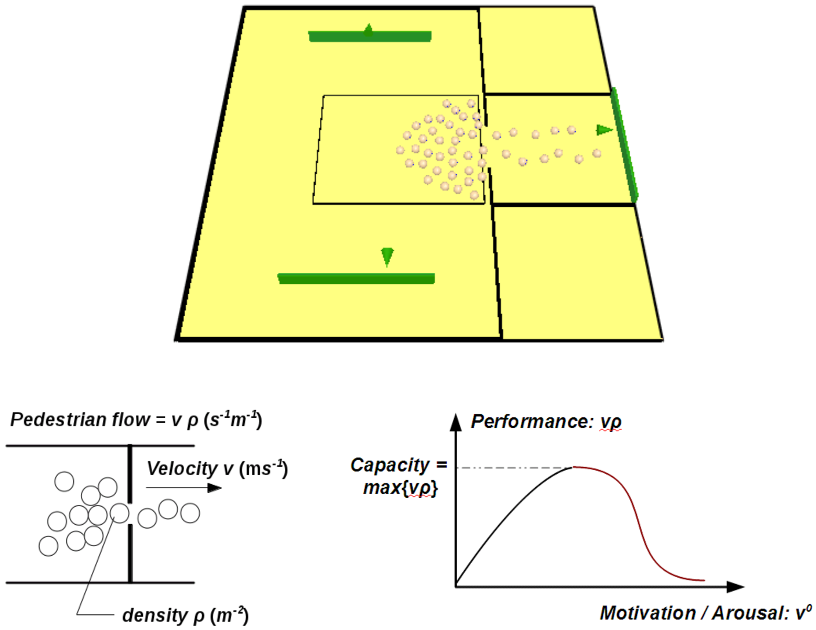


Figure 3. About crowd movement in a passageway

Yerkes–Dodson law states the relationship between arousal level and performance: performance increases with arousal, but only up to a point. Beyond the point the arousal becomes excessive and the situation is much stressful such that performance diminishes. The arousal level indicates the intensity of motivation and it depends on stimulus strength from environment (e.g. alarm or hazard). Motivation leads to behavioral response. In the social-force model, the arousal or motivation is represented by desired velocity $\boldsymbol{v}^0$, and the behavioral response is represented by actual velocity $\boldsymbol{v}$. The performance of crowd escape is measured by pedestrian flow ρv at the doorway, describing how many individuals pass through a doorway of unit width per time unit (See Figure 3, ρ and v are the crowd density and physical speed nearby the doorway). The pedestrian flow is limited by the passage capacity, which determines the maximal pedestrian flow that people are able to realize in collective motion (Wang et al., 2008). In other words, the passageway capacity determines the critical point in Yerkes–Dodson law, indicating whether the collective motivation is excessive or not.

(a) When the passage capacity is sufficient, v increases along with v^o while ρ can be adjusted such that the physical distance among people is psychologically comfortable. As a result, people are able to move as fast as desired while still keep proper interpersonal distance. This scenario corresponds to the increasing segment of the curve in Figure 3, and we can call it faster-is-faster effect in this paper. This effect has been partly observed in real-world experiments in Daamen and Hoogendoorn, 2012[1], and it is also justified by the simulation result in Høiland-Jørgensen et al., 2010. Although the phenomenon was not emphasized in Helbing, Farkas, and Vicsek, 2000, we can still infer this effect from the testing result in the paper[2].

(b) When the passage is saturate, the physical speed v and density ρ reach the maximum and the pedestrian flow ρv is the maximal. In this situation further increasing v^o will compress the crowd and increase the repulsion among people. As the repulsion increases, the risk of disorder and disaster at the bottleneck increases correspondingly (e.g., jamming and injury). If such disastrous events occur, the moving crowd will be significantly slowed down and the faster-is-slower effect comes into being, and this corresponds to the decreasing segment of the curve in Figure 3.

In sum, as motivation level v^0 increases, there are two scenarios as introduced above. The relationship between v^0 and performance ρv is depicted by an inverted-U curve as shown in Figure 3. Here the motivation level v^0 especially depends on environmental stressors, which are any event or stimulus perceived as threats or challenges to the individuals. For example, in

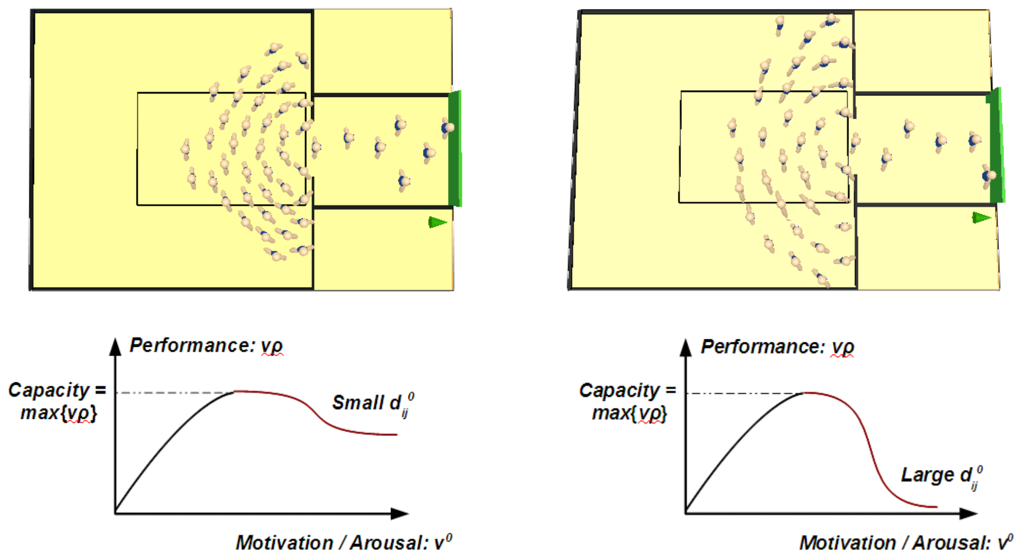
[1]In Daamen and Hoogendoorn, 2012 the crowd are excited and the designed experiment does not typically refer to the stress and time-pressure. Exciting the crowd is similar to heating the particles, which can also cause jamming in the bottleneck ("freezing-by-heating effect in Helbing et al., 2002). However, this effect can be considered as an extension of the faster-is-slower effect. Please refer to Section 5.4 for more details.

[2]In Helbing, Farkas, and Vicsek, 2000 the simulation results demonstrated how the desired velocity v^d affects the "physical flow ρv divided by the desired velocity" in order to emphasize the faster-is-slower effect (See Figure 1(d) in Helbing, Farkas, and Vicsek, 2000). Also, there is an ascending portion of the curve in Figure 1(d) as the desired velocity increases initially, and this portion actually exhibited the faster-is-fast effect. We infer that the ascending of the curve will be more significant if the physical flow ρv ($s^{-1}m^{-1}$) is directly plotted.

emergency evacuation a sort of important stressors are hazardous condition (e.g., fire and smoke). Perception of hazard will increase the arousal level so that the desired velocity v^0 increases. In addition, whether faster-is-slower effect emerges also depends on whether people tend to compete or cooperate with each other. If interpersonal repulsion does not significantly increase, the faster-is-slower phenomenon will not emerge (Høiland-Jørgensen et al., 2010). In other words, the competition or cooperation of people is another important issue, and the desired interpersonal distance d_{ij}^0 plays an important role here.

The desired interpersonal distance d_{ij}^0 critically represents the social relationship of individual i and individual j. The smaller d_{ij}^0 is, the closer is the relationship of individual i and individual j. In crowd evacuation, small value of d_{ij}^0 implies familiarity of evacuees and they tend to cooperate rather than compete with each other. As a result, when they pass through a bottleneck, even if they get close to each other, repulsion will not significantly increase. The faster-is-slower effect is thus mitigated and the relationship of motivation (i.e., v^o) and the pedestrian flow (i.e., ρv) should be replotted as shown in Figure 4(a). In contrast, large value of d_{ij}^0 implies people are mainly composed of strangers and it is more likely for them to compete than cooperate at the bottleneck, resulting in higher probability of faster-is-slower effect at a bottleneck (See Figure 4b).

