Homework 3 (Due on Oct. 9)

1. (Autocorrelation and cross-correlation function) (30pts)

   The random processes are given by $X(t) = n(t) + Acos(2\pi f_0 t + \theta), Y(t) = n(t) + Asin(2\pi f_0 t + \theta)$, where A and f_0 are positive constants and θ is a random variable uniformly distributed in the interval $[-\pi, \pi)$. The first term n(t) represents a stationary random noise process with autocorrelation function $R_n(\tau) = B\Lambda(\tau) + C$, where B and C are positive constants. We further assume the random process n(t) and $Acos(2\pi f_0 t + \theta)$ are uncorrelated, n(t) and $Asin(2\pi f_0 t + \theta)$ are also uncorrelated.

   1) Find the autocorrelation functions of X(t) and Y(t), respectively.
   2) Find the cross-correlation function of X(t) and Y(t).
   3) Find the power spectral densities of X(t) and Y(t), respectively.
   4) Find the cross power spectral density of X(t) and Y(t).
   5) Find the total powers of X(t) and Y(t), respectively.
   6) Find the DC powers of X(t) and Y(t), respectively.

   (Hint: $\Lambda(\tau) = \begin{cases} 1-|\tau|, & |t| \le 1 \\ 0, & otherwise \end{cases}$, the DC power of X(t) is $\overline{X(t)}^2 = E^2[X(t)]$)

2. (Gaussian random process transmission through a linear system) (30pts)

   The input to a lowpass filter with impulse response $h(t) = \exp(-10t)u(t)$ is white, Gaussian noise with two-sided power spectral density of 2W/Hz. Obtain the following:

   1) The mean of the output.
   2) The power spectral density of the output.
   3) The autocorrelation function of the output.
   4) The probability density function of the output at an arbitrary time t_1.
   5) The joint probability density function of the output at times t_1 and $t_1 + 2$.
   6) Find the noise equivalent bandwidth of the filter.

   (Hint: $\mathfrak{F}[\exp(-\alpha t)\, u(t), \alpha > 0] = \frac{1}{\alpha + j2\pi f}, \mathfrak{F}[\exp(-\alpha|t|), \alpha > 0] = \frac{2\alpha}{\alpha^2 + (2\pi f)^2}$)

3. (Narrowband noise) (40pts)

   Noise n(t) has the power spectral density shown in the figure 1. We write $n(t) = n_c(t)\cos(2\pi f_0 t + \theta) - n_s(t)\sin(2\pi f_0 t + \theta)$, find and plot $S_{n_c}(f), S_{n_s}(f)$ and $S_{n_c n_s}(f)$ for the following case.

   1) $f_0 = f_1$
   2) $f_0 = f_2$
   3) $f_0 = (f_1 + f_2)/2$
   4) For which of these cases are $\mathrm{n_c(t)}$ and $\mathrm{n_s(t)}$ uncorrelated.

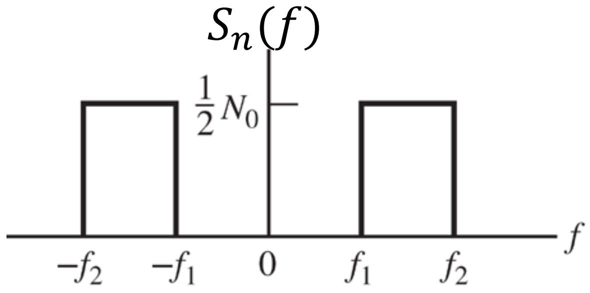

Figure 1