Name: ____________________

Student number: ____________________

1. What is the derivative of the function: $f(x) = \sin(nx + n)$, with respect to x; n is an integer number. [**Total Marks 3**]

   - [**1 Mark**] Define the change of variable
   - [**1 Mark**] Compute the change of variable, $\frac{du(x)}{d\,x}$
   - [**1 Mark**] Correct derivative

$$u(x) = nx + n, \quad \text{where} \quad \frac{du(x)}{d\,x} = n \tag{1}$$

$$f(u(x)) = \sin(u(x)) \tag{2}$$

$$\frac{d\,f(x)}{d\,x} = \frac{d\sin(u(x))}{d\,u}\frac{du(x)}{d\,x} = \cos(u(x))\;n = n\;\cos(nx + n) \tag{3}$$

Answer: ____________________

2. Compute the **total derivative** (or **total differential**) of the following state function,

$$f(T, P) = a_0 + b_0P + (a_1 + b_1P)T + (a_2 + b_2P)T^{-2}$$

[**Total Marks 4**]

   - [**1 Mark**] Define the total derivative without the partial derivatives
   - [**2 Marks**] Compute $\frac{\partial f(T,P)}{d\,T}$, and $\frac{\partial f(T,P)}{d\,P}$
   - [**1 Mark**] Correct answer

$$\text{Total derivative:} \quad df(T, P) = \left(\frac{\partial f(T, P)}{d\,T}\right)_P dT + \left(\frac{\partial f(T, P)}{d\,P}\right)_T dP \tag{4}$$

$$\frac{\partial f(T, P)}{d\,T} = (a_1 + b_1P) + (a_2 + b_2P)\left(\frac{\partial T^{-2}}{\partial T}\right) \quad \text{where} \quad \frac{\partial T^{-2}}{\partial T} = \frac{-2}{T^3} \tag{5}$$

$$= (a_1 + b_1P) - \frac{2(a_2 + b_2P)}{T^3} \tag{6}$$

$$\frac{\partial f(T, P)}{d\,P} = b_0 + b_1T + \frac{b_2}{T^2} \tag{7}$$

$$df(T, P) = \left((a_1 + b_1P) - \frac{2(a_2 + b_2P)}{T^3}\right) dT + \left(b_0 + b_1T + \frac{b_2}{T^2}\right) dP \tag{8}$$

Answer: ____________________

3. What is the value of the following integral? $\int \text{ReLU}(x)\, dx$
The ReLU function is defined as,

$$\text{ReLU}(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ x & \text{if} \quad x \geq 0 \end{cases}$$

- (Tip) split the integral into two integrals each with a different integration range.

[**Total Marks 3**]

- [**1 Mark**] Split the integration range
- [**1 Mark**] Compute each integral
- [**1 Mark**] Correct answer

$$\int \text{ReLU}(x)dx = \int_{-\infty}^{0} \text{ReLU}(x)dx + \int_{0}^{\infty} \text{ReLU}(x)dx \tag{9}$$

$$\int_{-\infty}^{0} \text{ReLU}(x)dx = \int_{-\infty}^{0} 0\, dx = C_1 = 0 \tag{10}$$

$$\int_{-\infty}^{0} \text{ReLU}(x)dx = \int_{0}^{\infty} x\, dx = \frac{x^2}{2} + C \tag{11}$$

$$\int \text{ReLU}(x)dx = \frac{x^2}{2} + C \tag{12}$$

Answer:____________________

4. Consider the following matrices and vectors with their specified dimensions:

- **A** is a (3,2) matrix
- **B** is a (2,2) matrix
- **C** is a (4,3) matrix
- **v** is a column vector with 2 elements
- **w** is a column vector with 3 elements
- **u** is a column vector with 4 elements

Mark all expressions that are **valid operations** (multiplication, addition, or subtraction) based on their dimensions:

A) $\mathbf{A}\,\mathbf{v}$ B) $\mathbf{B}\,\mathbf{v} + \mathbf{w}$ C) $\mathbf{C}\,\mathbf{u}$ D) $\mathbf{A}\,\mathbf{v} - \mathbf{B}\,\mathbf{v}$ E) $\mathbf{w} + \mathbf{u}$

[**Total Marks 1**]

