statop hw2_3

Let

$$X \in \mathbb{R}^{n \times m}$$

Calculate

$$\partial \|X\|_2$$

where $\|X\|_2$ is the spectral norm of X.

Give the explicit subdifferential for when $X = ($

$\partial f(x_0) \equiv$ the subdifferential of $f(x_0)$, the set of all subgradients of f at x_0.

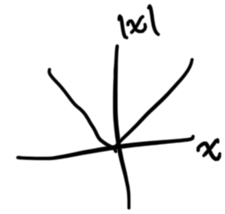
$g(x_0)$ is a subgradient of a real-valued convex function f at x_0, if:

$$f(x) - f(x_0) \geq \langle g(x_0), x - x_0 \rangle \quad \forall x$$

Example:

if

$$f(x) = |x|$$
$$x_0 = 0$$
$$x = [-1, 1]$$



then

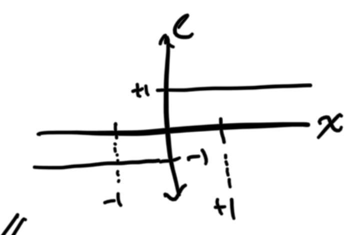
$$f(x) - f(x_0) \geq (g(x_0))^T (x - x_0)$$

$$f(x) - f(0) \geq g(0)(x - 0)$$

$$f(x) \qquad \geq g(0)\, x$$

$$g(0) \leq \frac{f(x)}{x}$$

$$g(0) \leq \frac{|x|}{x} \Rightarrow$$



$$\Downarrow$$

$$g(0) \in [-1, 1]$$

since this is a univariate case, we can let

$$g(x_0) = c$$

thus the subgradient $g(x_0)$ of $|x_0|$ in $[-1,1]$ is any constant c in $[-1,1]$. the set of subgradients is the subdifferential. So in this example

$$\partial |x_0| = [-1, 1]$$

~~So~~ X is real so

$$\|X\|_2 = \sqrt{\lambda_{max}(X^T X)}$$
$$= \sigma_{max}(X^T X)$$

$$\|X\|_2 - \|0_{m\times n}\| \geq g(0_{m\times n})^T (X - 0_{m\times n})$$

$$\|X\|_2 \geq g(0_{m\times n})^T X$$

$$\|X\|_2 X^{-1} \geq g(0_{m\times n})^T$$

$$g(0_{m\times n}) \leq (X^{-1})^T \|X\|_2^T$$

since $\|X\|_2^T$ is a scalar:

$$g(0_{m\times n}) \leq \|X\|_2 (X^{-1})^T$$

? Note Since all singular values are real, then

$$\|X\|_2 \in \mathbb{R}$$

then

$$\partial \|0_{m\times n}\| = \{ X \in \mathbb{R}^{m\times n} : X^{-1} \text{ exists}, \; ?? \}$$

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