Intro to Communication System
EE140
Fall, 2020

Assignment 10

Due time : 10:15, Dec 18, 2020 (Friday)

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Grade : ________

Problem 1 (5.12, Orthogonal subspace) Score: ________. For any subspace $\mathcal{S}$ of an inner product space $\mathcal{V}$, define $\mathcal{S}^{\perp}$ as the set of vectors $\boldsymbol{v} \in \mathcal{V}$ that are orthogonal to all $\boldsymbol{w} \in \mathcal{S}$.

(a) Show that $\mathcal{S}^{\perp}$ is a subspace of $\mathcal{V}$.

(b) Assuming that $\mathcal{S}$ is finite-dimensional, show that any $\boldsymbol{u} \in \mathcal{V}$ can be uniquely decomposed into $\boldsymbol{u} = \boldsymbol{u}_{|\mathcal{S}} + \boldsymbol{u}_{\perp\mathcal{S}}$, where $\boldsymbol{u}_{|\mathcal{S}} \in \mathcal{S}^{\perp}$.

(c) Assuming that $\mathcal{V}$ is finite-dimensional, show that $\mathcal{V}$ has an orthonormal basis where some of the basis vectors form a basis for $\mathcal{S}$ and the remaining basis vector form a basis for $\mathcal{S}^{\perp}$.

Solution: (a)

(b)

(c)

□

Problem 2 (5.13, Othonormal expansion) Score: ________. Expand the function $\operatorname{sinc}(3t/2)$ as an orthonormal expansion in the set of functions $\{\operatorname{sinc}(t-n); -\infty < n < \infty\}$.

Solution:

□

Problem 3 (6.3) Score: ________. (a) Assume that the received signal in a 4-PAM system is $V_k = U_k + Z_k$, where U_k is the transmitted 4-PAM signal at time k. Let Z_k be independent of U_k and Gaussian with density $f_Z(z) = \sqrt{1/2\pi}\exp(-z^2/2)$. Assume that the receiver chooses the signal $\tilde{U}_k$ closest to V_k. (It is shown in Chapter 8 that this detection rule minimizes P_e for equiprobable signals.) Find the probability P_e (in terms of Gaussian intervals) that $U_k \neq \tilde{U}_k$.

(b) Evaluate the partial derivative of P_e with respect to the third signal point a_3 (i.e. the positive inner signal point) at the point where a_3 is equal to its value $d/2$ in standard 4-PAM and all other signal points are kept at 4-PAM values. [Hint. This does not require any calculation.]

Solution: (a)

(b)

□

Problem 4 (6.4, Nyquist) Score: ________. Suppose that the PAM modulated baseband waveform $u(t) = \sum_{k=-\infty}^{\infty} u_k p(t-kT)$ is received. That is, $u(t)$ is known, T is known, and $p(t)$ is known. We want to determine the signals $\{u_k\}$ from $u(t)$. Assume only linear operations can be used. That is, we wish to find some waveform $d_k(t)$ for each integer k such that $\int_{-\infty}^{\infty} u(t)d_k(t)\,\mathrm{d}t = u_k$.

(a) What properties must be satisfied by $d_k(t)$ such that the above equation is satisfied no matter what values are taken by the other signals. $\cdots, u_{k-2}, u_{k-1}, u_{k+1}, u_{k+2}, \cdots$? These properties should take the from of constrains on the inner products $\langle p(t-kT), d_j(t)\rangle$. Do not worry about convergence, interchange of limits, etc.

(b) Suppose you find a function $d_0(t)$ that satisfies these constrains for $k = 0$. Shown that, for each k, a function $d_k(t)$ satisfying these constrains can be found simply in terms of $d_0(t)$.

(c) What is the relationship between $d_0(t)$ and a function $q(t)$ that avoids intersymbol interference in the approach taken in Section 6.3 (i.e. a function $q(t)$ such that $p(t) * q(t)$ is ideal Nyquist)?

You have shown that the filter/sample approach in Section 6.3 is no less general than the arbitrary linear operation approach here. Note that, in the absence of noise and with a known constellation, it must be possible to retrieve the signals from the waveform using nonlinear operations even in the presence of intersymbol interference.

Solution: (a)

(b)

(c)

□

Problem 5 (6.5, Nyquist) Score: ________. Let $v(t)$ be a continuous $\mathcal{L}_2$ waveform with $v(0) = 1$ and define $g(t) = v(t)\,\mathrm{sinc}\,(t/T)$.

