第四次作业

截止时间: 2022 年 10 月 17 日 (周一)

姓名 : 陈 稼 霖
学号 : SA21038052
成绩 : __________

第 1 题 (课本习题 2.24) 得分: _________. 考虑一维粒子被一个 δ 函数势

$$V(x)=-\nu_0\delta(x),\quad (\nu_0\text{ 为正实数})$$

束缚于一个固定的中心位置处, 求波函数和基态束缚能. 有激发的束缚态吗?

解: 该粒子的哈密顿量为

$$H=\frac{p^2}{2m}+V(x)=\begin{cases}\frac{p^2}{2m}, & x\neq 0,\\ \frac{p^2}{2m}-\nu_0\delta(x), & x=0.\end{cases}\tag{1}$$

在 $x\neq 0$ 处的薛定谔方程为

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=E\psi.\tag{2}$$

考虑到束缚态波函数在无穷远处的函数值为 0, $x<0$ 处和 $x>0$ 处波函数的通解分别为

$$\psi(x<0)=Ae^{kx},\tag{3}$$

$$\psi(x>0)=Be^{-kx},\tag{4}$$

其中 $k=\frac{\sqrt{2mE}}{\hbar}$. 利用连续性条件

$$\psi(x=0^-)=A=\psi(x=0^+)=B,\tag{5}$$

$$\psi'(x=0^+)-\psi'(x=0^-)=-Ak-Bk=\int_{0^-}^{0^+}\frac{\partial^2\psi}{\partial x'^2}\,\mathrm{d}x'=\frac{2m}{\hbar^2}\int_{0^-}^{0^+}[-\nu_0\delta(x)-E]\psi(0)\,\mathrm{d}x'=-\frac{2m\nu_0}{\hbar^2}A,\tag{6}$$

解得

$$A=B,\quad k=\frac{m\nu_0}{\hbar^2}.\tag{7}$$

考虑到波函数满足归一化条件, 得波函数

$$\psi(x)=\frac{\sqrt{m\nu_0}}{\hbar}e^{-m\nu_0|x|/\hbar^2},\tag{8}$$

基态束缚能为

$$E=-\frac{\hbar^2k^2}{2m}=-\frac{m\nu_0^2}{2\hbar^2}.\tag{9}$$

该系统仅有一个束缚基态, 没有激发的束缚态. □

第 2 题 (课本习题 2.26) 得分: _________. 一个一维粒子 $(-\infty<x<\infty)$ 受到一个可从

$$V=\lambda x,\quad (\lambda>0)$$

导出的恒力的作用.

(a) 其能谱是连续的还是分立的? 写出由 E 所确定的能量本征函数的近似表达式. 然后粗略地画出其示意图.

(b) 简略地讨论, 如果用

$$V=\lambda\left|x\right|.$$

代替 V, 什么地方需要改动?

解: (a) 在 $x\to-\infty$ 处必有 $E>V$, 粒子非束缚, 故其能谱必为连续的.

能量本征函数的近似表达式为

$$\psi(x)\sim\begin{cases}\frac{1}{[E-\lambda x]^{1/4}}e^{\pm\frac{i}{\hbar}\int_{-\infty}^{x}\sqrt{2m(E-\lambda x')}\,\mathrm{d}x'}, & \text{for } x<\frac{E}{\lambda},\\ \frac{1}{[\lambda x-E]^{1/4}}e^{-\frac{1}{\hbar}\int_{E/\lambda}^{x}\sqrt{2m(\lambda x'-E)}\,\mathrm{d}x'}, & \text{for } x>\frac{E}{\lambda}.\end{cases}\tag{10}$$

如图 1 所示.

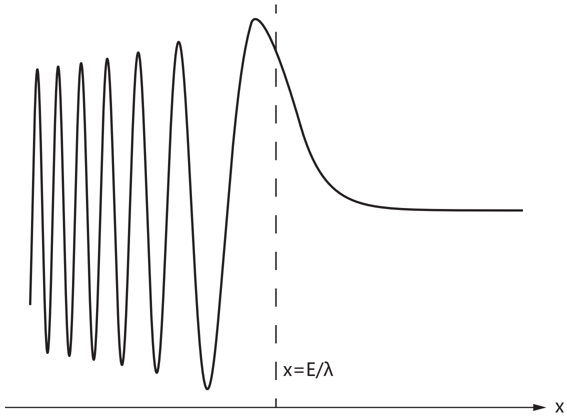


图 1: 能量本征函数的近似图像. 注意在 $x<E/\lambda$ 处波函数为波动形式, 且随着 $x\to-\infty$, 波动频率加快, 波动幅度变小; 在 $x>E/\lambda$ 处波函数为衰减形式.

(2) 若 $V=\lambda|x|$, 则粒子处于束缚态, 能谱为一系列分离的本征能量构成. 能量本征函数的近似表达式为

$$\psi(x)\sim\begin{cases}\frac{1}{[-\lambda x-E]^{1/4}}e^{\frac{1}{\hbar}\int_{-\infty}^{x}\sqrt{2m(-\lambda x'-E)}\,\mathrm{d}x'}, & \text{for } x<-\frac{E}{\lambda},\\ \frac{1}{[E-\lambda|x|]^{1/4}}\cos\left[\frac{1}{\hbar}\int_{-E/\lambda}^{x}\sqrt{2m(E-\lambda\,|x'|)}\,\mathrm{d}x'\right], & \text{for } -\frac{E}{\lambda}<x<\frac{E}{\lambda},\\ \frac{1}{[\lambda x-E]^{1/4}}e^{-\frac{1}{\hbar}\int_{E/\lambda}^{x}\sqrt{2m(\lambda x'-E)}\,\mathrm{d}x'}, & \text{for } x>\frac{E}{\lambda},\end{cases}\tag{11}$$

