

Problem 1 (Definition of FM, 10pts) Score: _____. An FM modulator has output $x_c(t) = 10 \cos[2\pi f_c t + 2\pi f_d \int_0^t m(\tau) d\tau]$, where $f_d = 20$ Hz/Volt. Assume that $m(t) = 3\Lambda\left(\frac{1}{3}(t-3)\right)$, as shown in Figure 1.

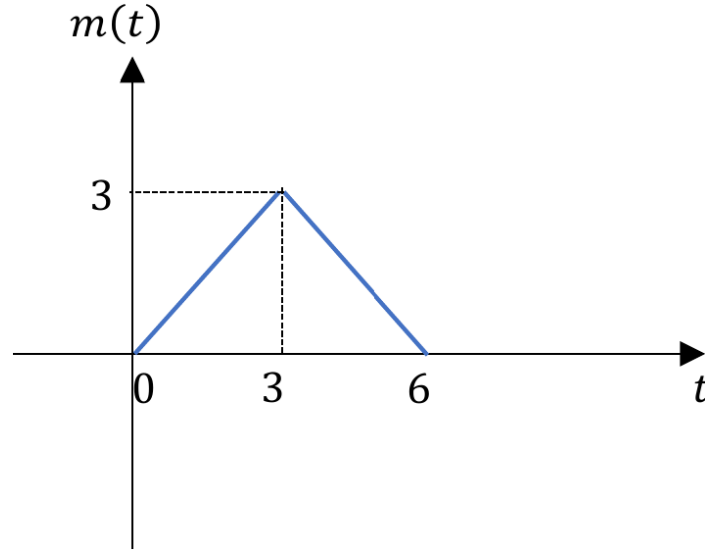


Figure 1:

- 1) Determine the phase deviation in radians.
- 2) Determine the frequency deviation in hertz.
- 3) Determine the peak frequency deviation in hertz.
- 4) Determine the peak phase deviation in radians.

Solution: 1) The phase deviation in radians is

$$\phi(t) = 2\pi f_d \int_0^t m(\tau) d\tau = 40\pi \int_0^t 3\Lambda\left(\frac{1}{3}(\tau-3)\right) d\tau = \begin{cases} 20\pi t^2, & 0 \leq t \leq 3, \\ -20\pi t^2 + 240\pi t - 360\pi, & 3 < t \leq 6, \\ 360\pi, & t > 6. \end{cases} \quad (\text{unit: rad}) \quad (1)$$

- 2) The frequency deviation in hertz is

$$\frac{1}{2\pi} \frac{d\phi}{dt} = \frac{1}{2\pi} \frac{d\left[2\pi f_d \int_0^t m(\tau) d\tau\right]}{dt} = f_d m(t) = 60\Lambda\left(\frac{1}{3}(t-3)\right) \quad (\text{unit: Hz}). \quad (2)$$

- 3) The peak frequency deviation in hertz is

$$\Delta f = \max \left[\left| \frac{1}{2\pi} \frac{d\phi}{dt} \right| \right] = 60 \text{ Hz}. \quad (3)$$

- 4) The peak phase deviation in radians is

$$\Delta \phi = \max[|\phi(t)|] = 360\pi \text{ rad}. \quad (4)$$

□

Problem 2 (Definition of PM, 10pts) Score: _____. A PM modulator has output $x_c(t) = 10 \cos[2\pi f_c t + k_p m(t)]$, where $k_p = 20$ radian/Volts. Assume that $m(t) = 3\Lambda\left(\frac{1}{3}(t-3)\right)$, as shown in Figure 1.

- 1) Determine the phase deviation in radians.
- 2) Determine the frequency deviation in hertz.
- 3) Determine the peak frequency deviation in hertz.
- 4) Determine the peak phase deviation in radians.