(a) Show that $g(t)$ is ideal Nyquist with interval T.

(b) Find $\hat{g}(f)$ as a function of $\hat{v}(f)$.

(c) Give a direct demonstration that $\hat{g}(f)$ satisfies the Nyquist criterion.

(d) If $v(t)$ is baseband-limited to B_b, what is $g(t)$ baseband-limited to?

Solution: (a)

(b)

(c)

(d)

□

Problem 6 (6.6, Nyquist) Score: ________. Consider a PAM baseband system in which the modulator is defined by a signal interval T and a waveform $p(t)$, the channel is defined by a filter $h(t)$, and the receiver is defined by a a filter $q(t)$ which is sampled at T-spaced intervals. The received waveform, after the receiver filter $q(t)$, is then given by $r(t) = \sum_k u_k g(t - kT)$, where $g(t) = p(t) * h(t) * q(t)$.

(a) What properties must $g(t)$ have so that $r(kT) = u_k$ for all k and for all choices of input $\{u_k\}$? What is the Nyquist criterion for $\hat{g}(f)$?

(b) Now assume that $T = 1/2$ and that $p(t)$, $h(t)$, $q(t)$ and all their Fourier transforms are restricted to be real. Assume further that $\hat{p}(f)$ and $\hat{h}(f)$ are specified by 1, i.e. by

$$\hat{p}(f) = \begin{cases} 1 & |f| \le 0.5; \\ 1.5 - t & 0.5 < |f| \le 1.5; \\ 0 & |f| > 1.5; \end{cases} \qquad \hat{h}(f) = \begin{cases} 1 & |f| \le 0.75; \\ 0 & 0.75 < |f| \le 1; \\ 1 & 1 < |f| \le 1.25; \\ 0 & |f| > 1.25. \end{cases}$$

Is it possible to choose a receiver filter transform $\hat{q}(f)$ so that there is no intersymbol interference? If so, give such a $\hat{q}(f)$ and indicate the regions in which your solution is nonunique.

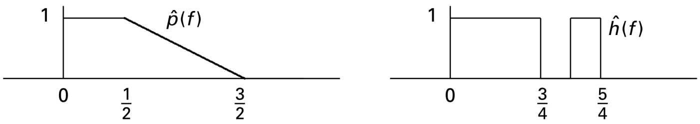

Figure 1:

(c) Redo part (b) with the modification that now $\hat{h}(f) = 1$ for $|f| \leq 0.75$ and $\hat{h}(f) = 0$ for $|f| > 0.75$.

(d) Explain the conditions on $\hat{p}(f)\hat{h}(f)$ under which intersymbol interference can be avoided by proper choice of $\hat{q}(f)$. (You may assume, as above, that $\hat{p}(f)$, $\hat{h}(f)$, $p(t)$, and $h(t)$ are all real.)

Solution: (a)

(b)

(c)

(d)

□

Problem 7 (6.16, Passband expansion) Score: _________. Prove Theorem 6.6.1. [Hint. First show that the set of functions $\{\hat{\psi}_{k,1}(f)\}$ and $\{\hat{\psi}_{k,2}(f)\}$ are orthogonal with energy 2 by comparing the integral over negative frequencies with that over positive frequencies.] Indicate explicitly why you need $f_c > B/2$.

Theorem 6.6.1 *Let $\{\theta_k(t) : k \in \mathbb{Z}\}$ be an orthonormal set limited to the frequency band $[-B/2, B/2]$. Let f_e be greater than $B/2$, and for each $k \in \mathbb{Z}$ let*

$$\psi_{k,1}(t) = Re\,[2\theta_k(t)e^{2\pi i f_c t}],$$
$$\psi_{k,2}(t) = Im\,[-2\theta_k(t)e^{2\pi i f_c t}].$$

The set $\{\psi_{k,j}; k \in \mathbb{Z}, j \in \{1,2\}\}$ is an orthogonal set of functions, each with energy 2. Furthermore if $u(f) = \sum_k u_k\theta_k(t)$, then the corresponding passband function $x(t) = 2\,Re\,[u(t)e^{2\pi i f_c t}]$ is given by

$$x(t) = \sum_k Re\,[u_k]\psi_{k,1}(t) + Im\,[u_k]\psi_{k,2}(t).$$

Solution: □