或

$$\psi(x)\sim\begin{cases}\frac{1}{[-\lambda x-E]^{1/4}}e^{\frac{1}{\hbar}\int_{-\infty}^{x}\sqrt{2m(-\lambda x'-E)}\,\mathrm{d}x'}, & \text{for } x<-\frac{E}{\lambda},\\ \frac{1}{[E-\lambda|x|]^{1/4}}\sin\left[\frac{1}{\hbar}\int_{-E/\lambda}^{x}\sqrt{2m(E-\lambda\,|x'|)}\,\mathrm{d}x'\right], & \text{for } -\frac{E}{\lambda}<x<\frac{E}{\lambda},\\ \frac{1}{[\lambda x-E]^{1/4}}e^{-\frac{1}{\hbar}\int_{E/\lambda}^{x}\sqrt{2m(\lambda x'-E)}\,\mathrm{d}x'}, & \text{for } x>\frac{E}{\lambda},\end{cases}\tag{12}$$

这些本征函数均为偶函数或奇函数. 能量本征值满足

$$\int_{-E/\lambda}^{E/\lambda}\sqrt{2m(E-\lambda\,|x'|)}\,\mathrm{d}x'=(n+\frac{1}{2})\pi\hbar,\quad n=0,1,2,3,\cdots\tag{13}$$

解得能量本征值为

$$E_n=\frac{\left[3\left(n+\frac{1}{4}\right)\pi\hbar\lambda\right]^{2/3}}{2m^{1/3}},\quad n=0,1,2,3,\cdots\tag{14}$$

□

第 3 题 (课本习题 2.31) 得分: _________. 导出 (2.6.16) 式, 并求得 (2.6.16) 式的三维推广.

证: 一维自由粒子的哈密顿量为

$$H=\frac{p^2}{2m},\tag{15}$$

显然其与动量算符 p 对易; $\{|p'\rangle\}$ 为哈密顿量 H 和动量算符 p 的共同本征态:

$$p|p'\rangle=p'|p'\rangle,\quad H|p'\rangle=\left(\frac{p'^2}{2m}\right)|p'\rangle.\tag{16}$$

由传播子的定义, 一维自由粒子的传播子为

$$\begin{aligned}
K(x'',t;x',t_0)=&\int_{-\infty}^{\infty}\mathrm{d}p'\,\langle x''|p'\rangle\langle p'|x'\rangle\exp\left[\frac{-iE_{p'}(t-t_0)}{\hbar}\right]\\
&(\text{利用 }\langle x'|p'\rangle=\frac{1}{\sqrt{2\pi\hbar}}\exp\left(\frac{ip'x'}{\hbar}\right))\\
=&\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}\mathrm{d}p'\,\exp\left[\frac{ip'(x''-x')}{\hbar}-\frac{ip'^2(t-t_0)}{2m\hbar}\right]\\
=&\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}\mathrm{d}p'\,\exp\left\{-\frac{i(t-t_0)}{2m\hbar}\left[p'-\frac{m(x''-x')}{t-t_0}\right]^2+\frac{im(x''-x')^2}{2\hbar(t-t_0)}\right\},\\
&(\text{令 }\xi'=\sqrt{\frac{i(t-t_0)}{2m\hbar}}\left[p'-\frac{m(x''-x')}{t-t_0}\right])\\
=&\frac{1}{2\pi\hbar}\sqrt{\frac{2m\hbar}{i(t-t_0)}}\int_{-\infty}^{\infty}\mathrm{d}\xi\,\exp(-\xi^2)\exp\left[\frac{im(x''-x')^2}{2\hbar(t-t_0)}\right],\\
&(\text{利用高斯积分 }\int_{-\infty}^{\infty}e^{-\xi^2}\,\mathrm{d}\xi=\sqrt{\pi})\\
=&\sqrt{\frac{m}{2\pi i\hbar(t-t_0)}}\exp\left[\frac{im(x''-x')^2}{2\hbar(t-t_0)}\right],
\end{aligned}\tag{17}$$

此即 (2.6.16) 式.

下面将上式推广至三维情况. 三维自由粒子的哈密顿量为

$$H=\frac{p_x^2+p_y^2+p_z^2}{2m},\tag{18}$$

显然与动量算符 p_x, p_y, p_z 对易; $\{|p'_x,p'_y,p'_z\rangle\}$ 为哈密顿量 H 和动量算符 p_x, p_y, p_z 的共同本征态:

$$p_x|p'_x,p'_y,p'_z\rangle=p'_x|p'_x,p'_y,p'_z\rangle,\quad p_y|p'_x,p'_y,p'_z\rangle=p'_y|p'_x,p'_y,p'_z\rangle,\quad p_z|p'_x,p'_y,p'_z\rangle=p'_z|p'_x,p'_y,p'_z\rangle,\tag{19}$$

$$H|p'_x,p'_y,p'_z\rangle=\frac{p'^2_x+p'^2_y+p'^2_z}{2m}|p'_x,p'_y,p'_z\rangle.\tag{20}$$

由传播子的定义, 三维自由粒子的传播子为

$$\begin{aligned}
K(\boldsymbol{x}'',t;\boldsymbol{x},t_0)=&\int_{-\infty}^{\infty}\mathrm{d}p'_x\int_{-\infty}^{\infty}\mathrm{d}p'_y\int_{-\infty}^{\infty}\mathrm{d}p'_z\,\langle\boldsymbol{x}''|p'_x,p'_y,p'_z\rangle\langle p'_x,p'_y,p'_z|\boldsymbol{x}'\rangle\exp\left[-\frac{iE_{p'_x,p'_y,p'_z}(t-t_0)}{\hbar}\right]\\
&(\text{利用 }\langle\boldsymbol{x}'|\boldsymbol{p}'\rangle=\frac{1}{(2\pi\hbar)^{3/2}}\exp\left(\frac{i\boldsymbol{p}'\cdot\boldsymbol{x}'}{\hbar}\right))\\
=&\frac{1}{(2\pi\hbar)^3}\int_{-\infty}^{\infty}\mathrm{d}p'_x\int_{-\infty}^{\infty}\mathrm{d}p'_y\int_{-\infty}^{\infty}\mathrm{d}p'_z\times\\
&\exp\left\{\frac{i[p'_x(x''-x')+p'_y(y''-y')+p'_z(z''-z')]}{\hbar}-\frac{i(p'^2_x+p'^2_y+p'^2_z)(t-t_0)}{2m\hbar}\right\}\\
=&\frac{1}{(2\pi\hbar)^3}\int_{-\infty}^{\infty}\mathrm{d}p'_x\,\exp\left\{-\frac{i(t-t_0)}{2m\hbar}[p'_x-\frac{m(x''-x')}{t-t_0}]^2+\frac{im(x''-x')}{2\hbar(t-t_0)}\right\}\times
\end{aligned}$$