To testify the above theory, we slightly modify the source program of FDS+Evac and implement the desired interpersonal distance in the program. Below is the simulation result by FDS+Evac. The left diagram corresponds to small d_{ij}^0, where we specify $d_{ij}^0 = 2\cdot r_{ij}$, while the right diagram corresponds to relatively large d_{ij}^0, where $d_{ij}^0 = 3\cdot r_{ij}$ is used. Here r_{ij} is the sum of the radii of individual i and j, namely, $r_{ij} = r_i + r_j$ (See Figure 1). The comparative results suggest that increasing desired distance d_{ij}^0 will decrease the pedestrian flow rate at the bottleneck.



(a) (b)

Figure 4. About Social Force and Faster-Is-Slower Effect

In sum, mismatch of psychological demand and physical reality results in a stressful condition. In emergency egress, such stress is aroused from environmental factors in two categories. A major kind of factors include hazard conditions and alarm, resulting in impatience of evacuees and causing time-pressure. The psychological model refers to the fight-or-flight response (Cannon, 1932), where hazardous stimulus motivate organisms to flee such that the desired velocity increases. Another kind of stress is aroused from surrounding people, resulting in interpersonal stress in collective behavior. Such stressor makes one repulsive with others, and it determines whether people tend to compete or cooperate with each other. In brief, the simulation result about the faster-is-slower effect reiterate Yerkes–Dodson law with respect to two-dimensional stressors.

From the perspective of stress Yerkes–Dodson law is also understood by dividing stress into eustress and distress (Selye, 1975): stress that enhances function is considered eustress. Excessive stress that is not resolved through coping or adaptation, deemed distress, may lead to anxiety or withdrawal behavior and degenerate the performance. Thus, stress could either improve or impair human performance. Traditionally, this psychological theorem mainly refers to performance at individual level, such as class performance of a student or fight-or-flight response of an organism. The simulation of social-force model reiterates this well-known psychological knowledge in collective behavior. In brief, the testing result of social-force model agrees with Yerkes–Dodson law and it provides a new perspective to understand this classic psychological principle.

IV. HERDING EFFECT AND GROUP DYNAMICS

Stress is perceived when we think the demand being placed on us exceed our ability to cope with, and it can be external and related to the environment, and it becomes effective by internal perceptions. The motivation level v_i^0 and d_{ij}^0 are thus the result of such perception, and are adapted to the environmental stressors. As a result, stress refers to agents' response and adaption to the environment, and it is feasible to extend social-force model to characterize the interplay between individuals and their surroundings. As below we present a diagram to describe the interplay between individuals and their surroundings based on the extended social-force model.

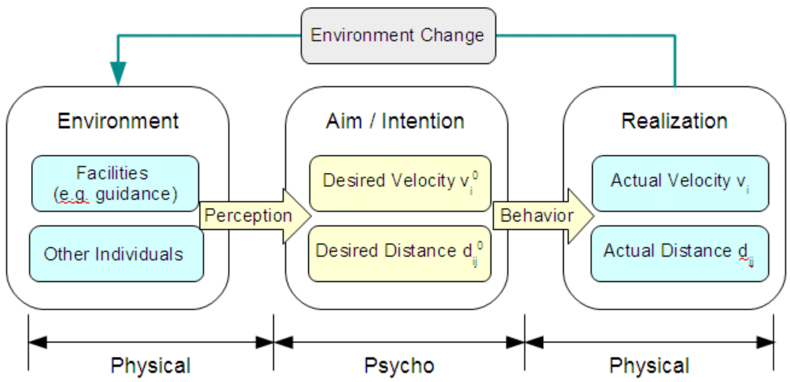


Figure 5. Perception and Behavior in a Feedback Mechanism

In the above diagram environmental factors include surrounding people and facilities (e.g., alarm, guidance and hazard in egress). The resulting pedestrian motion is a response to stressors in environmental conditions, and $\boldsymbol{v}_i^0$ and d_{ij}^0 could vary both temporally and spatially. As for the facilitates (e.g, hazard, exit signs), we will briefly explain how to apply the above model in simulation of crowd evacuation as below.

(a) About Hazard: When hazard propagates towards people, people normally desire moving faster to escape from danger or search for familiar ones to escape together. As a result, the desired velocity $\boldsymbol{v}^0$ increases in order to escape from danger while d_{ij} will decrease in order to make people cohesive to stay together. The direction of $\boldsymbol{v}^0$ is pointing to the place of safety where the hazard level decreases, but it also depends on the high-level decision making process such as exit choice in egress.

(b) Guidance: Guidance such as exit signs will guide people to place of safety and it changes the destination of escape. The guidance thus affects the direction of desired velocity $\boldsymbol{v}_i^0$ with certain probability. Existing research shows that people tend to use the familiar path and trust more on personalized guidance (Kuligowski, 2009). Such factors are integrated into a probabilistic graph model (Wang et al., 2008) to affect the direction of desired velocity.

(c) Social Norms: In the field of social psychology, social norms are defined as "representations of appropriate behavior" in a certain situation or environment. From the perspective of crowd modeling, the social norm is indicated by d_{ij}^0. For example in elevators or entrance of a passageway, people commonly accept smaller proximal distance. Thus, the desired interpersonal distance is smaller there, and d_{ij}^0 is to be scaled down proportionally in these places. In brief, d_{ij}^0 is occasion-dependent, and it varies along with locations. In emergencies the social norm is modified such that competitive behavior may emerge, and the model is thus useful to investigate crowd behavior in emergencies.

In addition to surrounding entities such as hazard, alarm and guidance (e.g., exit signs), another important factor is human factor. That is, how an individual's opinion and behavior are affected by surrounding people, and this issue leads to grouping dynamics and self-organizing phenomenon in pedestrian crowd and we will next elaborate this issue in detail.

4.1. Herding Behavior and Opinion Dynamics

Herding is especially evident when people are responding to an emergency (Low, 2000). Emergency implies time-pressure as mentioned before and excessive time-pressure weakens the ability of logical thinking and reasoning, and independent decision making is more difficult in stressful conditions. Thus, people are more inclined to follow others (e.g., neighbors' decisions) rather than make decisions by themselves. Based on the social-force model in Helbing, Farkas and Vicsek, 2000 and Helbing et al., 2002, the herding effect is modified as below.

$$\boldsymbol{e}_i^{\boldsymbol{0}}(t+1)=Norm[(1-p_i)\boldsymbol{e}_i^{\boldsymbol{0}}(t)+p_i[\boldsymbol{e}_j(t)]_i] \qquad [\boldsymbol{e}_j(t)]_i=\sum_{R_i}\boldsymbol{e}_j(t) \tag{5}$$

The above equation characterizes that an individual desired direction e^0_i is updated by mixing itself with the average direction $e_j(t)$ of his neighbors j within radius R_i. Norm[] represents normalization of a vector. Both options are weighted with some parameter $(1-p_i)$ and p_i, and two opinions follow two-point distribution with probability $(1-p_i)$ and p_i, and e^0_i is updated by the statistical average. As a consequence, individualistic behavior is dominant if p_i is low, but herding behavior emerges if p_i is high. In Helbing, Farkas and Vicsek, 2000 p_i is considered to indicate one's panic level. Similarly we can understand that p_i is a stress indicator and people are more inclined to follow others when they are stressed or under much time-pressure.