- $\mathbf{A}\,\mathbf{v} \to (3,2)(2,1) \to (3,1)$, **valid operation**
- $\mathbf{B}\,\mathbf{v} + \mathbf{w} \to (2,2)(2,1) + (3,1) \to (2,1) + (3,1)$, not valid operation
- $\mathbf{C}\,\mathbf{u} \to (4,3)(4,1)$, not valid operation
- $\mathbf{A}\,\mathbf{v} - \mathbf{B}\,\mathbf{v} \to (3,1) + (2,1)$, not valid operation
- $\mathbf{A}\,\mathbf{v} - \mathbf{B}\,\mathbf{v} \to (3,1) + (2,1)$, not valid operation
- $\mathbf{w} + \mathbf{u} \to (3,1) + (4,1)$, not valid operation

Answer:____________________

5. Let's consider a function of two variables, $f(x, y)$. Select the option that contains **all the unique** second order partial derivatives.

A) $\left[\frac{\partial^2 f(xy)}{\partial^2 x^2}, \frac{\partial^2 f(xy)}{\partial^2 y^2}\right]$

B) $\left[\frac{\partial^2 f(xy)}{\partial^2 y^2}, \frac{\partial^2 f(xy)}{\partial x \partial y}, \frac{\partial^2 f(xy)}{\partial y \partial x}\right]$

C) $\left[\frac{\partial^2 f(xy)}{\partial y \partial x}, \frac{\partial^2 f(xy)}{\partial^2 y^2}, \frac{\partial^2 f(xy)}{\partial^2 x^2}\right]$

D) $\left[\frac{\partial^2 f(xy)}{\partial^2 y^2}, \frac{\partial^2 f(xy)}{\partial x \partial y}, \frac{\partial^2 f(xy)}{\partial^2 x^2}, \frac{\partial^2 f(xy)}{\partial y \partial x}\right]$

(Tip): You can use $f(xy) = f_0(x) f_1(y)$ or $f(xy) = f_0(x) + f_1(y)$ to help you in this question; f_0 and f_1 are functions of single variable.

[**Total Marks 2**]

- Correct answer is C)

$$\frac{\partial^2 f(xy)}{\partial x \partial y} = \frac{\partial^2 f(xy)}{\partial y \partial x} \tag{13}$$

Answer:________________________

6. The **trace** of a matrix, $\mathrm{Tr}(\cdot)$, is defined as sum of the elements of the diagonal. Compute $\mathrm{Tr}(\mathbf{A})$ for,

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & -2 & 0 & -2 \\ -2 & -2 & 2 & 1 & -3 \\ -2 & 0 & -2 & -1 & 0 \\ -1 & 2 & 2 & 0 & -3 \\ -3 & -2 & 2 & 2 & -2 \end{bmatrix}$$

[**Total Marks 2**]

$$\mathrm{Tr}(\mathbf{A}) = \sum_{i=1}^{5} a_{ii} = \underbrace{2}_{a_{11}} + \underbrace{(-2)}_{a_{22}} + \underbrace{(-2)}_{a_{33}} + \underbrace{0}_{a_{44}} + \underbrace{(-2)}_{a_{55}} = -4 \tag{14}$$

Answer:________________________

7. Compute the determinant of the matrix $\mathbf{C}$,

$$\mathbf{C} = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$$

[**Total Marks 2**]

$$\det(\mathbf{C}) = (-2)(-1) - (0)(-3) = 2 \tag{15}$$

Answer:________________________

8. What are the number of rows and columns and the result of $\mathbf{u}\mathbf{v}^\top$, given

$$\mathbf{u} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

[Total Marks 2]

- **[1 Mark]** $\mathbf{u}\mathbf{v}^\top \rightarrow (2,1)(1,3) \rightarrow (2,3)$

$$\mathbf{u}\mathbf{v}^\top = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ 10 & 20 & 30 \end{bmatrix} \tag{16}$$

Answer:______________________________

9. Solve the following system of linear equations using any method.

$$\begin{aligned} x_0 - 7x_1 &= 11 \\ 5x_0 + 2x_1 &= -18 \end{aligned}$$

[Total Marks 4]

$$x_0 = 11 + 7x_1 \tag{17}$$

$$5(11 + 7x_1) + 2x_1 = -18 \tag{18}$$

$$37x_1 = -18 - 55 = -73 \tag{19}$$

$$x_1 = -\frac{73}{37} = -1.973 \tag{20}$$

$$x_0 = 11 + 7(1.973) = \frac{37 \times 11 - 7 \times 73}{37} = -\frac{104}{37} = -2.81 \tag{21}$$

Answer:______________________________

10. Select the correct answer. For a state function in thermodynamics, if the system returns to its initial state, the change in the state function is always greater than zero.

