Chap1电磁现象的普遍规律

库仑定律$\mathbf{F} = Q'\mathbf{E} = \frac{QQ'\hat{r}}{4\pi\varepsilon_0 r^2}$　　**电场叠加性**$\mathbf{E} = \sum_{i=1}^{n} \frac{Q_i \hat{r}_i}{4\pi\varepsilon_0 r_i^2} = \int_V \frac{\rho(\mathbf{x}')\mathbf{r}dV'}{4\pi\varepsilon_0 r^3}$

高斯定理&电场散度$\oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\sum_i Q_i = \frac{1}{\varepsilon_0}\int_V \rho dV$ 或 $\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$

静电场旋度$\nabla\times\mathbf{E} = 0$　　**电荷守恒定律**$\oint_S \mathbf{J}\cdot d\mathbf{S} = -\int_V \frac{\partial\rho}{\partial t}dV$ 或 $\nabla\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = 0$

毕奥-萨伐尔定律$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int_V \frac{\mathbf{J}(\mathbf{x}')\times\mathbf{r}}{r^3}dV' = \frac{\mu_0}{4\pi}\oint_L \frac{Id\mathbf{l}\times\mathbf{r}}{r^3}$

磁场环量&旋度$\oint_L \mathbf{B}\cdot d\mathbf{l} = \mu_0 I = \mu_0\int_S \mathbf{J}\cdot d\mathbf{S}$ 或 $\nabla\times\mathbf{B} = \mu_0\mathbf{J}$

电磁感应定律$\mathscr{E} = \oint_L \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{S}$ 或 $\nabla\times\mathbf{E} = -\frac{\partial B}{\partial t}$

位移电流$\mathbf{J}_D = \varepsilon\frac{\partial\mathbf{E}}{\partial t} \Rightarrow \nabla\times\mathbf{B} = \mu(\mathbf{J}+\mathbf{J}_D)$

介质的极化 电极化强度矢量$\mathbf{P} = \frac{\sum_i \mathbf{p_i}}{\Delta V}$　　其中**电偶极矩**$\mathbf{p} = q\mathbf{l}, \mathbf{l}$由负指向正电荷

　束缚电荷密度$\int_V \rho_P dV = -\oint_S \mathbf{P}\cdot d\mathbf{S}$ 或 $\rho_P = -\nabla\cdot\mathbf{P}$

　介质分界面束缚电荷面密度$\sigma_P = -\mathbf{e}_n\cdot(\mathbf{P}_2 - \mathbf{P}_1)$　　其中$\mathbf{e}_n$由介质1指向2

　高斯定理$\varepsilon_0\nabla\cdot\mathbf{E} = \rho_f + \rho_P \Rightarrow \nabla\cdot(\varepsilon_0\mathbf{E}+\mathbf{P}) = \rho_f$

　电位移矢量$\mathbf{D} = \varepsilon_0\mathbf{E}+\mathbf{P}$ 故 $\nabla\mathbf{D} = \rho_f$

　对各向同性线性介质$P = \chi_e\varepsilon_0\mathbf{E}$ 故 $\mathbf{D} = \varepsilon\mathbf{E}$

　　其中χ_e–极化率，$\varepsilon = \varepsilon_r\varepsilon_0$–电容率，$\varepsilon_r = 1+\chi_e$–相对电容率

　对各向异性介质$D_i = \sum_{j=1}^{3}\varepsilon_{ij}E_j$

　强场下介质非线性$D_i = \sum_j \varepsilon_{ij}E_j + \sum_{j,k}\varepsilon_{ijk}E_jE_k + \sum_{jkl}\varepsilon_{ijkl}E_jE_kE_l + \cdots$

介质的磁化 分子电流磁矩$\mathbf{m} = i\mathbf{a}$　　其中i–分子电流，$\mathbf{a}$–分子电流环绕面积

　磁化强度$\mathbf{M} = \frac{\sum_i \mathbf{m}_i}{\Delta V}$　　**磁化电流**$I_M = \oint_L \mathbf{M}\cdot d\mathbf{l}$　　**磁化电流密度**$\mathbf{J}_M = \nabla\times\mathbf{M}$

　当电场变化时,介质的极化强度矢量$\mathbf{P} = \frac{\sum_i e_i\mathbf{x}_i}{\Delta V}$($\Delta V$中各带电粒子位置为$\mathbf{x}_i$，电荷为$e_i$)变化，产生极化电流密度$\mathbf{J}_P = \frac{\partial\mathbf{P}}{\partial t} = \frac{\sum_i e_i\mathbf{v}_i}{\Delta V}$，此时**磁场旋度**$\frac{1}{\mu_0}\nabla\times\mathbf{B} = \mathbf{J}_f + \mathbf{J}_M + \mathbf{J}_P + \varepsilon_0\frac{\partial E}{\partial t}$ 或 $\nabla\times(\frac{\mathbf{B}}{\mu_0} - \mathbf{M}) = \mathbf{J}_f + \frac{\partial\mathbf{D}}{\partial t}$