In an addition, the mixture of two choices may not only refer to moving directions, but also moving speed. Therefore, the desired speed or actual speed are also taken into account in herding effect. For example, if one's neighbors all move fast towards somewhere, he or she probably also wants to speed up to carry on with others. In Lakoba, Kaup and Finkelstein, 2005, the speed was taken into account based on Helbing, Farkas and Vicsek, 2000. We present the following equation to describe how moving speed evolves in herding behavior.

$$v_i^0(t+1)=(1-p_i)v_i^0(t)+p_i[v_j(t)]_i \qquad [v_j(t)]_i=(\sum_{R_i}v_j(t))/N \tag{6}$$

where the magnitude of velocity is the moving speed, i.e., $|\boldsymbol{v}_i^0| = v_i^0$ and $|\boldsymbol{v}_i| = v_i$.

Based on Equation (5) and (6) we know how an individual's velocity is affected by his neighbors in terms of both direction and speed. Here we have the following remarks.

One may notice that e^0_i is updated based on the sum of others' moving directions while v^0_i is updated based on the arithmetic average of others' moving speed. The explanation is given as follows. If there are 6 people within radius R_i and they all move towards a common destination, the arithmetic average mean of e_j(t) is the same as anyone among them. However, from the perspective of social-psychological viewpoint 6 people should have more impact on one's opinion than 1 or 2 people. The more people are there, the more impact is on your opinion, and it is an established psychological finding which suggests that . social impact within a social structure is affected by strength (S), immediacy (I) and number of people (N), and this finding is named by the social impact theory developed by Bibb Latané in 1980's. In particular, the greater the number of people acting on the target. in a social situation, the greater the impact would be. Thus, as for the desired moving direction, the effect is addible in a sense that the arithmetic average cannot measure such impact because arithmetic average of 6 people is equal to one person in the above example. So we suggest that e_j(t) should be updated by the sum rather than the average in order to reflect such crowd effect. In contrast, as for the moving speed, it is clearly not a addible effect. If an individual is surrounded by many people moving faster than him, the individual may also want to speed up his pace, and the desired speed refers to the average speed of others, not the sum. Thus, it is reasonable to update v^0_i by the arithmetic average.

Equation (5) and (6) assume that one's opinion is weighted with parameter $(1-p_i)$ and p_i, and $\boldsymbol{v}^0_i$ is updated by the weighted average. An important issue relates to the meaning of parameter p_i and how it evolves in the simulation. In a statistical sense p_i means probability that individual expects to follow others. In Helbing, Farkas and Vicsek, 2000 and Helbing et al., 2002, p_i is given by ratio of $(v_i^0 - v_i)/v_i^0$, and it is called a “nervousness" parameter. This ratio critically affects several testing results in their work. Because the gap of $(v_i^0 - v_i)$ is measured in the parameter, it also can be understood as an indicator of one's stress level, and the parameter is normalized by dividing the gap by v_i^0. As a result, the “nervousness” parameter can be explained as a normalized stress indicator, and it shows that people are more inclined to follow others when they feel more stressed in an emergency situation. This is a reasonable assumption and is consistent with psychological findings.

From the perspective of opinion dynamics the effective range of p_i can be further extended as $p_i \in [-1, 1]$. As a result, p_i is not a probability measure, but a weight parameter which prompts the individuals' opinions to either converge or diverge. The negative value of p_i implies that others' standpoint has an inverse impact on one's opinion. Thus, the more others state their opinion to individual i, the more individual i will reject it and hold more firmly on his or her own standpoint. In sum, $p_i \in [-1, 1]$ implies an assumption that interactions bring opinions either closer to each other, or more apart from each other.

Table 1.

$-1<p<0$	$p=0$	$0<p<1$
Against others' opinions	Hold his own opinion and do not care about others' opinions	Support others' opinions

Very interestingly, Equation (5) and (6) implies that one affects the surrounding people and is also affected by surroundings. Such interaction is mutual in nature, but it is not symmetric, and thus does not obey Newton 3rd Law.

Last but not least, whether Equation (5) and (6) will result in convergence of desired motion in a collective sense is another interesting topic to study. Current simulation results seem to be chaotic. However, existing psychological studies suggest that individuals' opinions may have the tendency of converging to a crowd opinion when they interact in certain circumstances. This means that everyone's desired moving direction and speed may converge to a common value and emerge a crowd opinion. Extensive research has been conducted in the field of opinion dynamics.

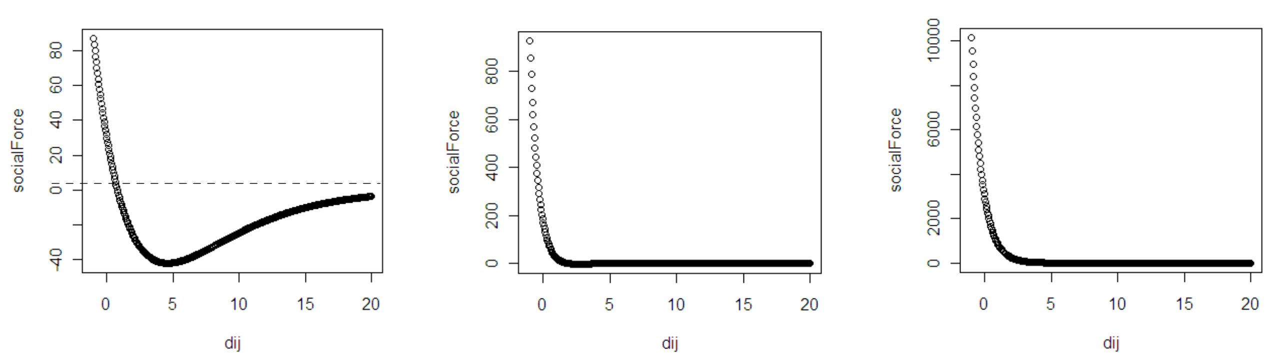
4.2 Cohesive Force

In a group individuals exhibit some degree of social cohesion based on their relationship and they are more than a simple collection or aggregate of individuals. To model group dynamics, attraction is necessarily taken into account in interaction of individuals. For example, attraction will make acquainted people to join together and possibly form a group. In Helbing and Molnar, 1995 and Helbing et al., 2002 attraction was considered, but separate from the social force. In this paper attraction and repulsion are put in the same social context: repulsion makes people to keep proper distance while attraction makes them cohesive and form groups. Thus, this subsection integrates attraction into the social force based on the concept of desired interpersonal distance. The resulting force is either repulsive or attractive, and it is capable to represent a cohesive force in grouping dynamics. The social-force is modified as below.

$$\boldsymbol{f}_{\boldsymbol{ij}}^{\boldsymbol{soc}} = A_{ij}(d_{ij}^0 - d_{ij})\exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_{ij}}\right]\boldsymbol{n}_{\boldsymbol{ij}} \tag{7}$$

When d_{ij} is sufficiently large, the social force tends to be zero so that individual *i* and individual *j* have no interaction. This trend is the same as the repulsive social force as given by Equation (3). If d_{ij} is comparable to d_{ij}^0, interaction of individual *i* and *j* comes into existence. If $d_{ij}^0 < d_{ij}$, the social force is attraction whereas it is repulsion if $d_{ij}^0 < d_{ij}$. The attraction reaches the maximal when $d_{ij}^0 - d_{ij} = \boldsymbol{B}_{\mathbf{ij}}$, and the maximal is $A_{\mathbf{ij}}B_{\mathbf{ij}}\exp(1)$. The desired distance d_{ij}^0 makes the curve move horizontally with a certain interval. The curve shape is affected by parameter $A_{\mathbf{ij}}$ and $B_{\mathbf{ij}}$. $A_{\mathbf{ij}}$ is a linear scaling factor which affects the strength of the force whereas $B_{\mathbf{ij}}$ determines the effective range of the interaction. In an addition, the force is approximated by a linear form when $d_{ij}^0 \approx d_{ij}$, and such a linear approximation will be useful in our later discussion.

Two plots of Equation (7) are illustrated as below: Figure (6a) shows that individual *i* is attracted by individual *j* when they stay close sufficiently, and thus individual *i* is probably familiar with individual *j*. In contrast Figure (5b) does not show such relationship because there is almost no attraction when they are close enough, and thus they are probably strangers.



