Assignment 5

Due time : 10:15, Oct 23, 2020 (Friday)

Name : 陈 稼 霖
Student ID : 45875852
Grade : ________

Problem 1 (Definition of FM, 10pts) Score: ________. An FM modulator has output $x_c(t) = 10\cos[2\pi f_c t + 2\pi f_d \int_0^t m(\tau)\,\mathrm{d}\tau]$, where $f_d = 20$ Hz/Volt. Assume that $m(t) = 3\Lambda\left(\frac{1}{3}(t-3)\right)$, as shown in Figure 1.

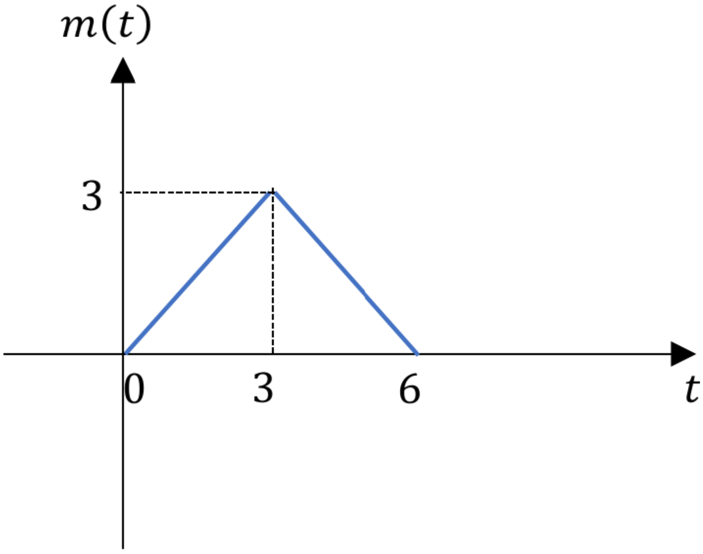

Figure 1:

1) Determine the phase deviation in radians.

2) Determine the frequency deviation in hertz.

3) Determine the peak frequency deviation in hertz.

4) Determine the peak phase deviation in radians.

Solution: 1) The phase deviation in radians is

$$\phi(t) = 2\pi f_d \int_0^t m(\tau)\,\mathrm{d}\tau = 40\pi \int_0^t 3\Lambda\left(\frac{1}{3}(\tau-3)\right)\,\mathrm{d}\tau = \begin{cases} 20\pi t^2, & 0 \leq t \leq 3, \\ -20\pi t^2 + 240\pi t - 360\pi, & 3 < t \leq 6, \\ 360\pi, & t > 6. \end{cases} \quad \text{(unit: rad)} \tag{1}$$

2) The frequency deviation in hertz is

$$\frac{1}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{1}{2\pi}\frac{\mathrm{d}\left[2\pi f_d \int_0^t m(\tau)\,d\tau\right]}{\mathrm{d}t} = f_d m(t) = 60\Lambda\left(\frac{1}{3}(t-3)\right) \quad \text{(unit: Hz)}. \tag{2}$$

3) The peak frequency deviation in hertz is

$$\Delta f = \max\left[\left|\frac{1}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}t}\right|\right] = 60 \text{ Hz}. \tag{3}$$

4) The peak phase deviation in radians is

$$\Delta\phi = \max[|\phi(t)|] = 360\pi \text{ rad}. \tag{4}$$

□

Problem 2 (Definition of PM, 10pts) Score: _________. A PM modulator has output $x_c(t) = 10\cos[2\pi f_c t + k_p m(t)]$, where $k_p = 20$ radian/Volts. Assume that $m(t) = 3\Lambda\left(\frac{1}{3}(t-3)\right)$, as shown in Figure 1.

1) Determine the phase deviation in radians.
2) Determine the frequency deviation in hertz.
3) Determine the peak frequency deviation in hertz.
4) Determine the peak phase deviation in radians.