$$\int_{-\infty}^{\infty}\mathrm{d}p_y'\,\exp\left\{-\frac{i(t-t_0)}{2m\hbar}[p_y'-\frac{m(y''-y')}{t-t_0}]^2+\frac{im(y''-y')}{2\hbar(t-t_0)}\right\}\times$$
$$\int_{-\infty}^{\infty}\mathrm{d}p_z'\,\exp\left\{-\frac{i(t-t_0)}{2m\hbar}[p_z'-\frac{m(z''-z')}{t-t_0}]^2+\frac{im(z''-z')}{2\hbar(t-t_0)}\right\}$$
$$=\left(\frac{m}{2\pi i\hbar(t-t_0)}\right)^{3/2}\exp\left\{\frac{im[(x''-x')^2+(y''-y')^2+(z''-z')^2]}{2\hbar(t-t_0)}\right\}. \tag{21}$$

□

第 4 题 (补充题) 得分: _________. 求一维自由粒子高斯波包坐标与动量测不准关系随时间的变化.

解: $t=0$ 时刻一维自由粒子高斯波包的波函数为

$$\psi(x',0)=e^{ip_0x'/\hbar}\frac{\exp\left(-\frac{x'^2}{2d_0^2}\right)}{(\pi d_0^2)^{1/4}}. \tag{22}$$

一维自由粒子的传播子为

$$K(x'',t;x',t_0)=\sqrt{\frac{m}{2\pi i\hbar(t-t_0)}}\exp\left[\frac{im(x''-x')^2}{2\hbar(t-t_0)}\right]. \tag{23}$$

t 时刻该波包的波函数演化为

$$\begin{aligned}
\psi(x,t)=&\int K(x,t;x',0)\psi(x',0)\,\mathrm{d}x'\\
=&\int_{-\infty}^{\infty}\sqrt{\frac{m}{2\pi i\hbar t}}\exp\left[\frac{im(x-x')^2}{2\hbar t}\right]e^{ip_0x'/\hbar}\frac{\exp\left(-\frac{x'^2}{2d_0^2}\right)}{(\pi d_0^2)^{1/4}}\,\mathrm{d}x'\\
=&\sqrt{\frac{m}{2\pi i\hbar t}}\frac{1}{(\pi d_0^2)^{1/4}}\int_{-\infty}^{\infty}\exp\left\{\left(\frac{im}{2\hbar t}-\frac{1}{2d_0^2}\right)x'^2-\left(\frac{imx}{\hbar t}-\frac{ip_0}{\hbar}\right)x'+\frac{imx^2}{2\hbar t}\right\}\mathrm{d}x'\\
=&\sqrt{\frac{m}{2\pi i\hbar t}}\frac{1}{(\pi d_0^2)^{1/4}}\int_{-\infty}^{\infty}\exp\left\{-\left(-\frac{im}{2\hbar t}+\frac{1}{2d_0^2}\right)\left[x'^2-2\frac{x-\frac{p_0t}{m}}{1+i\frac{\hbar t}{md_0^2}}+\frac{x^2}{1+i\frac{\hbar t}{md_0^2}}\right]\right\}\mathrm{d}x'\\
=&\sqrt{\frac{m}{2\pi i\hbar t}}\frac{1}{(\pi d_0^2)^{1/4}}\int_{-\infty}^{\infty}\exp\left\{-\left(-\frac{im}{2\hbar t}+\frac{1}{2d_0^2}\right)\left(x'-\frac{x-\frac{p_0t}{m}}{1+i\frac{\hbar t}{md_0^2}}\right)^2+\frac{-\frac{im}{2\hbar t}\left(x-\frac{p_0t}{m}\right)^2+\left(\frac{im}{2\hbar t}-\frac{1}{2d_0^2}\right)x^2}{1+\frac{i\hbar t}{md_0^2}}\right\}\mathrm{d}x'\\
&\left(\text{令 }\xi=\left(\frac{im}{2\hbar t}-\frac{1}{2d_0^2}\right)^{1/2}\left(x'-\frac{x-\frac{p_0t}{m}}{1+i\frac{\hbar t}{md_0^2}}\right)\right)\\
=&\sqrt{\frac{m}{2\pi i\hbar t}}\frac{1}{(\pi d_0^2)^{1/4}}\left(-\frac{im}{2\hbar t}+\frac{1}{2d_0^2}\right)^{-1/2}\int_{-\infty}^{\infty}\exp\left\{-\xi^2+\frac{-\frac{im}{2\hbar t}\left(x-\frac{p_0t}{m}\right)^2+\left(\frac{im}{2\hbar t}-\frac{1}{2d_0^2}\right)x^2}{1+\frac{i\hbar t}{md_0^2}}\right\}\mathrm{d}\xi\\
&\left(\text{利用高斯积分 }\int_{-\infty}^{\infty}e^{-\xi^2}=\sqrt{\pi}\right)\\
=&\left[\pi^{1/2}\left(d_0+\frac{i\hbar t}{md_0}\right)\right]^{-1/2}\exp\left[\frac{-\frac{im}{2\hbar t}\left(x-\frac{p_0t}{m}\right)^2+\left(\frac{im}{2\hbar t}-\frac{1}{2d_0^2}\right)x^2}{1+\frac{i\hbar t}{md_0^2}}\right]\\
=&\left[\pi^{1/2}\left(d_0+\frac{i\hbar t}{md_0}\right)\right]^{-1/2}\exp\left[\frac{-\left(x-\frac{p_0t}{m}\right)^2\left(1-\frac{i\hbar t}{md_0^2}\right)}{2d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\exp\left[\frac{ip_0}{\hbar}\left(x-\frac{p_0t}{2m}\right)\right].
\end{aligned} \tag{24}$$