$\boldsymbol{d_{ij}^0}$ *=0.8m* $\boldsymbol{A}_{\mathbf{ij}}$ =30 N $\boldsymbol{B}_{\mathbf{ij}}$ =3.8m
(a) Familiarity

$\boldsymbol{d_{ij}^0}$ *=1.8m* $\boldsymbol{A}_{\mathbf{ij}}$ = 10 N $\boldsymbol{B}_{\mathbf{ij}}$ = 0.8m
(b) Stranger

$\boldsymbol{r}_{ij}$ *=0.3m* $\boldsymbol{A}_{\mathbf{ij}}$ = 2000 N $\boldsymbol{B}_{\mathbf{ij}}$ = 0.8m
(c) Traditional Social Force

Figure 6. Social force from individual *j* to individual *i*

In the above curve the negative segment represents attraction (See Equation 3 and 7), or as called cohesive social-force. In contrast the positive segment denotes repulsion, or as called repulsive social-force. In an addition, when two individuals are strangers, there is almost no attraction as shown in Figure 5(b), and their interaction force is mainly repulsive.

Equally importantly, the gap between d_{ij}^0 and d_{ij} is expressed in Equation (7), and the interpersonal stress is characterized in consistency with our previous discussion. The gap of d_{ij}^0 and d_{ij} is either negative or positive, meaning that being too far away or too close to someone result in stress in proximity. Keeping proper distance with others is the way to protect us from too much arousal, and this is evident in psychological study because being isolated or overcrowded can both lead to stressful conditions.

In addition, compared with the new force given by Equation (7) the traditional formula of social force partly plays a role of collision avoidance because it is only calculated based on the physical size of individual agents (i.e., r_{ij}), and it fits to model crowd movement in high density. As for the new formula, the desired distance d_{ij}^0 is larger than r_{ij}, and parameter A_i and B_i are thus in different value. This issue will be further discussed in detail in numerical testing results.

Another important issue is that the exponential form of social-force is not well justified. Existing psychological research does not provide enough evidence to justify the exponential description as above. In this paper we will keep the assumption, but further justification is still necessary.

In addition, Equation (7) implies that d_{ij}^0 may be different from d_{ji}^0. As a result, the social-force between two individuals is not balanced, i.e., $d_{ij}^0 \neq d_{ji}^0$ and $f_{ij}^{soc} \neq f_{ji}^{soc}$. Thus, Newton third law does not hold for social force. Here an important issue is whether the social-force model means that pedestrian motion does not obey Newton 3rd law.

In a psychological sense d_{ij}^0 is a subjective concept which exists in people's mind, and it characterizes how an individual intends to interact with others. As a result, the social-force given by Equation (7) and the self-driven force are both subjective forces which are generated involving one's intention and opinion. In a physics sense the subjective forces are generated by the foot-floor friction, which exactly obey Newton's laws. In other words Newton 3rd law still stands in pedestrian modeling at the physical level where the social force is viewed as a part of foot-floor friction. At another level where consciousness and opinions are involved to characterize how the social-force is generated in one's mind, Newton's 3rd law is not applied. Thus, the social-force model exhibits a bridge between the physics laws and psychological principles regarding crowd motion.

4.3 Grouping Dynamics

With combination of social cohesion and herding behavior, a kind of convergent pattern is supposed to emerge in a crowd. Here the social cohesion and herding effect are related but different concepts in group formation. Social cohesion describes the social relationship of individuals, and it emphasizes whether there is a social tie between individuals, and such a social tie facilitates to form a group. The herding effect, or generally considered as opinion dynamics, emphasizes how an individuals' opinion interacts with others' to form a common motive or destination. You may meet your friend on the street, but if you do not have a common destination, you and your friend usually head to each destination individually after greeting or talking briefly. Another example is evacuation of a stadium where people follow the crowd flow to move to an exit. There are a multitude of small groups composed of friends or family members, and they keep together in egress because of their social relationship. These small groups also compose a large group of evacuees, and herding behavior widely exists among these small groups, contributing to form a collective pattern of motion. In brief, the cohesive force makes individuals socially bonded with each other, and it emphasizes the social relationship of individuals. Herding effect does not focus on such social relationship, but emphasizes people tend to follow their neighbors' characteristic, and thus help to form a common motive.

Considering a group composed by n individuals, the social relationship of the group members is described by a $n \times n$ matrix D^0, of which the element is d_{ij}^0. In a similar way, there are $n \times n$ matrices A and B, and the elements are A_{ij} and B_{ij}, respectively. Generally speaking, D^0, A and B are asymmetrical.

$$\boldsymbol{D^0} = [d_{ij}^0]_{n\times n} \qquad \boldsymbol{A} = [A_{ij}]_{n\times n} \qquad \boldsymbol{B} = [B_{ij}]_{n\times n} \tag{8}$$

By using the matrices D^0, A and B many kinds of social relationship can be modeled such as children-parents group, the leader-follower group and so forth. Below is a simulation implemented in FDS+Evac, and the built environment is from an testing example in Pan et. al., 2007. The aforementioned cohesive force and is applied to the simulation, and several groups are identified in the testing result. Some small groups merge into a large group and regrouping may occur at intersections or at bottleneck areas when many groups meet there. In sum, the groups are not static concepts in our model, but dynamically changing and self-organized during the movement.

Based on Equation (2) and (7), an individual's motion is classified into two types. One type of motion is mainly motivated by the self-driven force, not by surrounding people. The other type of motion is motivated by surrounding people, and thus the interactive force is predominant, either cohesive or repulsive social force. In a general sense, an individual's motion is a combination of both types, but we can usually differentiate such two types in simulation and identify whether one's motion is active or passive.

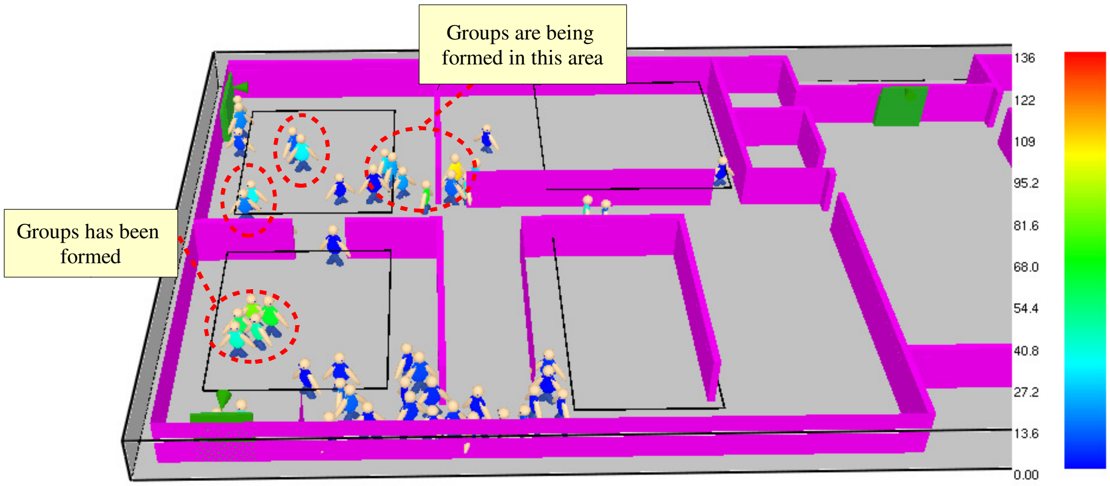


Figure 7. Simulation of Group Dynamics in FDS+Evac

In evacuation simulation Equation (7) and (8) can be better explained by flight-or-affiliation effect in psychological studies. The self-driven force motivates one to flee while the cohesive force makes one affiliated with others. This effect agrees with social attachment theory in psychological study (Mawson, 2007; Bañgate et al., 2017). The social attachment theory suggests that people usually seek for familiar ones (e.g., friends or parents) to relieve stress in face of danger, and this is rooted from our instinctive response to danger in childhood when a child seek for the parents for shelter. Affiliated with familiar and trust individuals relieves our stress. Thus, different from the fight-or-flight response (Cannon, 1932), the modified social model well agrees with the flight-or-affiliation effect. Therefore, the pedestrian model presented above is especially useful to model crowd behavior in pre-movement stage in crowd evacuation (Sorensen, 1991, Kuligowski, 2009). In brief, when the alarm or hazard is detected, people usually do not head to exits immediately, but go to find their friends or trust ones to form groups. Such grouping effect usually delays the movement towards exits, and thus is called pre-movement stage. Thus, the new model contributes to modeling the crowd behavior in pre-movement period and will be useful to investigate how the initial delay is formed and influenced by the group dynamics.