Solution: 1) The phase deviation in radians is

$$\phi(t) = k_p m(t) = 60\Lambda\left(\frac{1}{3}(t-3)\right) \quad \text{(unit: rad)}. \tag{5}$$

2) The frequency deviation in hertz is

$$\frac{1}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}t} = \begin{cases} \frac{10}{\pi}, & 0 \le t \le 3, \\ -\frac{10}{\pi}, & 3 < t \le 6, \\ 0, & t > 6. \end{cases} \quad \text{(unit: Hz)} \tag{6}$$

3) The peak frequency deviation in hertz is

$$\Delta f = \max\left[\left|\frac{1}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}t}\right|\right] = \frac{10}{\pi}\ \text{Hz}. \tag{7}$$

4) The peak phase deviation in radians is

$$\Delta\phi = \max[|\phi(t)|] = 60\ \text{rad}. \tag{8}$$

□

Problem 3 Score: _________. An FM modulator has $f_c = 2000$ Hz and $f_d = 20$ Hz/Volt. The modulating message signal is $m(t) = 5\cos 20\pi t$.

1) What's the peak frequency deviation?
2) What's the modulation index?
3) Is this narrow band FM? Why?
4) If the same $m(t)$ is used for a phase modulator, what must k_p be to yield the modulation index given in 1)?
5) Determine the approximate bandwidth of the FM signal, using Carson's rule.
6) Determine the bandwidth by transmitting only those side frequencies whose amplitude exceed 1 percent of the unmodulated carrier amplitude. Use the Table from Page 163 for this calculation. (Hint: find $n_{\max}$, which is the largest value of the integer that satisfies the requirement $J_n(\beta) > 0.01$. Then $B = 2n_{\max} f_m$.)
7) Repeat your calculation in 5), assuming that the amplitude of the modulating signal $m(t)$ is doubled. (Hint: $m(t) = 10\cos 20\pi t$.)
8) Repeat your calculation in 5), assuming that the frequency of the modulating signal $m(t)$ is doubled. (Hint: $m(t) = 5\cos 40\pi t$.)

Solution: 1) The peak frequency deviation is

$$\Delta f = f_d A_m = 20 \times 5 = 100 \text{ Hz}. \tag{9}$$

2) The modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{100}{10} = 10. \tag{10}$$

3) This is **not** narrow band FM, since narrow band FM requires that $\beta \ll 1$ but this modulation has $\beta = 10$.

4) If the same $m(t)$ is used for a phase modulator, then the modulation index is

$$\beta = k_p A_m = 5k_p. \tag{11}$$

To yield the same modulation index as in 1), we need

$$k_p = 2. \tag{12}$$

5) Using Carson's rule, the approximate bandwidth is

$$B = 2(1+\beta)f_m = 2 \times (1+10) \times 10 = 220 \text{ Hz}. \tag{13}$$

6) The modulated signal is

$$x_c(t) = A_c \sum_{n=-\infty}^{+\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]. \tag{14}$$

The spectrum of modulated signal is

$$X_c(f) = \mathscr{F}[x_c(t)] = \frac{A_c}{2} \sum_{n=-\infty}^{+\infty} J_n(\beta) \left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right], \tag{15}$$

where $\beta = 0$ as obtained in 2), the unmodulated carrier amplitude is $\left|\frac{A_c}{2} J_0(\beta)\right|$ and the amplitude of side frequency $f_c \pm nf_m$ is $\left|\frac{A_c}{2} J_n(\beta)\right|$. According to the table from page 163, ~~$|J_0(10)| = 0.246$ and $|J_n(10)| > 1\% \, |J_0(10)|, \forall 0 \leq n$~~ $n = 14$ is the maximum n such that $J_n(10) > 0.01$, so $n_{\max} = \cancel{15}14$. The bandwidth is

$$B = 2n_{\max} f_m = \cancel{300}280 \text{ Hz}. \tag{16}$$

7) If the amplitude of the modulating signal $m(t)$ is doubled, i.e., $A_m = 10$, then the peak frequency deviation is

$$\Delta f = f_d A_m = 200 \text{ Hz}. \tag{17}$$

The modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{200}{10} = 20. \tag{18}$$