此时位置坐标的期望值为

$$\langle x\rangle=\int_{-\infty}^{\infty}\psi^*(x',t)x\psi(x',t)\,\mathrm{d}x'=\frac{p_0t}{m}. \tag{25}$$

位置坐标的平方的期望值为

$$\langle x^2\rangle=\int_{-\infty}^{\infty}\psi^*(x',t)x^2\psi(x',t)\,\mathrm{d}x'$$

$$
\begin{aligned}
&=\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]x'^2\,\mathrm{d}x'\\
&=\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\left[\left(x'-\frac{p_0t}{m}\right)^2+2\frac{p_0t}{m}\left(x'-\frac{p_0t}{m}\right)+\left(\frac{p_0t}{m}\right)^2\right]\mathrm{d}x'\\
&=\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}\left\{\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\left(x'-\frac{p_0t}{m}\right)^2\mathrm{d}x'\right.\\
&\quad+2\frac{p_0t}{m}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\left(x'-\frac{p_0t}{m}\right)\mathrm{d}x'\\
&\quad\left.+\left(\frac{p_0t}{m}\right)^2\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\mathrm{d}x'\right\}\\
&\quad(\text{利用}\int_{-\infty}^{\infty}e^{-\alpha\xi^2}\,\mathrm{d}\xi'=\frac{\pi^{1/2}}{\alpha^{1/2}},\quad\int_{-\infty}^{\infty}e^{-\alpha\xi^2}\xi\,\mathrm{d}\xi=0,\quad\text{及}\quad\int_{-\infty}^{\infty}e^{-\alpha\xi^2}\xi^2\,\mathrm{d}\xi=\frac{\pi^{1/2}}{2\alpha^{3/2}})\\
&=\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}\left\{\frac{1}{2}\pi^{1/2}d_0^3\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)^{3/2}+\left(\frac{p_0t}{m}\right)^2\pi^{1/2}d_0\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)^{1/2}\right\}\\
&=\frac{1}{2}d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)+\left(\frac{p_0t}{m}\right)^2.
\end{aligned}\tag{26}
$$

位置坐标的不确定度为

$$
\langle(\Delta x)^2\rangle^{1/2}=[\langle x^2\rangle-\langle x\rangle^2]^{1/2}=\frac{1}{\sqrt{2}}d_0\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)^{1/2}.\tag{27}
$$

动量的期望值为

$$
\begin{aligned}
\langle p\rangle=&\int_{-\infty}^{\infty}\psi^*(x',t)\left(-i\hbar\frac{\partial}{\partial x'}\right)\psi(x',t)\,\mathrm{d}x'\\
=&\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{1/2}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x-\frac{p_0t}{m}\right)^2\left(1-\frac{i\hbar t}{md_0^2}\right)}{2d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\exp\left[\frac{ip_0}{\hbar}\left(x-\frac{p_0t}{2m}\right)\right]\times\\
&\left(-i\hbar\frac{\partial}{\partial x}\right)\exp\left[\frac{-\left(x-\frac{p_0t}{m}\right)^2\left(1+\frac{i\hbar t}{md_0^2}\right)}{2d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\exp\left[-\frac{ip_0}{\hbar}\left(x-\frac{p_0t}{2m}\right)\right]\mathrm{d}x'\\
=&\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\left[\frac{i\hbar\left(x-\frac{p_0t}{m}\right)\left(1+\frac{i\hbar t}{md_0^2}\right)}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}-p_0\right]\mathrm{d}x'\\
=&\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}\left\{\frac{i\hbar\left(1+\frac{i\hbar t}{md_0^2}\right)}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\left(x-\frac{p_0t}{m}\right)\mathrm{d}x'\right.\\
&\left.-p_0\int_{-\infty}^{\infty}\left[\frac{-\left(x-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\mathrm{d}x'\right\}\\
&(\text{利用}\int_{-\infty}^{\infty}e^{-\alpha\xi^2}\,\mathrm{d}\xi=\frac{\pi^{1/2}}{\alpha^{1/2}}\quad\text{及}\quad\int_{-\infty}^{\infty}e^{-\alpha\xi^2}\xi\,\mathrm{d}\xi=0)\\
=&p_0.
\end{aligned}\tag{28}
$$

