Intro to Communication System
EE140
Fall, 2020

Assignment 7

Due time : 10:15, Nov 20, 2020 (Friday)

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Grade : ________

Problem 1 (2.1 Coin flip.) Score: ________. A fair coin is flipped until the first head occurs. Let X denote the number of flips required.

(a) Find the entropy $H(X)$ in bits. The following expression may be useful:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad \sum_{n=0}^{\infty} nr^n = \frac{r}{(1-r)^2}.$$

(b) A random variable X is drawn according to this distribution. Find an "efficient" sequence of yes-no questions of the form, "Is X contained in the set S?" Compare $H(X)$ to the expected number of questions required to determine X.

Solution: $\square$

Problem 2 (2.2 Entropy of functions.) Score: ________. Let X be a random variable taking on a finite number of values. What is the (general) inequality relation of $H(X)$ and $H(Y)$ if

(a) $Y = 2^X$?

(b) $Y = \cos X$?

Solution: $\square$

Problem 3 (2.4 Entropy of functions of a random variable.) Score: ________. Let X be a discrete random variable. Show that the entropy of a function of X is less than or equal to the entropy of X by justifying the following steps:

$$\begin{aligned} H(X, g(X)) &\overset{(a)}{=} H(X) + H(g(X)|X) \\ &\overset{(b)}{=} H(X) \\ H(X, g(X)) &\overset{(c)}{=} H(g(X)) + H(X|g(X)) \\ &\overset{(d)}{\geq} H(g(X)). \end{aligned}$$

Thus, $H(g(X)) \leq H(X)$.

Solution: $\square$

Problem 4 (2.5 Zero conditional entropy.) Score: ________. Show that if $H(Y|X) = 0$, then Y is a function of X [i.e., for all x with $p(x) > 0$, there is only one possible value of y with $p(x, y) > 0$].

Solution: $\square$

Problem 5 (2.11 Measure of correlation.) Score: ________. Let X_1 and X_2 be identically distributed but not necessarily independent. Let

$$\rho = 1 - \frac{H(X_2|X_1)}{H(X_1)}.$$

(a) Show that $\rho = \frac{I(X_1;X_2)}{H(X_1)}$.

(b) Show that $0 \leq \rho \leq 1$.

(c) When is $\rho = 0$?

(d) When is $\rho = 1$?

Solution: □

Problem 6 (2.21Example of entropy.) Score: _________. Let $p(x, y)$ be given by

$X \backslash Y$	0	1
0	$\frac{1}{3}$	$\frac{1}{3}$
1	0	$\frac{1}{3}$

Find:

(a) $H(X)$, $H(Y)$.

(b) $H(X|Y)$, $H(Y|X)$.

(c) $H(X, Y)$.

(d) $H(Y) - H(Y|X)$.

(e) $I(X; Y)$.

(f) Draw a Venn diagram for the quantities in parts (a) through e.

Solution: □

Problem 7 (8.1 Diffrential entropy.) Score: _________. Evaluate the differential entropy $h(X) = -\int f \ln f$ for the following:

(a) The exponential density, $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$.

(b) The Laplace density, $f(x) = \frac{1}{2}\lambda e^{-\lambda |x|}$.

(c) The sum of X_1 and X_2, where X_1 and X_2 are independent random variables with means μ_i and variables σ_i^2, $i = 1, 2$.

Solution: □