V. Stress and Fluid Dynamics of Pedestrian Crowd

In this section the microscopic model of individual movement will be translated to a macroscopic description of crowd movement. The concepts of desired velocity, desired distance and stress measures will be abstracted to the macroscopic level correspondingly. A fluid-based analysis is essentially presented to describe how psychological intention of people interacts with physical characteristics of crowd motion (e.g., crowd speed and density), and it is an extension of Payne-Whitham flow model (Payne, 1971; Whitham, 1974, Helbing, 2001). Very importantly, fluid-based analysis enables us to have an energy viewpoint to understand crowd dynamics at the macroscopic level, and also provides a new practical perspective to prevent or mitigate the crowd disaster. The general framework of the fluid-based analysis is illustrated as below.

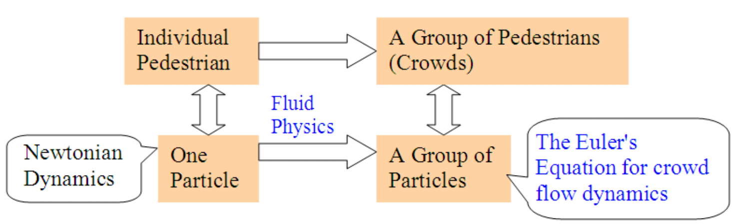
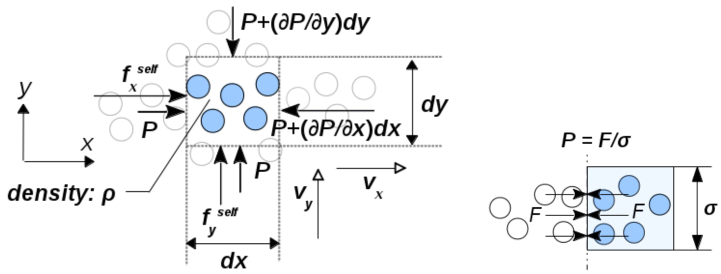


Figure 8. Fluid-Based Analysis of Crowd Movement

5.1. Crowd Fluid Dynamics in 2-Dimensional Space

As a crowd move on a planar surface, their movement can be considered as mass flowing with a specific rate in a two-dimensional space. The specific characteristics of crowd motion include:

(a) Flow density and mass: The flow density is the number of pedestrians per area unit, and it is defined by $\rho=(\mathrm{d}N)/(\mathrm{d}x\mathrm{d}y)$, where $\mathrm{d}N$ is the number of pedestrians in the area of $\mathrm{d}x\mathrm{d}y$ (Figure 9). The crowd density characterizes the distance of people. Let m_0 denote the average individual mass in the crowd, and the mass of the flow in area of $\mathrm{d}x\mathrm{d}y$ is $m=m_0\rho\mathrm{d}x\mathrm{d}y$.



(a) (b)

Figure 9. Crowd movement along a passageway

(b) Interactions: As people move collectively, their physical interactions of people are characterized by surface pressure $\boldsymbol{P}$. It is a two-dimensional analog of common pressure concept in three-dimensional space, and it is the lateral force per unit length applied on a line perpendicular to the force, namely $P=F/\sigma$ as shown in Figure 9(b). (Discontinuity of P in the crowd fluid)

(c) Physical motion: The velocity of the moving crowd is denoted by $\boldsymbol{v}$, and it can be decomposed orthogonally as v_x and y_y, characterizing the moving speed along x axis and y axis, respectively (See Figure 9).

(d) Motive Force: Human motion is self-driven. In physics, the motive force is commonly considered as frictions or pushing forces that people implement on the ground by feet. What differs creatures like human from non-creatures is that the motive force is intentionally generated by creatures such that they can decide where to go and how fast they move. Thus, the motive force is not only a physical concept, but also represents intentions of people in their mind. As a result, this paper presents the motive force by $\boldsymbol{f}^{\mathrm{self}}=m\boldsymbol{a}^{\mathrm{self}}$, where $\boldsymbol{a}^{\mathrm{self}}$ is called self-acceleration and it indicates intentions of people. In a similar way, $\boldsymbol{a}^{\mathrm{self}}$ can be decomposed as a_x^{self} and a_y^{self}.

Next, we will present crowd fluid equation in 2D space in a general sense. If readers are not familiar with fluid mechanics or not interested in knowing the mathematical description in 2D space, please move to subsection 5.2 on the next page to read the simplified analysis in 1D space.

The flow characteristics as presented above (i.e., ρ, P, v, a^{self}) are all functions of position (x,y) and time t, and the crowd flow is unsteady flow in our analysis. With the assumption of the conservation of flow mass (i.e., mass continuity equation), we now study the moving crowd in the flow section of $\mathrm{d}x\mathrm{d}y$, where the mass is $m=m_0\rho\mathrm{d}x\mathrm{d}y$. By the Newton Second Law we have

$$m_0\rho\left(\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right)=\boldsymbol{a}^{self}m_0\rho-\nabla\boldsymbol{P} \tag{10}$$

Equation (10) corresponds to Euler's Equation in fluid mechanics and it demonstrates the Newton Second Law ($\sum F=ma$) for the crowd flow. Here we note that the derivation steps as above do not require any mathematical form of $\boldsymbol{a}^{self}$. Learning from the self-driven force and social-force as presented before, this paper specifies the subjective targets in people's mind by desired velocity $\boldsymbol{v}^d$ and desired density ρ^d, and the self-acceleration is given in a feedback manner as below, and $\boldsymbol{a}_v^{self}$ and $\boldsymbol{a}_\rho^{self}$ corresponds to the self-driven force and the social-force in the pedestrian model in Section 3 and 4.

$$\begin{gathered}
\boldsymbol{a}^{self}=(\boldsymbol{a}_v^{self}+\boldsymbol{a}_\rho^{self})\\
\boldsymbol{a}_v^{self}=k_1(\boldsymbol{v}^d-\boldsymbol{v})=k_1[(v_x^d-v_x)\boldsymbol{i}+(v_y^d-v_y)\boldsymbol{j}]\\
\boldsymbol{a}_\rho^{self}=k_2(\rho^d-\rho)\exp\left(\frac{\rho^d-\rho}{k_3}\right)\nabla\rho=k_2(\rho^d-\rho)\exp\left(\frac{\rho^d-\rho}{k_3}\right)\left(\frac{\partial\rho}{\partial x}\boldsymbol{i}+\frac{\partial\rho}{\partial y}\boldsymbol{j}\right)
\end{gathered} \tag{11}$$

where k_1 k_2 and k_3 are parameters that weigh differently on targets of v^d and ρ^d, and they have specific units to form the acceleration. Besides, the desired density ρ^d and actual density ρ correspond to the desired distance d^0 and actual distance d in Equation (7).

The desired speed v^d and desired density ρ^d represent the psychological target in people's mind, specifying the speed and interpersonal distance that people desire to realize. The physical speed v and density ρ are the physical entities in the reality. As a result, the difference v^d -v and ρ^d -ρ show the gap between the human subjective wish and realistic situation, and they form the motive force to make the physical variables approaching towards the psychological targets.