动量的平方的期望值为

$$
\begin{aligned}
\langle p^2\rangle =&\int_{-\infty}^{\infty}\psi^*(x',t)\left(-\hbar^2\frac{\partial^2}{\partial x'^2}\right)\psi(x',t)\,\mathrm{d}x'\\
=&\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2\left(1-\frac{i\hbar t}{md_0^2}\right)}{2d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\exp\left[\frac{ip_0}{\hbar}\left(x'-\frac{p_0t}{2m}\right)\right]\times\\
&\left(-\hbar^2\frac{\partial^2}{\partial x'^2}\right)\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2\left(1+\frac{i\hbar t}{md_0^2}\right)}{2d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\exp\left[-\frac{ip_0}{\hbar}\left(x'-\frac{p_0t}{2m}\right)\right]\,\mathrm{d}x'\\
=&\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}(-\hbar^2)\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\times\\
&\left\{-\frac{1+\frac{i\hbar t}{md_0^2}}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}+\left[\frac{\left(x'-\frac{p_0t}{m}\right)\left(1+\frac{i\hbar t}{md_0^2}\right)}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}+\frac{ip_0}{\hbar}\right]^2\right\}\\
=&\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}(-\hbar^2)\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\times\\
&\left\{\frac{\left(1+\frac{i\hbar t}{md_0^2}\right)^2}{d_0^4\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)^2}\left(x'-\frac{p_0t}{m}\right)^2+\frac{2ip_0\left(1+\frac{i\hbar t}{md_0^2}\right)}{\hbar d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\left(x'-\frac{p_0t}{m}\right)-\frac{p_0^2}{\hbar^2}-\frac{1+\frac{i\hbar t}{md_0^2}}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right\}\mathrm{d}x'\\
=&\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}(-\hbar^2)\left\{\frac{\left(1+\frac{i\hbar t}{md_0^2}\right)^2}{d_0^4\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)^2}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\left(x'-\frac{p_0t}{m}\right)^2\mathrm{d}x'\right.\\
&+\frac{2ip\left(1+\frac{i\hbar t}{md_0^2}\right)}{\hbar d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\left(x'-\frac{p_0t}{m}\right)\mathrm{d}x'\\
&\left.-\left[\frac{p_0^2}{\hbar^2}+\frac{1+\frac{i\hbar t}{md_0^2}}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\int_{-\infty}^{\infty}\exp\left[\frac{-\left(x'-\frac{p_0t}{m}\right)^2}{d_0^2\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)}\right]\mathrm{d}x'\right\}\\
&(\text{利用}\int_{-\infty}^{\infty}e^{-\alpha\xi^2}\,\mathrm{d}\xi'=\frac{\pi^{1/2}}{\alpha^{1/2}},\quad\int_{-\infty}^{\infty}e^{-\alpha\xi^2}\xi\,\mathrm{d}\xi=0,\quad\text{及}\quad\int_{-\infty}^{\infty}e^{-\alpha\xi^2}\xi^2\,\mathrm{d}\xi=\frac{\pi^{1/2}}{2\alpha^{3/2}})\\
=&\pi^{-1/2}\left(d_0^2+\frac{\hbar^2t^2}{m^2d_0^2}\right)^{-1/2}(-\hbar^2)\left[-\frac{\pi^{1/2}\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)^{1/2}}{2d_0}-\frac{\pi^{1/2}p_0^2d_0\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)^{1/2}}{\hbar^2}\right]\\
=&\frac{\hbar^2}{2d_0^2}+p_0^2.
\end{aligned}\tag{29}
$$

动量的不确定度为

$$
\langle(\Delta p)^2\rangle^{1/2}=[\langle p^2\rangle-\langle p\rangle^2]^{1/2}=\frac{\hbar}{\sqrt{2}d_0}.\tag{30}
$$

此时位置坐标和动量满足不确定性关系:

$$
\langle(\Delta x)^2\rangle^{1/2}\langle(\Delta p)^2\rangle^{1/2}=\frac{\hbar}{2}\left(1+\frac{\hbar^2t^2}{m^2d_0^4}\right)^{1/2}\geq\frac{\hbar}{2}.\tag{31}
$$

□

第 5 题 (课本习题 2.34) 得分: ________. (a) 写出一个简谐振子对于一个有限时间间隔的经典作用量.

(b) 对于一个简谐振子, 利用费曼方法构造出微小的 $t_n - t_{n-1} = \Delta t$ 情况下的 $\langle x_n, t_n | x_{n-1}, t_{n-1}\rangle$. 只保留到 $(\Delta t)^2$ 量级的项, 证明它与由 (2.6.26) 式给出的传播子在 $t - t_0 \to 0$ 时的极限完全一致.

解: (a) 简谐振子的经典拉格朗日量为

$$L(x, \dot{x}) = \frac{m\dot{x}}{2} - \frac{m\omega^2 x^2}{2}. \tag{32}$$

对于一个有限时间间隔 (t_{n-1}, t_n) 的经典作用量为

$$\begin{aligned} S(n, n-1) =& \int_{t_{n-1}}^{t_n} \mathrm{d}t\, L(x, \dot{x}) \\ \approx& (t_n - t_{n-1}) \left[\frac{m}{2}\left(\frac{x_n - x_{n-1}}{t_n - t_{n-1}}\right)^2 - \frac{m\omega^2}{2}\left(\frac{x_n + x_{n-1}}{2}\right)^2\right] \\ \approx& \frac{m(\Delta x)^2}{2\Delta t} - \frac{m\omega^2 x_n^2 \Delta t^2}{2}, \end{aligned} \tag{33}$$

其中时间间隔 $\Delta t = t_n - t_{n-1}$, $\Delta x = x_n - x_{n-1} = x(t_n) - x(t_{n-1})$.

(b) 微小的 $t_n - t_{n-1} \equiv \Delta t$ 情况下,

$$\begin{aligned} \langle x_n, t_n | x_{n-1}, t_{n-1}\rangle =& \sqrt{\frac{m}{2\pi i\hbar\Delta t}} \exp\left[\frac{iS(n, n-1)}{\hbar}\right] \\ =& \sqrt{\frac{m}{2\pi i\hbar\Delta t}} \exp\left\{\frac{i}{\hbar}\left[\frac{m(\Delta x)^2}{2\Delta t} - \frac{m\omega^2 x_n^2 \Delta t}{2}\right]\right\}. \end{aligned} \tag{34}$$

由课本 (2.6.26) 式和 (2.6.18),

$$\begin{aligned} \langle x'', t | x', t_0\rangle =& K(x'', t; x', t_0) \\ =& \sqrt{\frac{m\omega}{2\pi i\hbar \sin[\omega(t - t_0)]}} \exp\left[\left\{\frac{im\omega}{2\hbar\sin[\omega(t - t_0)]}\right\} \times \right. \\ & \left. \{(x''^2 + x'^2)\cos[\omega(t - t_0)] - 2x''x'\}\right], \end{aligned} \tag{35}$$