　定义**磁场强度**$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ 从而有$\nabla\times\mathbf{H} = \mathbf{J}_f + \frac{\partial\mathbf{D}}{\partial t}$

　对各向同性非铁磁物质$\mathbf{M} = \chi_M\mathbf{H} \Rightarrow \mathbf{B} = \mu\mathbf{H}$

　　其中χ_M–磁化率，$\mu = \mu_r\mu_0$–磁导率，相对磁导率$\mu_r = 1+\chi_M$

　对铁磁性物质，$\mathbf{B}$与$\mathbf{H}$的关系依赖于磁化过程，常用磁化曲线和磁滞回线表示

麦克斯韦方程组$\begin{cases}\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}\\ \nabla\times\mathbf{H} = \mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\\ \nabla\cdot\mathbf{D} = \rho\\ \nabla\cdot\mathbf{B} = 0\end{cases}$ (积分形式)$\begin{cases}\oint_L \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{S}\\ \oint_L \mathbf{H}\cdot d\mathbf{l} = I_f + \frac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{S}\\ \oint_S \mathbf{D}\cdot d\mathbf{S} = Q_f\\ \oint_S \cdot d\mathbf{S} = 0\end{cases}$

洛伦兹力密度$\mathbf{f} = \rho\mathbf{E}+\mathbf{J}\times\mathbf{B}$　**点电荷受洛伦兹力**$\mathbf{F} = q\mathbf{E}+q\mathbf{v}\times\mathbf{B}$　**欧姆定律**$J = \sigma\mathbf{E}$

边值关系$\begin{cases}\mathbf{e}_n\cdot(D_2+D_1 = \sigma_f & \mathbf{e}_n\times(\mathbf{E}_2-\mathbf{E}_1) = 0\\ \mathbf{e}_n\cdot(\mathbf{B}_2-\mathbf{B}_1) = 0 & \mathbf{e}_n\times(\mathbf{H}_2-\mathbf{H}_1) = \boldsymbol{\alpha}_f\\ \mathbf{e}_n\cdot(\mathbf{P}_2-\mathbf{P}_1) = -\sigma_P & \mathbf{e}_n\times(\mathbf{M}_2-\mathbf{M}_1) = \boldsymbol{\alpha}_M\\ \mathbf{e}_n\cdot(\mathbf{J}_2-\mathbf{J}_1) = -\frac{\partial\sigma}{\partial t}\end{cases}$

　其中$\sigma = \sigma_f+\sigma_P$，$\alpha$–自由电流面密度，$\boldsymbol{\alpha}_M$–磁化电流面密度

能量守恒定律$-\oint_S \mathbf{S}\cdot d\boldsymbol{\sigma} = \int_V \mathbf{f}\cdot\mathbf{v}dV + \frac{d}{dt}\int_V wdV$ 或 $-\nabla\cdot\mathbf{S} = \frac{\partial w}{\partial t}+\mathbf{f}\cdot\mathbf{v}$

　其中w–能量密度　$\mathbf{S}$–能流密度(坡印廷矢量)　$\mathbf{f}$–场对电荷作用力密度　$\mathbf{v}$–电荷速度

　能流密度$\mathbf{S} = \mathbf{E}\times\mathbf{H}$　　**能量密度变化率**$\frac{\partial w}{\partial t} = \mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t} + \mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}$

　　真空中$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}, w = \frac{1}{2}(\varepsilon_0E^2 + \frac{1}{\mu_0}B^2)$

　　线性介质中(包含极化能和磁化能，不计介质热损耗)$w = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D}+\mathbf{H}\cdot\mathbf{B})$

Chap2静电场

定义**电势**$\mathbf{E} = -\nabla\varphi$

点电荷在距离r处的电势$\varphi = \frac{Q}{4\pi\varepsilon_0 r}$　　**电势叠加**$\varphi = \sum_i \frac{Q}{4\pi\varepsilon_o r_i} = \int_V \frac{\rho(\mathbf{x}')dV'}{4\pi\varepsilon_0 r}$

泊松方程(均匀各向同性线性介质)$\nabla^2\varphi = -\frac{\rho}{\varepsilon}$　**边值关系**$\varphi_1 = \varphi_2, \varepsilon_2\frac{\partial\varphi_2}{\partial n} - \varepsilon_1\frac{\partial\varphi_1}{\partial n} = -\sigma_f$

