This document has some practice questions to help you prepare for the midterm.

1 Derivatives

1.1 Differentiation

1. Use the limit definition to compute the derivative of $f(x) = \frac{x+1}{2-x}$.

2. Use the limit definition to compute the derivative of $f(x) = \cos(3x)$.
   Try to simplify your answer as much as possible, you may need this trigonometric identity $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$.

3. Use the limit definition to compute the derivative of $f(x) = \frac{a}{x}$.

4. Which of the following functions does not have a **continuos** derivative?
   a) $x^2 - 2x + 1$, b) $\cos(x^2)$, c) $|x-1|$

1.2 Total derivative

1. Compute the total derivative of the Beattie-Bridgeman equation of state for the pressure,

$$p(T, \nu) = \frac{RT}{\nu^2}\left(1 - \frac{c}{\nu T^3}\right)(\nu + B) - \frac{A}{\nu^2}, \tag{1}$$

   where A and B are some functions of the molar volume ν, here we will consider them as constants.

2. What is the total derivative of a linear model of the form,

$$f(\mathbf{x}) = \sum_{i=1}^{m} w_i x_i, \quad \text{where} \quad \mathbf{x} = [x_1, x_2, \cdots, x_m]. \tag{2}$$

3. What is the total derivative of a polynomial expansion of the form,

$$f(x) = \sum_{i=1}^{m} w_i x^i \quad \text{where} \quad x^i \quad \text{is x to the power of } i. \tag{3}$$

1.3 Partial derivatives

1. The virial expansion is a model of thermodynamic equations of state, and it is given by,

$$P(V_m, T) = \frac{R}{T}\left(1 + \frac{B(T)}{V_m} + \frac{B(T)}{V_m^2} + \cdots\right), \tag{4}$$

   where V_m is the molar volume.
   Compute the partial derivate of $P(V_m, T)$ with respect to the molar volume.

2. Compute the partial derivative of $f(x, y) = (2x + 6y)^4$ with respect to the variable y

$$\frac{\partial f(x, y)}{\partial y} = \frac{\partial}{\partial y}(2x + 6y)^4 \tag{5}$$

3. Compute the partial derivative of $f(x_1, x_2) = \frac{e^{x_1}}{e^{x_1}+e^{x_2}}$ with respect to the variable x_1

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{\partial}{\partial x_1}\left(\frac{e^{x_1}}{e^{x_1} + e^{x_2}}\right) \tag{6}$$

2 Linear Algebra

1. What is the result of $\mathbf{u}\mathbf{v}^\top$, given

$$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \quad . \tag{7}$$

   Remember that $\mathbf{v}^\top$ is a row vector, meaning the transpose of a column vector.

2. Compute the Manhattan distance of the vector

$$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{8}$$

   The Manhattan distance is defined as, $d_M(\mathbf{x}) = \sum_i |x_i|$.

3. Given the following two matrices $\mathbf{A}$ and $\mathbf{B}$, where $\mathbf{A}$ has 10 rows and 100 columns, and $\mathbf{B}$ has 200 rows and 10 columns. Which of the following operations is doable,
   a) $\mathbf{A}\,\mathbf{B}$, b) $\mathbf{B}\,\mathbf{A}$
   For the selected option, what is the size of the output tensor.

4. What is $B^\top$ ($\top$: transpose),

$$B = \begin{pmatrix} 5 & 7 & 1 \\ 2 & 0 & 1 \end{pmatrix} \tag{9}$$

5. What is the output of $\mathbf{A}\,\mathbf{A}^\top$?

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{10}$$

6. How many number of rows and columns does the matrix $\mathbf{C}$ has and what is its value? $\mathbf{C}$ is computed by,

$$\mathbf{C} = \mathbf{A}\,\mathbf{V}\,\mathbf{B}, \tag{11}$$

where,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 6 \\ -10 & -10 & -10 \\ 1 & -3 & 3 \\ 3 & -1 & -9 \\ -2 & -1 & -4 \end{pmatrix} \quad \mathbf{V} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 9 & -7 & -8 & 4 \\ -6 & 7 & -7 & -6 \\ -7 & 0 & -8 & -10 \end{pmatrix} \tag{12}$$

3 Linear Equations

1. Solve the following system of linear equations either using Gauss or Gauss-Jordan method.
   Set of equations (1):

$$\begin{aligned} -x_0 + 3x_1 + 2x_2 &= 0 \\ 2x_1 - x_2 &= 1 \\ 2x_0 - x_1 + 4x_2 &= 2 \end{aligned}$$

   Set of equations (2):

$$\begin{aligned} 3x_0 + 3x_1 + x_2 &= 0 \\ 4x_0 + 2x_1 - x_2 &= 1 \\ -3x_0 - x_1 + 2x_2 &= 0 \end{aligned}$$

   Set of equations (3):

$$\begin{aligned} 3x_0 + 3x_1 + x_2 &= 1 \\ 4x_0 + x_1 &= 1 \\ x_0 + 5x_1 + 2x_2 &= 2 \end{aligned}$$

   Set of equations (4):

$$\begin{aligned} 4w + x + 2y - 3z &= -16 \\ -3w + 3x - y + 4z &= 20 \\ -w + 2x + 5y + z &= -4 \\ 5w + 4x + 3y - z &= -10 \end{aligned}$$

2. For the previous systems of linear equations, state if they could have a unique solution, multiple or no solution at all, use the determinant of a matrix in your answer.

The End.