在 $t - t_0 \to 0$ 的极限下, $\sin[\omega(t - t_0)] \approx \omega(t - t_0)$, $\cos[\omega(t - t_0)] \approx 1 - \frac{\omega^2(t - t_0)^2}{2}$, 值保留 $(\Delta t)^2$ 量级的项, 上式可简化为

$$\begin{aligned} \langle x'', t | x', t_0\rangle \approx& \sqrt{\frac{m}{2\pi i\hbar(t - t_0)}} \exp\left[\frac{im}{2\hbar(t - t_0)}\left\{[x'^2 + (x' + (x'' - x'))^2]\left[1 - \frac{\omega^2(t - t_0)^2}{2}\right] - 2[x' + (x'' - x')]x'\right\}\right] \\ \approx& \sqrt{\frac{m}{2\pi i\hbar(t - t_0)}} \exp\left[\frac{im}{\hbar}\left(\frac{(x'' - x')^2}{2(t - t_0)} - \frac{\omega^2 x'^2 (t - t_0)}{2}\right)\right]. \end{aligned} \tag{36}$$

可见, 利用高斯方法构造的微小的 $t_n - t_{n-1} = \Delta t$ 情况下的 $\langle x_n, t_n | x_{n-1}, t_{n-1}\rangle$ 与传播子在 $t - t_0 \to 0$ 时的极限具有一致的形式.

□

第 6 题 (课本习题 2.37) 得分: ________. (a) 证明 (2.7.25) 式和 (2.7.27) 式的正确性.

(b) 证明具有由 (2.7.31) 式给定的 $\boldsymbol{j}$ 的连续性方程 (2.7.30) 的正确性.

证: (a) 课本 (2.7.25) 式:

$$\begin{aligned}[\Pi_i,\Pi_j]=&[p_i-\frac{eA_i}{c},p_j-\frac{eA_j}{c}]\\=&-\frac{e}{c}([p_i,A_j]+[A_i,p_j])\\=&i\hbar\frac{e}{c}\left(\frac{\partial A_j}{\partial x_i}-\frac{\partial A_i}{\partial x_j}\right)\\=&\frac{i\hbar e}{c}\varepsilon_{ijk}(\nabla\times\boldsymbol{A})_k\\=&\frac{i\hbar e}{c}\varepsilon_{ijk}B_k,\end{aligned}\tag{37}$$

其中 $\varepsilon_{ijk}=\begin{cases}1, & i,j,k\text{ 为偶排列},\\-1, & i,j,k\text{ 为奇排列},\\0, & i,j,k\text{ 中有重复}.\end{cases}$

课本式 (2.7.27):

$$\begin{aligned}m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2}=&\frac{\mathrm{d}\boldsymbol{\Pi}}{\mathrm{d}t}=\frac{1}{i\hbar}[\boldsymbol{\Pi},H]\\=&\frac{1}{i\hbar}[\boldsymbol{\Pi},\frac{\boldsymbol{\Pi}^2}{2m}+e\phi]\\=&\frac{1}{2i\hbar m}[\Pi_x\hat{x}+\Pi_y\hat{y}+\Pi_z\hat{z},\boldsymbol{\Pi}^2]+\frac{e}{i\hbar}[\Pi_x\hat{x}+\Pi_y\hat{y}+\Pi_z\hat{z},\phi]\\=&\frac{1}{2i\hbar m}([\Pi_x,\boldsymbol{\Pi}^2]\hat{x}+[\Pi_y,\boldsymbol{\Pi}^2]\hat{y}+[\Pi_z,\boldsymbol{\Pi}^2]\hat{z})+\frac{e}{i\hbar}([\Pi_x,\phi]\hat{x}+[\Pi_y,\phi]\hat{y}+[\Pi_z,\phi]\hat{z}),\end{aligned}\tag{38}$$

其中

$$[\Pi_i,\Pi_j^2]=\Pi_j[\Pi_i,\Pi_j]+[\Pi_i,\Pi_j]\Pi_j=\frac{i\hbar e}{c}\varepsilon_{ijk}(\Pi_jB_k+B_k\Pi_j),\tag{39}$$

$$\begin{aligned}\Longrightarrow[\Pi_x,\boldsymbol{\Pi}^2]=&[\Pi_x,\Pi_x^2+\Pi_y^2+\Pi_z^2]\\=&[\Pi_x,\Pi_x^2]+[\Pi_x,\Pi_y^2]+[\Pi_x,\Pi_z^2]\\=&\frac{i\hbar e}{c}[(\Pi_yB_z+B_z\Pi_y)-(\Pi_zB_y+B_y\Pi_z)]\\=&\frac{i\hbar e}{c}[(\boldsymbol{\Pi}\times\boldsymbol{B})_x-(\boldsymbol{B}\times\boldsymbol{\Pi})_x],\end{aligned}\tag{40}$$

以及

$$[\Pi_i,\phi]=[p_i-\frac{e\boldsymbol{A}(\boldsymbol{x})}{c},\phi(\boldsymbol{x})]=[p_i,\phi]=-i\hbar\frac{\partial\phi}{\partial x_i},\tag{41}$$

从而

$$\begin{aligned}m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2}=&\frac{1}{2i\hbar m}\frac{i\hbar e}{c}[(\boldsymbol{\Pi}\times\boldsymbol{B})_x\hat{x}-(\boldsymbol{B}\times\boldsymbol{\Pi})_x\hat{x}+(\boldsymbol{\Pi}\times\boldsymbol{B})_y\hat{y}-(\boldsymbol{B}\times\boldsymbol{\Pi})_y\hat{y}+(\boldsymbol{\Pi}\times\boldsymbol{B})_z\hat{z}-(\boldsymbol{B}\times\boldsymbol{\Pi})_z\hat{z}]\\&+\frac{e}{i\hbar}(-i\hbar)\left(\frac{\partial\phi}{\partial x}\hat{x}+\frac{\partial\phi}{\partial y}\hat{y}+\frac{\partial\phi}{\partial z}\hat{z}\right)\\=&\frac{e}{2mc}(\boldsymbol{\Pi}\times\boldsymbol{B}-\boldsymbol{B}\times\boldsymbol{\Pi})-e\nabla\phi\\=&e\left[\boldsymbol{E}+\frac{1}{2c}\left(\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}\times\boldsymbol{B}-\boldsymbol{B}\times\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}\right)\right].\end{aligned}\tag{42}$$

