CHEM 3PC3

Quiz #7

November 11, 2025

Name: ______________________________

Student Number: ______________________________

SUBMISSION INSTRUCTIONS — READ CAREFULLY

To receive full credit, you must follow these steps:

1. Answer all the questions.
2. Write your solutions on new, separate pages. Do not write your solutions in the margins of this paper.
3. At the top of each solution page, clearly write the corresponding question number (e.g., **"Question 5"**). If you use more than one page for a question, write the question number on each page (e.g., **"Question 5 (Page 1 of 2)"**).
4. **If you cannot solve a question, still attach a page with the question number and write "Blank" or "No Answer"** to indicate you attempted it.
5. **If you are unsure of a complete answer, still attempt the question:** attach a page with the question number and write down any relevant thoughts, formulas, or initial steps. Partial credit may be awarded for demonstrated effort and correct reasoning, whereas a blank answer will receive no credit.
6. Show all your work clearly and legibly. Unorganized or illegible work may not receive credit.

1 Problems

1. Answer with true or false. Give reasons.

a) If every element of 2×2 matrix A is a positive number, then the determinant of A is a positive number.

b) The solution of the equation: $\begin{vmatrix} \sin(x) & \cos(x) \\ -\cos(x) & \sin(x) \end{vmatrix} = 0$ is the empty set.

2. Expand the determinant and then solve for x.

a)

$$\begin{vmatrix} -2x & 1 & 0 \\ 0 & 3 & 2 \\ -x & 1 & 5 \end{vmatrix} = 4$$

b)

$$\begin{vmatrix} x & 4 & 0 \\ 2 & 2 & -x \\ 1 & 1 & 1 \end{vmatrix} = 0$$

3. Let $\mathbf{X} \in \mathbb{R}^{N\times d}$ be the **design matrix** whose i-th row is the feature vector $\mathbf{x}_i^\top \in \mathbb{R}^{1\times d}$, and let $\mathbf{Y} = [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_N]^\top \in \mathbb{R}^N$ be the vector of target values.
The mean squared error loss is defined as

$$\mathcal{L}(\mathbf{w}) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{2}\left(\hat{y}_i - \mathbf{w}^\top\mathbf{x}_i\right)^2.$$

Show that this expression can be equivalently written in compact matrix form as

$$\mathcal{L}(\mathbf{w}) = \frac{1}{2N}\left(\mathbf{Y}^\top\mathbf{Y} - 2\mathbf{Y}^\top\mathbf{X}\mathbf{w} + \mathbf{w}^\top\mathbf{X}^\top\mathbf{X}\mathbf{w}\right).$$

Provide a complete proof of this identity.
4. Find the cross product of the vectors $\mathbf{u}$ and $\mathbf{v}$.
a) $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
b) $\mathbf{u} = \mathbf{i} - \mathbf{j}$, $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$