Using Carson's rule, the approximate bandwidth is

$$B = 2(1+\beta)f_m = 2 \times (1+20) \times 10 = 420 \text{ Hz}. \tag{19}$$

8) If the frequency of the modulating signal $m(t)$ is doubled, i.e., $f_m = 20$, then the peak frequency deviation is still

$$\Delta f = f_d A_m = 100 \text{ Hz} \tag{20}$$

The modulating index is

$$\beta = \frac{\Delta f}{f_m} = \frac{100}{20} = 5. \tag{21}$$

Using Carson's rule, the approximate bandwidth is

$$B = 2(1+\beta)f_m = 2 \times (1+5) \times 20 = 240 \text{ Hz}. \tag{22}$$

□

Problem 4 (Bandwidth of Wideband PM, 20pts) Score: ________. Consider a PM signal produced by a sinusoidal modulating wave $m(t) = A_m \cos 2\pi f_m t$ using a modulator with a phase deviation constant equal to k_p radians per volt. The unmodulated carrier wave has frequency f_c and amplitude A_c.

1) Show that if the maximum phase deviation of the PM signal is much larger than 1 radian, the bandwidth of the PM signal varies linearly with the modulation frequency f_m.

2) Compare this characteristic of a wideband PM signal with that of the corresponding wideband FM signal.

Solution: 1) The phase deviation of the PM signal is

$$\phi(t) = k_p m(t) = k_p A_m \cos 2\pi f_m t. \tag{23}$$

The frequency deviation of the PM signal is

$$\frac{1}{2\pi}\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = -k_p A_m f_m \sin 2\pi f_m t. \tag{24}$$

The peak frequency deviation of the PM signal is

$$\Delta f = \max\left[\frac{1}{2\pi}\frac{\mathrm{d}\phi(t)}{\mathrm{d}t}\right] = k_p A_m f_m. \tag{25}$$

The bandwidth of the modulating wave is

$$W = f_m. \tag{26}$$

The deviation ratio of the PM signal is

$$D = \frac{\Delta f}{W} = k_p A_m. \tag{27}$$

The bandwidth of the PM signal is

$$B = 2(1+D)W = 2(1+k_p A_m)f_m. \tag{28}$$

If the maximum phase deviation of the PM signal is much larger than 1, i.e., $\max[\phi(t)] = k_p A_m \gg 1$, then

$$B \approx 2k_p A_m f_m, \tag{29}$$

i.e., the bandwidth of the PM signal varies linearly with the modulation frequency f_m.

2) For FM signal, using Carson's rule, its bandwidth is

$$B = 2(1+\beta)f_m = 2\left(1 + \frac{f_d}{f_m}A_m\right)f_m. \tag{30}$$

The phase deviation of the FM signal is

$$\phi(t) = 2\pi\int_0^t f_d m(\tau)\,\mathrm{d}\tau = 2\pi\int_0^t f_d A_m \cos 2\pi f_m\tau\,\mathrm{d}\tau = \frac{f_d A_m}{f_m}\sin 2\pi f_m t. \tag{31}$$

If the maximum phase deviation of the FM signal is much larger than 1, i.e., $\max[\phi(t)] = \frac{f_d A_m}{f_m} \gg 1$, then

$$B \approx 2f_d A_m, \tag{32}$$

i.e., the bandwidth of the FM does **not** vary with the modulation frequency f_m (but varies linearly with the amplitude of the modulation wave A_m).

□

Problem 5 (Generation of Wideband FM Signal, 20pts) Score: ________. A narrowband FM has a carrier frequency of 110 kHz and a deviation ratio of 0.05. The bandwidth of the modulating signal is 10 kHz. This narrowband FM signal is used to generate a wideband FM signal with a deviation ratio of 20 and a carrier frequency of 100 MHz. We use the Armstrong indirect FM transmitter in Figure 2 to accomplish this. Give the required value of frequency multiplication n. Also, fully define the mixer by giving two permissible frequencies for local oscillator, and define the required bandpass filter (the center frequency and the bandwidth using Carson's rule).