(b) 电磁场中粒子的薛定谔方程为

$$
\begin{aligned}
i\hbar\frac{\partial}{\partial t}\langle\boldsymbol{x}'|\alpha,t_0;t\rangle =&\langle\boldsymbol{x}'|H|\alpha,t_0;t\rangle\\
=&\langle\boldsymbol{x}'|\left\{\frac{1}{2m}\left[p-\frac{e\boldsymbol{A}(\boldsymbol{x})}{c}\right]^2+e\phi(\boldsymbol{x})\right\}|\alpha,t_0;t\rangle\\
=&\frac{1}{2m}\left[-i\hbar\nabla'-\frac{e\boldsymbol{A}(\boldsymbol{x}')}{c}\right]\cdot\left[-i\hbar\nabla'-\frac{e\boldsymbol{A}(\boldsymbol{x}')}{c}\right]\langle\boldsymbol{x}'|\alpha,t_0;t\rangle+e\phi(\boldsymbol{x}')\langle\boldsymbol{x}'|\alpha,t_0;t\rangle\\
=&\frac{1}{2m}\left\{-\hbar^2\nabla'^2+\frac{i\hbar e}{c}\nabla'\cdot\boldsymbol{A}(\boldsymbol{x}')+\frac{i\hbar e\boldsymbol{A}(\boldsymbol{x}')}{c}\cdot\nabla'+\frac{e^2\boldsymbol{A}^2(\boldsymbol{x}')}{c}+e\phi(\boldsymbol{x}')\right\}\langle\boldsymbol{x}'|\alpha,t_0;t\rangle,
\end{aligned}
\tag{43}
$$

即

$$
i\hbar\frac{\partial}{\partial t}\psi=\frac{1}{2m}\left\{-\hbar^2\nabla'^2+\frac{i\hbar e}{c}\nabla'\cdot\boldsymbol{A}(\boldsymbol{x}')+\frac{i\hbar e\boldsymbol{A}(\boldsymbol{x}')}{c}\cdot\nabla'+\frac{e^2\boldsymbol{A}^2(\boldsymbol{x}')}{c}+e\phi(\boldsymbol{x}')\right\}\psi,\tag{44}
$$

其中 $\psi=\langle\boldsymbol{x}'|\alpha,t_0;t\rangle$, 注意大括号中第二项的 ∇' 是同时作用在 $\boldsymbol{A}'(\boldsymbol{x}')\psi$ 上. 上式的厄米共轭为

$$
-i\hbar\frac{\partial}{\partial t}\psi^*=\frac{1}{2m}\left\{-\hbar^2\nabla'^2-\frac{i\hbar e}{c}\nabla'\cdot\boldsymbol{A}(\boldsymbol{x}')-\frac{i\hbar e\boldsymbol{A}(\boldsymbol{x}')}{c}\cdot\nabla'+\frac{e^2\boldsymbol{A}^2(\boldsymbol{x}')}{c}+e\phi(\boldsymbol{x}')\right\}\psi^*.\tag{45}
$$

$-$ 式 (45) $\times\psi+\psi^*\times$ 式 (44) 得

$$
\begin{aligned}
i\hbar\frac{\partial\psi^*}{\partial t}\psi+i\hbar\psi^*\frac{\partial\psi}{\partial t}=&\frac{\hbar^2}{2m}[(\nabla'^2\psi^*)\psi-\psi^*\nabla'^2\psi]\\
&+\frac{i\hbar e}{2mc}\left\{(\nabla'\cdot[\boldsymbol{A}(\boldsymbol{x}')\psi^*])\psi+\boldsymbol{A}(\boldsymbol{x}')\cdot(\nabla'\psi^*)\psi+\psi^*\nabla'\cdot[\boldsymbol{A}'(\boldsymbol{x}')\psi]+\psi^*\boldsymbol{A}(\boldsymbol{x}')\cdot\nabla'\psi\right\},
\end{aligned}
\tag{46}
$$

其中,

$$
\text{式 (46) 左边}=i\hbar\frac{\partial(\psi^*\psi)}{\partial t}=i\hbar\frac{\partial\psi}{\partial t},\tag{47}
$$

此处 $\rho=\psi^*\psi$,

$$
\begin{aligned}
\frac{\hbar^2}{2m}[(\nabla'^2\psi^*)\psi-\psi^*\nabla'^2\psi]=&\frac{\hbar^2}{2m}[(\nabla'^2\psi^*)\psi+\nabla'\psi^*\cdot\nabla'\psi-\nabla'\psi^*\cdot\nabla'\psi-\psi^*\nabla'^2\psi]\\
=&\frac{\hbar^2}{2m}\nabla'\cdot[(\nabla'\psi^*)\psi-\psi^*\nabla'\psi]\\
=&\frac{\hbar^2}{2m}\nabla'\cdot[-2i\,\mathrm{Im}(\psi^*\nabla'\psi)]\\
=&-\frac{i\hbar^2}{m}\nabla'\cdot\mathrm{Im}(\psi^*\nabla'\psi),
\end{aligned}
\tag{48}
$$

$$
\frac{i\hbar e}{2mc}\left\{(\nabla'\cdot[\boldsymbol{A}(\boldsymbol{x}')\psi^*])\psi+\boldsymbol{A}(\boldsymbol{x}')\cdot(\nabla'\psi^*)\psi+\psi^*\nabla'\cdot[\boldsymbol{A}'(\boldsymbol{x}')\psi]+\psi^*\boldsymbol{A}(\boldsymbol{x}')\cdot\nabla'\psi\right\}=\frac{i\hbar e}{mc}\nabla'\cdot[\boldsymbol{A}(\boldsymbol{x}')\psi^*\psi].\tag{49}
$$