Solution: 1) The phase deviation in radians is

$$\phi(t) = k_p m(t) = 60\Lambda\left(\frac{1}{3}(t-3)\right) \quad (\text{unit: rad}). \quad (5)$$

- 2) The frequency deviation in hertz is

$$\frac{1}{2\pi} \frac{d\phi}{dt} = \begin{cases} \frac{10}{\pi}, & 0 \leq t \leq 3, \\ -\frac{10}{\pi}, & 3 < t \leq 6, \\ 0, & t > 6. \end{cases} \quad (\text{unit: Hz}) \quad (6)$$

- 3) The peak frequency deviation in hertz is

$$\Delta f = \max \left[\left| \frac{1}{2\pi} \frac{d\phi}{dt} \right| \right] = \frac{10}{\pi} \text{ Hz}. \quad (7)$$

- 4) The peak phase deviation in radians is

$$\Delta\phi = \max[|\phi(t)|] = 60 \text{ rad}. \quad (8)$$

□

Problem 3 Score: _____. An FM modulator has $f_c = 2000$ Hz and $f_d = 20$ Hz/Volt. The modulating message signal is $m(t) = 5 \cos 20\pi t$.

- 1) What's the peak frequency deviation?
- 2) What's the modulation index?
- 3) Is this narrow band FM? Why?
- 4) If the same $m(t)$ is used for a phase modulator, what must k_p be to yield the modulation index given in 1)?
- 5) Determine the approximate bandwidth of the FM signal, using Carson's rule.
- 6) Determine the bandwidth by transmitting only those side frequencies whose amplitude exceed 1 percent of the unmodulated carrier amplitude. Use the Table from Page 163 for this calculation. (Hint: find n_{\max} , which is the largest value of the integer that satisfies the requirement $J_n(\beta) > 0.01$. Then $B = 2n_{\max}f_m$.)
- 7) Repeat your calculation in 5), assuming that the amplitude of the modulating signal $m(t)$ is doubled. (Hint: $m(t) = 10 \cos 20\pi t$.)
- 8) Repeat your calculation in 5), assuming that the frequency of the modulating signal $m(t)$ is doubled. (Hint: $m(t) = 5 \cos 40\pi t$.)

Solution: 1) The peak frequency deviation is

$$\Delta f = f_d A_m = 20 \times 5 = 100 \text{ Hz.} \quad (9)$$

2) The modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{100}{10} = 10. \quad (10)$$

3) This is **not** narrow band FM, since narrow band FM requires that $\beta \ll 1$ but this modulation has $\beta = 10$.

4) If the same $m(t)$ is used for a phase modulator, then the modulation index is

$$\beta = k_p A_m = 5k_p. \quad (11)$$

To yield the same modulation index as in 1), we need

$$k_p = 2. \quad (12)$$

5) Using Carson's rule, the approximate bandwidth is

$$B = 2(1 + \beta)f_m = 2 \times (1 + 10) \times 10 = 220 \text{ Hz.} \quad (13)$$

6) The modulated signal is

$$x_c(t) = A_c \sum_{n=-\infty}^{+\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]. \quad (14)$$

The spectrum of modulated signal is

$$X_c(f) = \mathcal{F}[x_c(t)] = \frac{A_c}{2} \sum_{n=-\infty}^{+\infty} J_n(\beta) [\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)], \quad (15)$$

where $\beta = 10$ as obtained in 2), the unmodulated carrier amplitude is $|\frac{A_c}{2} J_0(\beta)|$ and the amplitude of side frequency $f_c \pm nf_m$ is $|\frac{A_c}{2} J_n(\beta)|$. According to the table from page 163, $|J_0(10)| = 0.246$ and $|J_n(10)| > 1\% |J_0(10)|, \forall 0 \leq n \leq 15$, so $n_{\max} = 15$. The bandwidth is

$$B = 2n_{\max}f_m = 300 \text{ Hz.} \quad (16)$$

7) If the amplitude of the modulating signal $m(t)$ is doubled, i.e., $A_m = 10$, then the peak frequency deviation is

$$\Delta f = f_d A_m = 200 \text{ Hz.} \quad (17)$$

The modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{200}{10} = 20. \quad (18)$$