唯一性定理 区域V内给定自由电荷分布$\rho(\mathbf{x})$，V的边界S上给定 1. 电势$\varphi|_S$ 或 2. 电势的法线方向偏导数$\frac{\partial\varphi}{\partial n}|_S$，则$V$内电场唯一地确定　　**有导体存在时的唯一性定理** 区域V内有一些导体，给定去除导体后的区域V'内的电荷密度ρ，给定各导体上电势φ_i或各导体上总电荷Q_i，且给定V的外边界S上的φ_S或$\frac{\partial\varphi}{\partial n}|_S$，则$V'$内电场唯一地确定

若区域V内无自由电荷$\rho = 0$，泊松方程化为**拉普拉斯方程**$\nabla^2\varphi = 0$

分离变量法 直角坐标系$\varphi(x,y,z) = X(x)Y(y)Z(z) \Rightarrow \frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} = -\frac{1}{Z}\frac{d^2Z}{dz^2}$

　设$\frac{1}{X}\frac{d^2X}{dx^2} = -\alpha^2, \frac{1}{Y}\frac{d^2Y}{dy^2} = -\beta^2, \frac{1}{Z}\frac{d^2Z}{dz^2} = \gamma^2$　其中$\gamma^2 = \alpha^2+\beta^2$　通解为$X(x) = \mathrm{Re}(A_\alpha e^{i\alpha x} + B_\alpha e^{-i\alpha x}), Y(y) = \mathrm{Re}(A_\beta e^{i\beta y} + B_\beta e^{-i\beta y}), Z(z) = \mathrm{Re}(A_\gamma e^{i\gamma z} + B_\gamma e^{-i\gamma z})$

柱坐标系 通解为$\varphi(r,\theta) = \sum_{n=1}^{\infty}[r^n(A_n\cos n\theta + B_n\sin n\theta) + r^{-n}(C_n\cos n\theta + D_n\sin n\theta)]$

　若轴对称，则为$\varphi(r) = A + B\ln r$

球坐标系 通解为$\varphi(R,\theta,\varphi) = \sum_{n,m}(a_{nm}R^n + \frac{b_{nm}}{R^{n+1}})P_n^m(\cos\theta)\cos m\varphi + \sum_{n,m}(c_{nm}R^n + \frac{d_{nm}}{R^{n+1}})P_n^m(\cos\theta)\sin m\varphi$　　若有对称轴且以之为极轴，则$\varphi = \sum_n(a_nR^n + \frac{b_n}{R^{n+1}})P_n(\cos\theta)$

　其中低阶勒氏多项式：　　低阶连带勒氏多项式：

$\begin{cases}P_0(x) = 1\\ P_1(x) = x\\ P_2(x) = \frac{1}{2}(3x^2-1)\\ P_3(x) = \frac{1}{2}(5x^3-3x)\\ P_4(x) = \frac{1}{8}(35x^4\\ -30x^2+3)\\ P_5(x) = \frac{1}{8}(63x^5\\ -70x^3+15x)\end{cases}$　$\begin{cases}P_l^0(x) = P_l(x) & \\ P_1^1(x) = \sqrt{1-x^2} & P_1^{-1}(x) = -\frac{1}{2}\sqrt{1-x^2}\\ P_2^1(x) = 3x\sqrt{1-x^2} & P_2^{-1}(x) = -\frac{1}{2}x\sqrt{1-x^2}\\ P_2^2(x) = 3(1-x^2) & P_2^{-2}(x) = \frac{1}{8}(1-x^2)\\ P_3^1(x) = \frac{3}{2}(1-x^2)^{\frac{1}{2}}(5x^2-1) & P_3^{-1}(x) = -\frac{1}{12}P_3^1(x)\\ P_3^2(x) = 15(1-x^2)x & P_3^{-2}(x) = \frac{1}{8}(1-x^2)x\\ P_3^3(x) = 15(1-x^2)^{\frac{3}{2}} & P_3^{-3}(x) = -\frac{1}{48}(1-x^2)^{\frac{3}{2}}\end{cases}$

接地无限大平面导体板附近有一点电荷Q，镜像电荷$Q' = -Q$位于点电荷关于导体板对称的位置，电势$\varphi = \frac{1}{4\pi\varepsilon_0}(\frac{Q}{r} - \frac{Q}{r'}) = \frac{1}{4\pi\varepsilon_0}[Q/\sqrt{x^2+y^2+(z-a)^2} - Q/\sqrt{x^2+y^2+(z+a)^2}]$