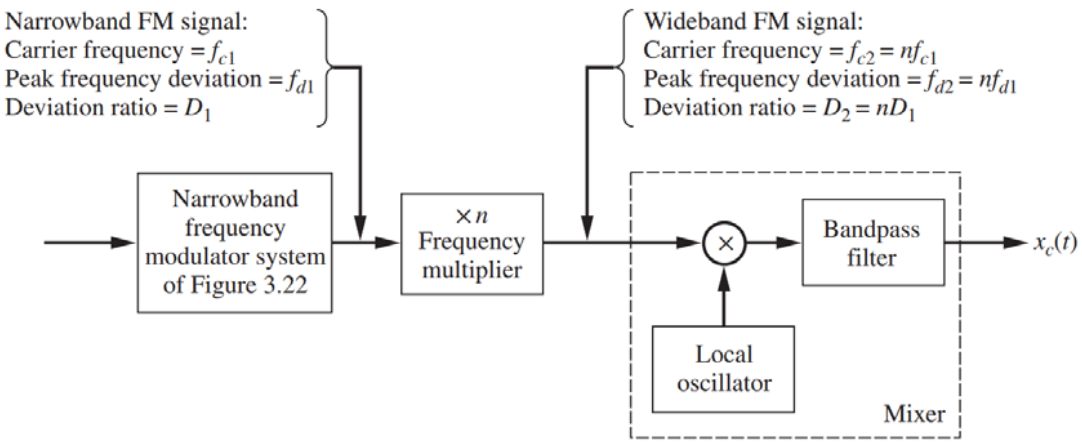

Figure 2:

Solution: The narrowband FM signal is

$$x(t) = A_c\cos[2\pi f_{c1}t + \phi(t)] = A_c\cos[2\pi\cdot 110000t + \phi(t)]. \tag{33}$$

The wideband FM signal after frequency multiplier is

$$y(t) = A_c\cos[2\pi f_{c2}t + n\phi(t)] = A_c\cos[2\pi n f_{c1}t + n\phi(t)] = A_c\cos[2\pi\cdot 110000nt + n\phi(t)]. \tag{34}$$

Suppose that the frequency of the local oscillator is f_{LO}. After multiplying with local oscillator, the signal becomes

$$e(t) = y(t)\cdot 2\cos 2\pi f_{\mathrm{LO}}t = 2A_c\cos[2\pi\cdot 110000nt + n\phi(t)]\cos 2\pi f_{\mathrm{LO}}t$$

$$=A_c\{\cos[2\pi(110000n+f_{\mathrm{LO}})t+n\phi(t)]+\cos[2\pi(110000n-f_{\mathrm{LO}})t+n\phi(t)]\}. \tag{35}$$

To generate a wideband FM signal with a deviation ratio of 20, we require the value of frequency multiplication n to be

$$n=\frac{D_2}{D_1}=\frac{20}{0.05}=400. \tag{36}$$

In this way, the signal after multiplying with local oscillator is

$$e(t)=A_c\{\cos[2\pi(44000000+f_{\mathrm{LO}})t+400\phi(t)]+\cos[2\pi(44000000-f_{\mathrm{LO}})t+400\phi(t)]\}. \tag{37}$$

To generate a wideband FM signal with a carrier frequency of 100 MHz, we require the frequency of local oscillator to be

$$f_{\mathrm{LO}}=f_c-f_{c2}=100000000\mathrm{Hz}-44000000\mathrm{Hz}=56000000\mathrm{Hz}=56\mathrm{MHz}, \tag{38}$$

or

$$f_{\mathrm{LO}}=f_c+f_{c2}=100000000\mathrm{Hz}+44000000\mathrm{Hz}=144000000\mathrm{Hz}=144\mathrm{MHz}. \tag{39}$$

The center frequency of the bandpass filter should be 100 MHz, and according to Carson's rule, its bandwidth should be

$$B=2(1+D)W=420\mathrm{kHz}. \tag{40}$$

(Actually, the bandwidth within range $[2(1+D)W, f_{c2}-2(1+D)W]=[420\mathrm{kHz}, 43580\mathrm{kHz}]$ should be OK.) □