Simple Monte Carlo sampling

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total points: 10

1. *Sample Mean Method.* Alternative to the *Hit or Miss* algorithm, one can calculate an integral from the mean-value theorem of calculus.

$$F = \int_a^b dx f(x) = (b-a)\langle f \rangle \ . \tag{1}$$

$\langle f \rangle$ is the average value of the function $f(x)$ in the range $a \leq x \leq b$. The sampled mean value method estimates the average $\langle f \rangle$ as follows:

   (a) We choose n random number x_i from the interval $[a, b]$, which are distributed uniformly.

   (b) We compute the values of the function $f(x)$ at these points.

   (c) We take their average $\langle f \rangle = \frac{1}{n}\sum_{i=1}^{n} f(x_i)$. Please implement the above method parallelly and evaluate $\int_0^3 e^x dx$ as a function of n. **(5 points)**

2. *Sample Mean method for higher dimensional integration* Please perform the following three dimensional integration with two different methods: one uses the conventional integration on equally discretized points, the other one uses the Monte Carlo method. By using the same number of points in the two methods, compare the accuracy and speed of the two algorithms. (**5 points**)

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = \frac{648\pi}{5} a^5 \tag{2}$$