According to Equation (7), if d_{ij} is close to d_{ij}^0, the social force could be approximated in a linear form as below.

$$\boldsymbol{f}_{ij}^{soc} \approx A_i(d_{ij}^0 - d_{ij})\boldsymbol{n}_{ij} \quad \text{if} \quad d_{ij} \to d_{ij}^0 \tag{12}$$

Correspondingly, the motive force regarding the desired density can also be approximated in a linear manner when ρ^d is close to ρ. As a result, Equation (11) is rewritten as below. In general, assumption of fluid only holds for high-density crowd (Helbing, et al, 2002). In other words, when people get sufficiently close with each other, they can be treated in analog with fluid and fluid analysis can be applied.

$$\begin{aligned} \boldsymbol{a}^{self} &= (\boldsymbol{a}_v^{self} + \boldsymbol{a}_\rho^{self}) \\ \boldsymbol{a}_v^{self} &= k_1(\boldsymbol{v}^d - \boldsymbol{v}) = k_1[(v_x^d - v_x)\boldsymbol{i} + (v_y^d - v_y)\boldsymbol{j}] \\ \boldsymbol{a}_\rho^{self} &= k_2(\rho^d - \rho)\nabla\rho = k_2(\rho^d - \rho)(\frac{\partial\rho}{\partial x}\boldsymbol{i} + \frac{\partial\rho}{\partial y}\boldsymbol{j}) \end{aligned} \tag{13}$$

Equation (10) and (13) should be jointly used with other equations (e.g., the conservation of mass; boundary conditions) in order to fully describe the flow characteristics in a given geometric setting. By plugging Equation (13) into Equation (10) and using orthogonal decomposition. Equation (10) and (13) can be rewritten as below with the equation of mass continuity.

$$m_0\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y}\right) = \left(k_1(v_x^d - v_x) + k_2(\rho^d - \rho)\frac{\partial\rho}{\partial x}\right)m_0\rho - \frac{\partial P}{\partial x} \tag{14}$$

$$m_0\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y}\right) = \left(k_1(v_y^d - v_y) + k_2(\rho^d - \rho)\frac{\partial\rho}{\partial y}\right)m_0\rho - \frac{\partial P}{\partial y} \tag{15}$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} = 0 \tag{16}$$

From the perspective of psychology studies, the gap between the psychological desire (i.e., v^d and ρ^d) and the physical reality (i.e., v and ρ) relates to how much stress people will be experiencing (Staal, 2004). In other words, the difference v^d -v and ρ^d -ρ relates to the psychological concept of stress, and Equation (10) and (13) shows that accumulation of such stress will motivate certain behavior of people. In particular, the gap of speeds characterizes the time-related stress, which relates to the time-pressure in psychological research. The gap of density reflects the stress about the interpersonal relationship, and it focuses on social characteristics in the crowd. In particular, Equation (14) (15) (16) implies a feedback mechanism by which people's mind functions like a controller of their behavior: the targets in mind (i.e., v^d and ρ^d) guide people to change their physical characteristics (i.e., v and ρ), and such changes in the physical world are also feedback to people's mind. As a result, mind and reality interact in a closed-loop such that a balance can be reached between the psychological world of human mind and physical world of reality. Control theory can help to explore how v and ρ are changed given v^d and ρ^d, and cognition science focuses on how people perceive the reality and thus adjust v^d and ρ^d based on their perceptions.

5.2. Fluid-Based Analysis of Pedestrians at Bottlenecks

Next we will discuss a scenario when people move through a passageway as shown in Figure 10. Here we assume that the width of the passageway is relatively small compared to the length of the crowd flow. The flow is thus assumed to be homogeneous in the direction perpendicular to the passageway direction (i.e., along y axis in Figure 9(a)), and the flow characteristics vary only along the passageway direction (i.e., along x axis in Figure 9(a)). In brief, this section focuses on the crowd movement in a passageway like in a one-dimensional space, and the possible physical forces from walls f_w are assumed to be and equal in magnitude and opposite in direction (See Figure 10). As a result, it yields $\partial P/\partial y=0$, $\partial\rho/\partial y=0$, $\partial v_x/\partial y=0$, $v^d_y=0$ and $v_y=0$. In other words, $v=v_x$, $v^d=v_x^d$ and $f^{self}=f_x^{self}$. As shown in Figure 10, we now study the moving crowd in the flow section of Ydx, where the flow mass is $m=m_0\rho Y\mathrm{d}x$. As people try to accelerate along the positive direction of x axis, the resistance comes from the surrounding people, and thus dF = $P(x+\mathrm{d}x)Y$-$P(x)Y$ is resistant to crowd motion. The motive force is $\boldsymbol{f}^{self}=m\boldsymbol{a}^{self}=\boldsymbol{a}^{self}m_0\rho Y\mathrm{d}x$. By Newton Second Law we have

$$m_0 \rho\, Y\, \mathrm{d}x \frac{\mathrm{d}v}{\mathrm{d}t} = f^{self} - [P(x+\mathrm{d}x)Y - P(x)Y] \tag{17}$$

The motive force is given by $\boldsymbol{f}^{self} = m\boldsymbol{a}^{self}$, where $\boldsymbol{a}^{self}$ is called self-acceleration and it indicates intentions of people. In a similar way, $\boldsymbol{a}^{self} = a_x^{self}$ in one-dimensional space.

$$m_0 \rho\, Y\, \mathrm{d}x \frac{\mathrm{d}v}{\mathrm{d}t} = a^{self} m_0 \rho\, Y\, \mathrm{d}x - [P(x+\mathrm{d}x)Y - P(x)Y] \tag{18}$$

Because $\partial P/\partial x = [P(x+\mathrm{d}x) - P(x)]/\mathrm{d}x$, then it follows

$$m_0 \rho \frac{\mathrm{d}v}{\mathrm{d}t} = a^{self} m_0 \rho - \frac{\partial P}{\partial x} \tag{19}$$

The above equation corresponds to Euler's Equation in fluid mechanics and it demonstrates the Newton Second Law ($ma = \sum F$) for crowd flow. Also, in one-dimensional space the flow characteristics as presented above (i.e., ρ, P, v, a^{self}) are all functions of position x and time t, and it follows

$$m_0 \rho \frac{1}{\mathrm{d}t} \left(\frac{\partial v}{\partial t} \mathrm{d}t + \frac{\partial v}{\partial x} \mathrm{d}x \right) = a^{self} m_0 \rho - \frac{\partial P}{\partial x} \tag{20}$$

In fluid mechanics dv/dt is the Lagrangian derivative (material derivative) – the derivative following moving parcels in the fluid, and ∂v/∂t is the Eulerian derivative, which is the derivative of flow speed with respect to a fixed position.

$$m_0 \rho \left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \frac{\mathrm{d}x}{\mathrm{d}t} \right) = a^{self} m_0 \rho - \frac{\partial P}{\partial x} \tag{21}$$

Note that $v = \mathrm{d}x/\mathrm{d}t$, it gives

$$m_0 \rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right) = a^{self} m_0 \rho - \frac{\partial P}{\partial x} \tag{22}$$

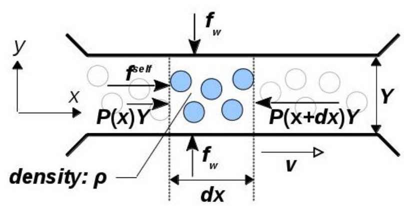


Figure 10. Crowd movement along a passageway

The above derivation steps do not specify any mathematical form of $\boldsymbol{a}^{self}$. As aforementioned, $\boldsymbol{a}^{self}$ should reflect the cognition process by which people perceive the physical world and form their motives. Based on Equation (13) the self-acceleration is given in a feedback manner as

$$a^{self} = k_1 (v^d - v) + k_2 (\rho^d - \rho) \frac{\partial \rho}{\partial x} \tag{23}$$

where k_1 k_2 are parameters that weigh differently on targets of v^d and ρ^d, and they have specific units to form the acceleration. As aforementioned, the difference v^d -v and ρ^d -ρ show the gap between the human subjective wish and realistic situation, and they form the motive force to make the physical entities approaching towards the psychological targets. From the perspective of psychology, the gap between the psychological desire (i.e., v^d and ρ^d) and the physical reality (i.e., v and ρ) relates to how much stress people are experiencing (Staal, 2004). The gap of speeds v^d -v characterizes the time-related stress, which relates to time-pressure in psychological research, and it corresponds to the self-driven force in Helbing, Farkas, and Vicsek, 2000. The gap of density ρ^d -ρ reflects the stress about the interpersonal relationship, and it corresponds to the social force which focuses on social characteristics in the crowd.