Using Carson's rule, the approximate bandwidth is

$$B = 2(1 + \beta)f_m = 2 \times (1 + 20) \times 10 = 420 \text{ Hz.} \quad (19)$$

- 8) If the frequency of the modulating signal $m(t)$ is doubled, i.e., $f_m = 20$, then the peak frequency deviation is still

$$\Delta f = f_d A_m = 100 \text{ Hz} \quad (20)$$

The modulating index is

$$\beta = \frac{\Delta f}{f_m} = \frac{100}{20} = 5. \quad (21)$$

Using Carson's rule, the approximate bandwidth is

$$B = 2(1 + \beta)f_m = 2 \times (1 + 5) \times 20 = 240 \text{ Hz}. \quad (22)$$

□

Problem 4 (Bandwidth of Wideband PM, 20pts) Score: _____. Consider a PM signal produced by a sinusoidal modulating wave $m(t) = A_m \cos 2\pi f_m t$ using a modulator with a phase deviation constant equal to k_p radians per volt. The unmodulated carrier wave has frequency f_c and amplitude A_c .

- 1) Show that if the maximum phase deviation of the PM signal is much larger than 1 radian, the bandwidth of the PM signal varies linearly with the modulation frequency f_m .
- 2) Compare this characteristic of a wideband PM signal with that of the corresponding wideband FM signal.

Solution: 1) The phase deviation of the PM signal is

$$\phi(t) = k_p m(t) = k_p A_m \cos 2\pi f_m t. \quad (23)$$

The frequency deviation of the PM signal is

$$\frac{1}{2\pi} \frac{d\phi(t)}{dt} = -k_p A_m f_m \sin 2\pi f_m t. \quad (24)$$

The peak frequency deviation of the PM signal is

$$\Delta f = \max \left[\frac{1}{2\pi} \frac{d\phi(t)}{dt} \right] = k_p A_m f_m. \quad (25)$$

The bandwidth of the modulating wave is

$$W = f_m. \quad (26)$$

The deviation ratio of the PM signal is

$$D = \frac{\Delta f}{W} = k_p A_m. \quad (27)$$

The bandwidth of the PM signal is

$$B = 2(1 + D)W = 2(1 + k_p A_m)f_m. \quad (28)$$

If the maximum phase deviation of the PM signal is much larger than 1, i.e., $\max[\phi(t)] = k_p A_m \gg 1$, then

$$B \approx 2k_p A_m f_m, \quad (29)$$

i.e., the bandwidth of the PM signal varies linearly with the modulation frequency f_m .

2) For FM signal, using Carson's rule, its bandwidth is

$$B = 2(1 + \beta)f_m = 2 \left(1 + \frac{f_d}{f_m} A_m \right) f_m. \quad (30)$$

The phase deviation of the FM signal is

$$\phi(t) = 2\pi \int_0^t f_d m(\tau) d\tau = 2\pi \int_0^t f_d A_m \cos 2\pi f_m \tau d\tau = \frac{f_d A_m}{f_m} \sin 2\pi f_m t. \quad (31)$$

If the maximum phase deviation of the FM signal is much larger than 1, i.e., $\max[\phi(t)] = \frac{f_d A_m}{f_m} \gg 1$, then

$$B \approx 2f_d A_m, \quad (32)$$

i.e., the bandwidth of the FM does **not** vary with the modulation frequency f_m (but varies linearly with the amplitude of the modulation wave A_m).

□

Problem 5 (Generation of Wideband FM Signal, 20pts) Score: _____. A narrowband FM has a carrier frequency of 110 kHz and a deviation ratio of 0.05. The bandwidth of the modulating signal is 10 kHz. This narrowband FM signal is used to generate a wideband FM signal with a deviation ratio of 20 and a carrier frequency of 100 MHz. We use the Armstrong indirect FM transmitter in Figure 2 to accomplish this. Give the required value of frequency multiplication n . Also, fully define the mixer by giving two permissible frequencies for local oscillator, and define the required bandpass filter (the center frequency and the bandwidth using Carson's rule).