A) True **B)** False

[Total Marks 1]

- **B)** is the correct answer.

State functions are path independent, and only depend between the initial and final state.

Answer:______________________________

Cont.

11. According to the second law of thermodynamics, which of the following must be true for a process to be spontaneous?

   A) $\Delta S_{\text{system}} > 0$

   B) $\Delta S_{\text{surroundings}} > 0$

   C) $\Delta S_{\text{universe}} > 0$

   D) $\Delta S_{\text{system}} = 0$

   [**Total Marks 1**]

   - **C)** is the correct answer.

   The $\Delta S_{\text{universe}}$ should always increase for a process to be spontaneous.

   Answer:______________________________

12. Which of the following thermodynamic quantities are **not** state functions?

   A) H (Enthalpy)

   B) G (Gibbs free energy)

   C) T (Temperature)

   D) q (Heat)

   E) S (Entropy)

   F) P (Pressure)

   G) W (Work)

   H) U (Internal energy)

   I) V (Volume)

   [**Total Marks 2**]

   - **D)** and **G)** are the correct answers.

   Answer:______________________________

13. Which of the following statements about isobaric, isochoric, and isothermal processes is **True**?

   A) In an isobaric process, the pressure remains constant while the volume changes.

   B) In an isochoric process, both temperature and pressure remain constant.

   C) In an isothermal process, the volume remains constant and temperature changes.

   D) In an isobaric process, the internal energy of the system does not change.

   [**Total Marks 1**]

   - **A)** is the correct answer.

   An an isobaric process means, there is no change in the pressure.

   Answer:______________________________

14. Which of the following is true for a **reversible adiabatic process**?

The first law of thermodynamics is given by $\Delta U = q + w$, where ΔU is the change in internal energy, q is the heat added to the system, and w is the work done on the system.

A) $q = 0$, so $\Delta U = w$

B) $w = 0$, so $\Delta U = q$

C) $\Delta U = 0$, so $q = -w$

D) $\Delta U = q + w$, as both heat and work are nonzero

[**Total Marks 1**]

- **A)** is the correct answer.

An adiabatic process means, there is no exchange of heat, $\delta q = 0$, therefore, $\Delta U = w$.

Answer:____________________________

15. Let the matrix $\mathbf{A}$ be defined as:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 5 & 0 \end{bmatrix}$$

Answer the following

[**Total Marks 5**]

- What is $\mathbf{A}^\top$ and its number of rows and columns?

[**1 Mark**] $\mathbf{A}^\top \rightarrow (2, 3)$

$$\mathbf{A}^\top = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 4 & 0 \end{bmatrix} \tag{22}$$

- Compute $\mathbf{A}^\top \mathbf{A}$, and state the number of rows and columns of this matrix-matrix multiplication.

[**1 Mark**] $\mathbf{A}^\top \mathbf{A} \rightarrow (2, 3)(3, 2) \rightarrow (2, 2)$

[**1 Mark**]

$$\mathbf{A}^\top \mathbf{A} = \begin{bmatrix} 26 & 2 \\ 2 & 20 \end{bmatrix} \tag{23}$$

- Compute $\mathbf{A}\ \mathbf{A}^\top$, and state the number of rows and columns of this matrix-matrix multiplication.

[**1 Mark**] $\mathbf{A}\,\mathbf{A}^\top \rightarrow (3, 2)(2, 3) \rightarrow (3, 3)$

[**1 Mark**]

$$\mathbf{A}\,\mathbf{A}^\top = \begin{bmatrix} 5 & 8 & 5 \\ 8 & 16 & 0 \\ 5 & 0 & 25 \end{bmatrix} \tag{24}$$

Answer:____________________________

The End.