$$m_0 \rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right) = \left(k_1 (v^d - v) + k_2 (\rho^d - \rho) \frac{\partial \rho}{\partial x} \right) m_0 \rho - \frac{\partial P}{\partial x} \tag{24}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0 \tag{25}$$

Mathematically, Equation (24) and (25) are to be jointly used with boundary conditions in order to fully describe the flow characteristics in a given geometric setting. Based on the existing theory in partial differential equations, the solution should be a wave function, which implies that the gap of v^d -v and ρ^d -ρ are periodic functions. In other words, v and ρ will sway around the target v^d and ρ^d, and finally converge to the target value v^d and ρ^d. If v^d and ρ^d are dynamically changed with time, then v and ρ will also track such changes with a time delay. Next, we will brief discuss the solution of Equation (24) and (25). If there is no physical interactions among people, then $\partial P/\partial x=0$, and it yields

$$\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)=\left(k_1(v^d-v)+k_2(\rho^d-\rho)\frac{\partial \rho}{\partial x}\right) \tag{26}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0 \tag{27}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}-k_2(\rho^d-\rho)\frac{\partial \rho}{\partial x}=k_1(v^d-v) \tag{28}$$

$$\frac{\partial \rho}{\partial t}+\rho\frac{\partial v}{\partial x}+v\frac{\partial \rho}{\partial x}=0 \tag{29}$$

The above equation is a hyperbolic partial differential equation. As below is the typical approach for analysis of the equation.

$$\frac{\partial v}{\partial t}=-v\frac{\partial v}{\partial x}+k_2(\rho^d-\rho)\frac{\partial \rho}{\partial x}+k_1(v^d-v) \tag{30}$$

$$\frac{\partial \rho}{\partial t}=-\rho\frac{\partial v}{\partial x}-v\frac{\partial \rho}{\partial x} \tag{31}$$

$$\begin{pmatrix}\frac{\partial v}{\partial t}\\ \frac{\partial \rho}{\partial t}\end{pmatrix}=\begin{pmatrix}-v & -k_2(\rho-\rho_d)\\ -\rho & -v\end{pmatrix}\begin{pmatrix}\frac{\partial v}{\partial x}\\ \frac{\partial \rho}{\partial x}\end{pmatrix}+\begin{pmatrix}k_1(v^d-v)\\ 0\end{pmatrix} \tag{32}$$

Here we take the state variable as $U=(v, \rho)^{\mathrm{T}}$, and the Joccobian Jacobian matrix is

$$A(U)=\begin{pmatrix}v & k_2(\rho-\rho_d)\\ \rho & v\end{pmatrix} \qquad |A(U)-I\lambda|=\begin{vmatrix}v-\lambda & k_2(\rho-\rho_d)\\ \rho & v-\lambda\end{vmatrix} \tag{33}$$

Then the eigenvalue are calculated as

$$\lambda=v\pm\sqrt{k_2\rho(\rho-\rho^d)} \tag{34}$$

Due to the limitation of the article length, we will not further discuss the analytical solution of hyperbolic partial differential equation as above. If readers are interested, please refer to Whitham, 1974 for detailed discussion.

5.3 Energy-Based Analysis of Pedestrian Crowd

Further, when the crowd flow converges into the steady state, it implies $\partial v/\partial t=0$ and the Eulerian derivative becomes zero. For such steady flow, the flow characteristics (i.e., ρ, F, v, a^{self}) are time-invariant, and energy-based analysis is commonly applied. By taking the dot product with ds – the element of moving distance – on both sides of Equation (22), an energy equation can be derived, which corresponds to the well-known Bernoulli equation in fluid mechanics. In particular, the element of distance in one-dimensional space is ds=dx, and thus we have

$$m_0\rho v\frac{\partial v}{\partial x}\mathrm{d}x=a^{self}m_0\rho\,\mathrm{d}x-\frac{\partial P}{\partial x}\mathrm{d}x \tag{35}$$

Because the flow characteristics (i.e., ρ, F, v, a^{self}) are only functions of position x for steady flow, it gives

$$m_0 v\,\mathrm{d}v=a^{self}m_0\,\mathrm{d}x-\frac{\mathrm{d}P}{\rho} \tag{36}$$

$$m_0 \mathrm{d}\left(\frac{v^2}{2}\right)+\frac{\mathrm{d}P}{\rho}=a^{self} m_0 \,\mathrm{d}x \tag{37}$$

Because the element of moving distance is the product of instantaneous speed and element time, i.e., dx=vdt, it gives

$$m_0 \mathrm{d}\left(\frac{v^2}{2}\right)+\frac{\mathrm{d}P}{\rho}=m_0 a^{self}\, v\,\mathrm{d}t \tag{38}$$

Since m_0 is the average individual mass and does not depend on moving speed v, the above equation can be integrated. The physical interactions are repulsive among people, and $P\geq 0$. Given the initial time of crowd movement t_0, an energy balance equation is obtained as below

$$\frac{m_0 v^2}{2}+\int_0^P \frac{\mathrm{d}P}{\rho}=\int_{t_0}^{t} m_0 a^{self}\, v\,\mathrm{d}t+C \tag{39}$$

Based on Equation (23) it further gives

$$\frac{m_0 v^2}{2}+\int_0^P \frac{\mathrm{d}P}{\rho}=\int_{t_0}^{t} m_0 k_1\left(v^d-v\right) v\,\mathrm{d}t+\int_{t_0}^{t} m_0 k_2\left(\rho^d-\rho\right)\frac{\partial \rho}{\partial x} v\,\mathrm{d}t+C \tag{40}$$

Similar to the well-known Bernoulli Equation in fluid mechanics, Equation (40) can be interpreted by the principle of energy conservation, where the psychological drive can be considered as a special form of potential energy that arises from crowd opinion, and its behavioral manifestations are energy in physical forms.

The left side of Equation (40) includes energy in physical forms – kinetic energy and static energy. In addition, if the crowd movement is not horizontal, but on a slope, the gravity should be taken into account by including the gravitational potential m_0gh on the left side of Equation (40), where h is the altitude of the crowd position and g is the gravitational acceleration. The kinetic energy is the common form that describes the energy regarding crowd motion. The static energy is an integral form that characterizes the physical interactions of people. The physical interaction comes into existence when the crowd density exceeds a certain limit. Therefore, P can also be considered as a function of crowd density ρ. A mathematical expression of the static energy is exemplified as below. Let ρ_0 represent the crowd density when people start to have physical contact, and the interaction force is given by

$$P=K\,g\left(\rho-\rho_0\right) \qquad \int_0^P \frac{\mathrm{d}P}{\rho}=K\,g\left(\ln\rho-\ln\rho_0\right) \tag{41}$$

where K is a positive parameter. $g(\cdot)$ is a piecewise function such that g(x)=0 if x<0 and g(x)=x if x>=0. The interaction force thus becomes nonzero when $\rho>\rho_0$. The resulting static energy is given by Equation (41). In general, the moving crowd is a compressible flow: the crowd density varies in different places of the flow. Thus, the static energy is a function of density ρ.