将以上三式代回式 (46) 得

$$
i\hbar\frac{\partial\rho}{\partial t}=-\frac{i\hbar^2}{m}\nabla'\cdot\mathrm{Im}(\psi^*\nabla\psi)+\frac{i\hbar e}{mc}\nabla'\cdot[\boldsymbol{A}(\boldsymbol{x}')\psi^*\psi],\tag{50}
$$

$$
\Longrightarrow\frac{\partial\rho}{\partial t}+\nabla'\cdot\boldsymbol{j}=0,\tag{51}
$$

此即课本式 (2.7.30), 其中概率流 j 为课本式 (2.7.31) 式所给定的形式:

$$
\boldsymbol{j}=\left(\frac{\hbar}{m}\right)\mathrm{Im}(\psi^*\nabla'\psi)-\left(\frac{e}{mc}\right)\boldsymbol{A}\left|\psi\right|^2.\tag{52}
$$

□

第 7 题 (课本习题 2.39) 得分: ________. 一个电子在一个均匀的、沿 z 方向的磁场 ($\boldsymbol{B} = B\hat{z}$) 中运动.

(a) 求

$$[\Pi_x, \Pi_y],$$

其中

$$\Pi_x \equiv p_x - \frac{eA_x}{c}, \quad \Pi_y \equiv p_y - \left(\frac{eA_y}{c}\right).$$

(b) 通过将哈密顿量及 (a) 中得到的对易关系与一维谐振子问题中相应的结果比较, 展示我们怎样能够立即写出能量本征值

$$E_{k,n} = \frac{\hbar^2 k^2}{2m} + \left(\frac{|eB|\,\hbar}{mc}\right)\left(n + \frac{1}{2}\right).$$

解: (a) 由上一题 (a) 中的结论,

$$[\Pi_x, \Pi_y] = \frac{i\hbar e}{c} B_z = \frac{i\hbar e}{c} B. \tag{53}$$

(b) 由于磁场 $\boldsymbol{B} = B\hat{z} = \nabla \times \boldsymbol{A}$ 沿 z 方向, 故 $\boldsymbol{A}$ 仅有沿 y 和 x 方向的分量. 该电子的哈密顿量为

$$H = \frac{\Pi_x^2}{2m} + \frac{\Pi_y^2}{2m} + \frac{p_z^2}{2m}. \tag{54}$$

回顾一维谐振子问题: 哈密顿量为

$$H_{\text{1D HO}} = \frac{p^2}{2m} + \frac{m\omega x^2}{2}, \tag{55}$$

且有对易关系

$$[x, p] = i\hbar. \tag{56}$$

受此启发, 我们可以定义 $Y = \frac{c\Pi_x}{eB}$, 从而该电子的哈密顿量为

$$H = \frac{\left(\frac{eBY}{c}\right)^2}{2m} + \frac{\Pi_y^2}{2m} + \frac{p_z^2}{2m} = \frac{m\left(\frac{eB}{mc}\right)^2 Y^2}{2} + \frac{\Pi_y^2}{2m} + \frac{p_z^2}{2m}, \tag{57}$$

且有对易关系

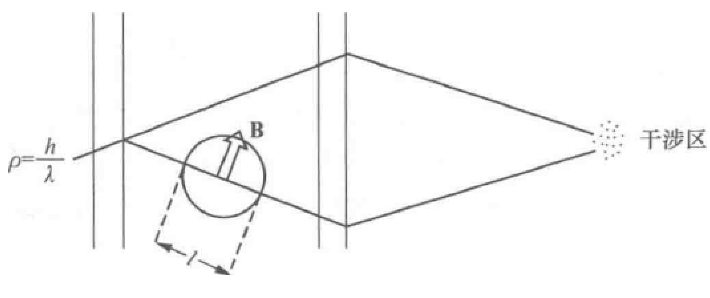
$$[Y, \Pi_y] = i\hbar. \tag{58}$$

可见, 该电子的哈密顿量相当于一个本征频率 $\omega = \frac{|eB|}{mc}$ 的一维谐振子的哈密顿量和沿 z 方向运动的自由粒子的哈密顿量之和, 从而其能量本征值为

$$E_{k,n} = \frac{\hbar^2 k^2}{2m} + \frac{|eB|\,\hbar}{mc}\left(n + \frac{1}{2}\right). \tag{59}$$

□

第 8 题 (课本习题 2.40) 得分: ________. 考虑中子干涉仪



证明在计数率中产生两个相继极大值的磁场差由下式给出

$$\Delta B = \frac{4\pi\hbar c}{|e|g_n\bar{\lambda}l},$$

其中 $g_n = (-1.91)$ 是中子磁矩. 单位为 $-e\hbar/2m_nc$. (假如你在 1967 年解出了这个问题的话, 你就会在 Physical Review Letters 上发表你的解!)

证: 中子磁矩在磁场中的势能为

$$V = -g_n\left(-\frac{e\hbar}{2m_nc}\right)B = \frac{g_ne\hbar}{2m_nc}B. \tag{60}$$

由上下两臂到达干涉区的中子束的相位差为

$$\Delta\phi = -\frac{V}{\hbar}\frac{l}{p/m_n} = -\frac{g_ne}{2m_nc}B\frac{l}{(\hbar/\bar{\lambda})/m_n}. \tag{61}$$

计数中产生两个相继极大值的情形的相位差为 2π:

$$\Delta\phi_2 - \Delta\phi_1 = -\frac{g_ne}{2m_nc}B_2\frac{l}{(\hbar/\bar{\lambda})/m_n} + \frac{g_ne}{2m_nc}B_1\frac{l}{(\hbar/\bar{\lambda})/m_n} = 2\pi, \tag{62}$$

对应的磁场差为

$$\Delta B = |B_2 - B_1| = \frac{4\pi\hbar c}{|eg_n|\,\bar{\lambda}l}. \tag{63}$$

$\square$