真空中一半径为R_0的接地导体球，距球心$a(>R_0)$处有一点电荷Q，镜像电荷$Q' = -\frac{R_0}{a}Q$位于距球心$b = \frac{R_0^2}{a}$处

格林函数法 给定区域V内电荷密度$\rho(\mathbf{x}')$和边界条件，泊松方程的解为

　$\varphi(\mathbf{x}) = \int_V G(\mathbf{x}',\mathbf{x})dV' + \varepsilon_0\oint_S[G(\mathbf{x}',\mathbf{x})\frac{\partial\varphi}{\partial n'} - \varphi(\mathbf{x}')\frac{\partial G(\mathbf{x}',\mathbf{x})}{\partial n'}]dS'$

　其中格林函数$G(\mathbf{x}',\mathbf{x})$是该问题边界条件下格林方程$\nabla^2G(\mathbf{x}',\mathbf{x}) = -\frac{\delta(\mathbf{x}-\mathbf{x}')}{\varepsilon_0}$的解

　第1类边界条件$G(\mathbf{x}',\mathbf{x})|_{\mathbf{x}'\in S} = 0$下

　　$\varphi(\mathbf{x}) = \int_V G(\mathbf{x}',\mathbf{x})\rho(\mathbf{x}')dV' - \varepsilon_0\oint_S[\varphi(\mathbf{x}')\frac{\partial G(\mathbf{x}',\mathbf{x})}{\partial n'}]dS'$

　第2类边界条件$\frac{\partial G(\mathbf{x}',\mathbf{x})}{\partial n'}|_{\mathbf{x}'\in S} = -\frac{1}{\varepsilon_0 S}$下，$\varphi(\mathbf{x}) = \int_V G(\mathbf{x}',\mathbf{x})\rho(\mathbf{x}')dV'$

　　$+\varepsilon_0\oint_S G(\mathbf{x}',\mathbf{x})\frac{\partial\varphi}{\partial n'}dS' + <\varphi>_S$　其中$<\varphi>_S$为界面电势平均值

电荷体系电势多级展开式$\varphi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}[\frac{q}{R} + \frac{\mathbf{p}\cdot\mathbf{R}}{R^3} + \frac{1}{6}\sum_{i,j=1}^{3}\mathcal{D}\frac{\partial^2}{\partial x_i\partial x_j}\frac{1}{R} + \cdots]$

　其中**电单极矩**$q = \int_V\rho(\mathbf{x}')dV'$，**电偶极矩**$\mathbf{p} = \int_V\rho(\mathbf{x}')\mathbf{x}'dV'$

　　电四极矩$\mathcal{D}_{ij} = \int_V 3x_i'x_j'\rho(\mathbf{x}')dV'$

　产生电势分别为$\varphi^{(0)}(\mathbf{x}) = \frac{q}{4\pi\varepsilon_0 R}, \varphi^{(1)} = \frac{\mathbf{p}\cdot\mathbf{R}}{4\pi\varepsilon_0 R^3}, \varphi^{(2)}(\mathbf{x}) =$

　　$\frac{1}{24\pi\varepsilon_0}\sum_{i,j=1}^{3}\mathcal{D}_{ij}\frac{\partial^2}{\partial x_i\partial x_j}\frac{1}{R}$　当电荷分布中心对称，$\mathbf{p} = 0$；当反对称，$\mathcal{D} = 0$

静电场总能量(线性介质)$W = \frac{1}{2}\int_\infty \mathbf{E}\cdot\mathbf{D}dV = \frac{1}{2}\int_V\rho\varphi dV = \frac{1}{8\pi\varepsilon}\int dV\int dV'\frac{\rho(\mathbf{x})\rho(\mathbf{x}')}{r}$

电势为φ_e的外场对电荷体系的作用能$W = \int_V\rho(\mathbf{x})\varphi_e(\mathbf{x})dV = q\varphi_e(0) + \mathbf{p}\cdot\nabla\varphi_e(0) + \frac{1}{6}\sum_{i,j=1}^{3}\mathcal{D}_{ij}\frac{\partial^2}{\partial x_i\partial x_j}\varphi_e(0) + \cdots$　其中$\varphi_e(0)$为外场在原点的电势

外场对电荷体系作用能$W = \int_V\rho(\mathbf{x})\varphi_e(\mathbf{x})dV$

　外场对电偶极子作用能$W^{(1)} = \mathbf{p}\cdot\nabla\varphi_e = -\mathbf{p}\cdot\mathbf{E}_e$