The right side of Equation (40) is energy involving crowd opinion, and it is called self-motivated energy or stress energy in this paper. This special kind of energy is expressed by an integral term, and it follows the typical formula that is the integral product of a force and moving distance along the direction of the force. The self-motivated energy in Equation (40) consists of two terms: one term drives people to adjust speed in a temporal space and the other one motivates people to adjust their social distance with others. This corresponds to the self-driven for and social force in the social force model. An interesting topic is that Equation (40) extends the law of energy conservation from the physical world of universe to the psychological world of human mind, implying potential transformation of self-energy into certain physical form. In fact, the energy-based equation shows that energy arises in mind when people have desire doing something, and it will be ultimately transformed into certain physical energy in reality. In other words, energy in mind cannot vanish by itself, but must find an outlet to the physical world.

Tab. 2 Conservation of Energy in Crowd Self-Motivated Motion

Kinetic Energy	$\frac{m_0 v^2}{2}$	Physical Energy ↑↓ Self-Motivated Energy
Static Energy	$\int_0^P \frac{dP}{\rho}$	
Self-Motivated Energy (Psychological Drive)	$\int_{t_0}^{t} m_0 a^{self}\, v\, dt$	

The self-motivated energy characterizes the collective opinion of people in mind. From the perspective of psychological principles, the variables of v^d and ρ^d thus have much freedom because they exist in people's mind. However, as people realize v^d and ρ^d, their behavior are not free any more since certain realistic factors confine their deed in the physical world (e.g, the size of a passage may not permit all the people to move as fast as desired). If the physical variables reach the maximum while people still desire increasing them, the self-motivated energy will not be transformed to the physical forms as desired. In this situation, the stronger is the subjective wish in people's mind, the worse may become the situation in the reality, showing a paradoxical relationship of subjective wishes of human and objective result in reality. An example in this kind is the "faster-is-slower" effect as shown in Helbing, Farkas, and Vicsek, 2000.

5.4 Doorway Scenario and Egress Performance

The energy-based analysis provides us a new perspective to reinterpret the faster-is-slower effect and Yerkes–Dodson law. If the self-motivated energy is transformed properly so that people are able to speed up, the psychological drive of motion will accelerate the crowd, and faster-is-faster effect shows up. In contrast, when the self-motivated energy cannot be transformed to the physical forms as desired, the faster-is-slower effect comes to being. Very importantly, the passage capacity determines the maximal amount of self-motivated energy that can be transformed to the physical forms, and it determines a critical threshold: below the threshold the psychological drive as expressed by the self-motivated energy is transformed to the kinetic energy and the crowd can accelerate as they desire. Above the threshold the kinetic energy reaches the maximum, and the excessive psychological drive will be transformed to the static form, resulting in an increase of crowd density. This answers the question about when the self-motivated energy is transformed to the kinetic form, and when to the static form.

If the crowd density exceeds a certain limit, the pedestrian flow will decrease. Further increase of the crowd density could result in disorder events (e.g., jamming or stampeding). In this light our crowd flow model and energy-based analysis also reiterates the Yerkes–Dodson law: moderate stress improves the performance (i.e., speeding up crowd flow) while excessive stress impairs it (e.g., disordering and jamming). The relationship of the desired flow $\rho^d v^d$ and physical flow ρv is plotted as shown in Figure 11, where $\rho^d v^d$ indicates the collective demand of crowd movement and ρv indicates the physical motion that the crowd realize. Corresponding to Figure 2 this figure reiterates the Yerkes-Dodson law at the macroscopic level, and $\rho^d v^d$ is the stress indicator, which represents the motivation level at the macroscopic level.

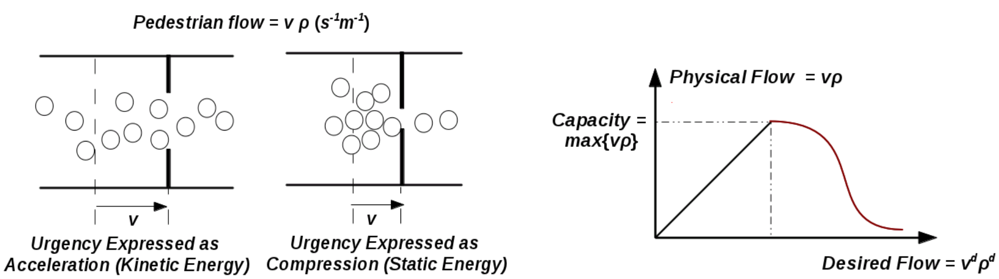


Figure 11. Relationship of the desired flow $\rho^d v^d$ and physical flow ρv

Continuing with the above energy-based analysis we will next take the gravitational force and fluctuation random force into account, and explain the risk of "down-slope" and "heating effect" in crowd egress. Down-slopes are commonly known as stairways, where the gravity is effective. As a result, the gravitational potential is necessarily taken into account in Equation (40), and it can either facilitate or impede acceleration or compression of a crowd. When people desire moving faster ($v^d > v$), the gravity facilitates acceleration on a down-slope while resists such acceleration on an up-slope. A similar effect is also for compression. Because people usually move downstairs in egress, acceleration and compression will be more intensive in a down-slope, and the risk of disorder and blocking is thus increased.

The heating effect was known for the "heating-by-freezing" in Helbing et al., 2002. Such a heating effect implies that the fluctuation force in Equation (1) is in higher magnitude (Helbing et al., 2002). From the perspective of energy-based analysis people are often more emotional when watching sports games, attending concert or joining parties, and they usually exhibit more energy in these occasions. Even if they are not in physical motion, they will release such energy by shouting or other behavior. So stadium, concert or nightclubs are a kind of public places where people's inner energy will find an outlet to the physical world, and such effect on crowd is similar to heating process as mentioned in Helbing et al., 2002. Corresponding to the fluctuation force in Equation (1), a fluctuation acceleration is added in Equation (22), and it indicates the heating effect on the crowd. For heating effect the fluctuation force is not toward a certain direction and it represents irrational mind of people. In contrast the desired velocities and desired distance are deterministic in directions and represent rational mind of people.

$$\frac{m_0 v^2}{2}+\int_0^P \frac{dP}{\rho}+m_0 gh=\int_{t_0}^{t} m_0 a^{self} v\, dt+\int_{t_0}^{t} m_0 a^{\xi} v\, dt+C \tag{42}$$

When the “heated” crowd are further compressed at a downstairs bottleneck in egress, the risk of disorder and stampede is much higher than in other places. In a list of historical events of stampeding in Still, 2016, it suggests that certain gathering places such as stadium are more frequent to occur stampede in emergency egress. Similar evidence is also listed in Helbing et. al., 2002, and one of the crucial reasons is that bottlenecks in a stadium are often not horizontal, but on stairways (See Figure 12). Another important reason is that people usually feel excited when watching football games, and the crowd are thus “heated” as Helbing described. From the practical viewpoint, it is almost impossible to clam down the crowd who join a sport event in stadium, but it is feasible to design egress facilities such that the bottleneck (e.g, a narrow passageway) are not placed on down-slope areas. In other words, it is better to design the downstairs ways in a wide or open area, and when people get gathered at the bottleneck, the passageway should be horizontal.

VI. Conclusion

Human motion is self-driven and self-adapted to the environment, and the forces are generated by intention of people in a psychological sense. How to capture this characteristic in consistency with known physics laws is meaningful to understand crowd behaviors. By integrating psychological principles to Newtonian motion of pedestrian crowd, we reinterpret the well-known social-force model by using the psychological concept of stress, and this individual-based model is further aggregated into crowd fluid dynamics at the macroscopic level. The concept of stress at both micro-and-macro scales is interpreted by the difference between the desired entities in human mind (i.e., desired velocity, desired interpersonal distance, desired crowd density) and the physical entities in reality (i.e., actual velocity, actual interpersonal distance, actual crowd density), and the resulting crowd dynamics describes how environmental stressors (e.g., surrounding people, hazard and exit signs) influence collective behavior in both normal and emergency condition. In brief, our study at micro-and-macro level helps to bridge a gap among psychological findings, pedestrian models and simulation results, and it further provides a new perspective to understand the paradoxical relationship of subjective wishes of human and objective result of their deeds in reality.

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