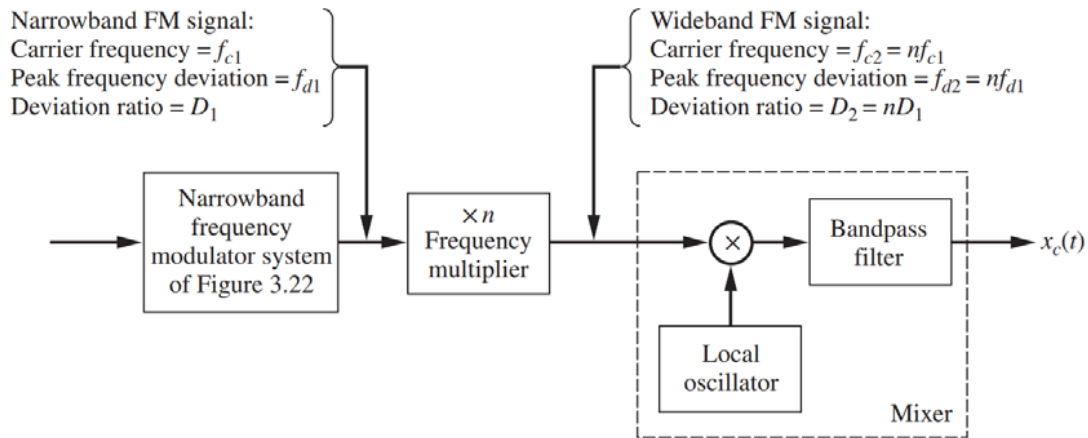


Figure 2:

Solution: The narrowband FM signal is

$$x(t) = A_c \cos[2\pi f_{c1}t + \phi(t)] = A_c \cos[2\pi \cdot 110000t + \phi(t)]. \quad (33)$$

The wideband FM signal after frequency multiplier is

$$y(t) = A_c \cos[2\pi f_{c2}t + n\phi(t)] = A_c \cos[2\pi n f_{c1}t + n\phi(t)] = A_c \cos[2\pi \cdot 110000nt + n\phi(t)]. \quad (34)$$

Suppose that the frequency of the local oscillator is f_{LO} . After multiplying with local oscillator, the signal becomes

$$e(t) = y(t) \cdot 2 \cos 2\pi f_{LO}t = 2A_c \cos[2\pi \cdot 110000nt + n\phi(t)] \cos 2\pi f_{LO}t$$

$$=A_c\{\cos[2\pi(110000n + f_{LO})t + n\phi(t)] + \cos[2\pi(110000n - f_{LO})t + n\phi(t)]\}. \quad (35)$$

To generate a wideband FM signal with a deviation ratio of 20, we require the value of frequency multiplication n to be

$$n = \frac{D_2}{D_1} = \frac{20}{0.05} = 400. \quad (36)$$

In this way, the signal after multiplying with local oscillator is

$$e(t) = A_c\{\cos[2\pi(44000000 + f_{LO})t + 400\phi(t)] + \cos[2\pi(44000000 - f_{LO})t + 400\phi(t)]\}. \quad (37)$$

To generate a wideband FM signal with a carrier frequency of 100 MHz, we require the frequency of local oscillator to be

$$f_{LO} = f_c - f_{c2} = 100000000\text{Hz} - 44000000\text{Hz} = 56000000\text{Hz} = 56\text{MHz}, \quad (38)$$

or

$$f_{LO} = f_c + f_{c2} = 100000000\text{Hz} + 44000000\text{Hz} = 144000000\text{Hz} = 144\text{MHz}. \quad (39)$$

The center frequency of the bandpass filter should be 100 MHz, and according to Carson's rule, its bandwidth should be

$$B = 2(1 + D)W = 420\text{kHz}. \quad (40)$$

(Actually, the bandwidth within range $[2(1 + D)W, f_{c2} - 2(1 + D)W] = [420\text{kHz}, 43580\text{kHz}]$ should be OK.) \square