CHEM 3PC3

Quiz #9

November 25, 2025

Name: ______________________________

Student Number: ______________________________

SUBMISSION INSTRUCTIONS — READ CAREFULLY

To receive full credit, you must follow these steps:

1. Answer all the questions.
2. Write your solutions on new, separate pages. Do not write your solutions in the margins of this paper.
3. At the top of each solution page, clearly write the corresponding question number (e.g., **"Question 5"**). If you use more than one page for a question, write the question number on each page (e.g., **"Question 5 (Page 1 of 2)"**).
4. **If you cannot solve a question, still attach a page with the question number and write "Blank" or "No Answer"** to indicate you attempted it.
5. **If you are unsure of a complete answer, still attempt the question:** attach a page with the question number and write down any relevant thoughts, formulas, or initial steps. Partial credit may be awarded for demonstrated effort and correct reasoning, whereas a blank answer will receive no credit.
6. Show all your work clearly and legibly. Unorganized or illegible work may not receive credit.

1 Problems

1. A hydrogen atom is in its ground state. The electron state is described by the function (usually refered as wavefunction),

$$\psi(r) = A\exp(-r/a),$$

where r is the radial distance of the electron from the nucleus. Determine the normalization constant, A, by enforcing the following normalization condition:

$$4\pi \int_0^\infty dr(r^2\psi^2(r)) = 1.$$

Note that the integrand has an r^2 factor because this is a radial integral in three dimensional space.

Hint:

1. For this use $\exp(-x)x^2|_0^\infty = 0$ and $[-\exp(-x)|_0^\infty = 1]$

2. Consider the following 2×2 matrix,

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

a. Show that this matrix is orthogonal (A square matrix is orthogonal if its transpose equals its inverse).
b. What is the inverse of this matrix?
c. Determine the eigenvalues of this matrix.

3. Find the critical point of

$$f(x,y) = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

and determine whether it is a maximum, minimum or saddle point. Note that

$$\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$$

is the determinant of the second derivative matrix.

Hint: For a critical point (x_0, y_0) with Hessian determinant $D = f_{xx}f_{yy} - (f_{xy})^2$:

1. If $D > 0$ and $f_{xx} > 0$, then (x_0, y_0) is a local minimum
2. If $D > 0$ and $f_{xx} < 0$, then (x_0, y_0) is a local maximum
3. If $D < 0$, then (x_0, y_0) is a saddle point