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PHYS1304
Electrodynamics
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Homework 3

Problem 1. Find the relation between the current and the field in the gap of a quadrupole magnet, assuming $\mu_{iron} = \infty$. (Hint: $B_x = ky, B_y = kx$)

Solution: As Figure 1 shows, we construct a rectangular coordinate system, and let R be the distance from the origin to one of the iron poles tips.
For any close curve, we have

$$\oint \vec{H} \cdot d\vec{l} = NI$$

For the curve shown in Figure 1, we get

$$\int_{path1} \vec{H} \cdot d\vec{l} + \int_{path2} \vec{H} \cdot d\vec{l} = NI$$

From the boundary conditions, at the interface, we have

$$B_{gap}(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}) = B_{iron}(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}})$$

From the definition of magnetic field, we have

$$H_{gap}(x,y) = \frac{B_{gap}(x,y)}{\mu_0}, \quad H_{iron}(x,y) = \frac{B_{iron}(x,y)}{\mu_0 \mu_{iron}}$$

According to the hint, on path 1 we have

$$H_{gap}(x,y) = \sqrt{2}kx$$

As a result,

$$NI = \frac{\sqrt{2}kR^2}{4} + \frac{B_{iron}}{\mu_0 \mu_{iron}}$$

Because $\mu_{iron} = \infty$, we have

$$NI = \frac{\sqrt{2}kR^2}{4}$$
$$\Longrightarrow k = \frac{2\sqrt{2}NI}{R^2}$$

Therefore, the relation ship between the current and the field in the gap of a quadrupole magnet is

$$\vec{B} = B_x\hat{x} + B_y\hat{y} = ky\hat{x} + kx\hat{y} = \frac{2\sqrt{2}NI}{R^2}(y\hat{x} + x\hat{y})$$

□

Problem 2. Find the potential of a uniformly charged ring with radius R and line charge density τ. Find the explicit function of the potential on the symmetry axis.

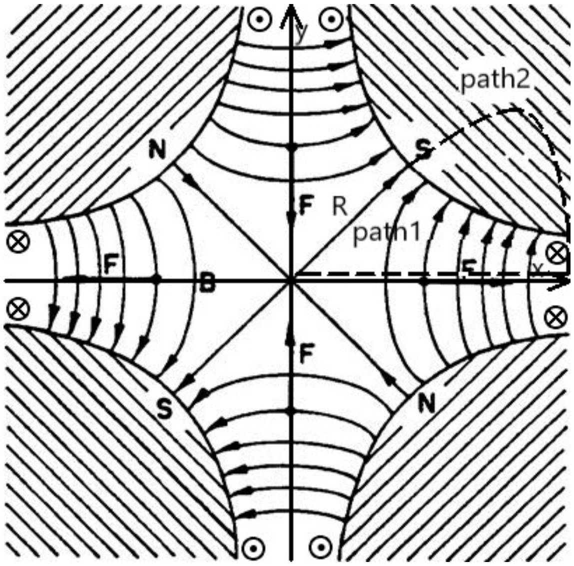

图 1: Problem 1

Solution: We construct a rectangular coordinate system whose origin is the center of the charged ring, as shown in Figure 2.The potential of the charged ring is

$$\begin{aligned}\phi(x,y,z)=&\frac{1}{4\pi\epsilon_0}\oint_{charged\ ring}\frac{\tau dl}{r}\\=&\frac{\tau}{4\pi\epsilon_0}\int_0^{2\pi}\frac{d\theta}{\sqrt{(x-R\cos\theta)^2+(y-R\sin\theta)^2+z^2}}\end{aligned}$$

The potential on the system axis (z axis) is

$$\phi(0,0,z)=\frac{\tau}{4\pi\epsilon_0}\int_0^{2\pi}\frac{d\theta}{\sqrt{R^2+R^2+z^2}}=\frac{\tau}{2\epsilon_0\sqrt{2R^2+z^2}}$$

□

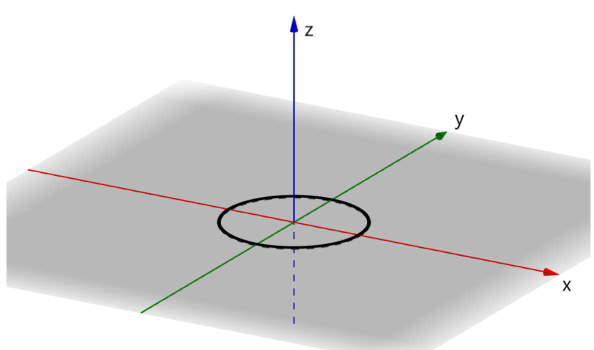

图 2: Problem 2