Here are some points I would like to address about problem 3.

1. **Definition of subgradient**

   A matrix $\boldsymbol{G} \in \mathbb{R}^{m\times n}$ is a subgradient for a matrix function at $\boldsymbol{X}$ if for any $\boldsymbol{Z} \in \mathbb{R}^{m\times n}$,

   $$f(\boldsymbol{Z}) - f(\boldsymbol{X}) \geq \langle \boldsymbol{G}, \boldsymbol{Z} - \boldsymbol{X} \rangle := \sum_{i,j} g_{ij}(z_{ij} - x_{ij}) = \text{Trace}(\boldsymbol{G}^\mathsf{T}(\boldsymbol{Z} - \boldsymbol{X})). \quad (1)$$

   For vectors, we have

   $$\langle \boldsymbol{g}, \boldsymbol{z} - \boldsymbol{x} \rangle = \boldsymbol{g}^\mathsf{T}(\boldsymbol{z} - \boldsymbol{x}). \quad (2)$$

   But (2) does not hold generally for matrices.

2. **Your solution for the scalar case is not correct**:

   Whe $a < 0$, we actually have $ga \leq |a| \iff g \geq |a|/a$.

3. **Solution steps**:

   (a) It would be much easier if you consider the form $\|\boldsymbol{X}\|_2 = \sup_{\|\boldsymbol{u}\|_2 = ,\|\boldsymbol{v}\|_2 = 1} \boldsymbol{u}^\mathsf{T}\boldsymbol{X}\boldsymbol{v}$.

   (b) Use matrix differentiation (formula is in topic 3's slides) to get the subdifferential of $\boldsymbol{u}^\mathsf{T}\boldsymbol{X}\boldsymbol{v}$ at $\boldsymbol{0}$.

   (c) Use the result on page 39 in topic 3.

   (d) Write the resulting convex hull in terms of the nuclear norm. (Hint: consider the SVD form on page 5 in topic 1)