　外场对电偶极子作用力$\mathbf{F} = -\nabla W^{(1)} = \mathbf{p}\cdot\nabla\mathbf{E}_e$

　外场对电偶极子力矩$L_\theta = -\frac{\partial W^{(1)}}{\partial\theta} = \mathbf{p}\times\mathbf{E}_e$

　外场对电四极矩作用能$W^{(2)} = -\frac{1}{6}\sum_{i,j=1}^{3}\mathcal{D}_{ij}\frac{\partial^2}{\partial x_i\partial x_j}\mathbf{E}_e$

Chap3静磁场

定义**矢势**$\mathbf{B} = \nabla\times\mathbf{A}$ 故 $\int_S\mathbf{B}\cdot d\mathbf{S} = \oint_L\mathbf{A}\cdot d\mathbf{l}$

库伦规范$\nabla^2\mathbf{A} = -\mu_0\mathbf{J}, \nabla\cdot\mathbf{A} = 0$下，$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int_V\frac{\mathbf{J}(\mathbf{x}')}{r}dV'$

边值关系$\mathbf{A}_1 = \mathbf{A}_2, \mathbf{e}_n\times(\frac{1}{\mu_2}\nabla\times\mathbf{A}_2 - \frac{1}{\mu_1}\nabla\times\mathbf{A}_1)$

定义**磁标势**$\mathbf{H} = -\nabla\varphi_m$和**假想磁荷密度**$\rho_m = -\mu_0\nabla\cdot\mathbf{M}$得**磁标势方程**$\nabla^2\varphi_m = -\frac{\rho_m}{\mu_0}$

边值关系$\mathbf{e}_n\times(-\nabla\varphi_2+\nabla\varphi_1) = \boldsymbol{\alpha}_f, B_{2n} = B_{1n}$

　对线性均匀介质，且界面上$\boldsymbol{\alpha}_f = 0$，有$\varphi_2 = \varphi_1, \mu_2\frac{\partial\varphi_2}{\partial n} = \mu_1\frac{\partial\varphi_1}{\partial n}$

磁标势法用静电场类比静磁场

$\begin{cases}\nabla\times\mathbf{E} = 0 & \nabla\times\mathbf{H} = 0\\ \nabla\cdot\mathbf{E} = \frac{(\rho_f+\rho_P)}{\varepsilon_0} & \nabla\cdot\mathbf{H} = \frac{\rho_m}{\mu_0}\\ \rho_P = -\nabla\cdot\mathbf{P} & \rho_m = -\mu_0\nabla\cdot\mathbf{M}\end{cases}$　$\begin{cases}\mathbf{D} = \varepsilon_0\mathbf{E}+\mathbf{P} & \mathbf{B} = \mu_0\mathbf{H}+\mu_0\mathbf{M}\\ \mathbf{E} = -\nabla\varphi & \mathbf{H} = -\nabla\varphi_m\\ \nabla^2\varphi = -\frac{(\rho_f+\rho_P)}{\varepsilon_0} & \nabla^2\varphi_m = -\frac{\rho_m}{\mu_0}\end{cases}$

磁矢势多级展开式$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int_V\mathbf{J}(\mathbf{x}')[\frac{1}{R} - \mathbf{x}'\cdot\nabla\frac{1}{R} + \cdots]dV'$

　其中无磁单极矩$\int_V J(\mathbf{x})dV' = 0, \mathbf{A}^{(1)} = 0$

　　磁偶极矩$\mathbf{m} = \frac{1}{2}\int_V\mathbf{x}'\times\mathbf{J}(\mathbf{x}')dV' = \frac{I}{2}\oint_L\mathbf{x}'\times d\mathbf{l}' = I\Delta\mathbf{S}$

　　磁偶极矩产生的矢势$\mathbf{A}^{(1)} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{R}}{R^3}$

　　磁偶极矩产生的磁场$\mathbf{B}(\mathbf{x}) = \nabla\times\mathbf{A}^{(1)} = \frac{\mu_0}{4\pi}[\frac{3(\mathbf{m}\cdot\mathbf{R})\mathbf{R}}{R^5} - \frac{\mathbf{m}}{R^3}]$

静磁场总能量$W = \int_\infty\frac{1}{2}\mathbf{B}\cdot\mathbf{H}dV = \int_V\mathbf{A}\cdot\mathbf{J}dV$(后者仅需对电流分布区域积分)

　外磁场对电流作用能(不考虑电磁感应)$W = \int_V\mathbf{J}(\mathbf{x})\mathbf{A}_e(\mathbf{x})dV$

　外场对磁偶极子作用能$W^{(1)} = -\mathbf{m}\cdot\mathbf{B}_e$

　外场对磁偶极子作用力$\mathbf{F} = -\nabla W^{(1)} = \mathbf{m}\cdot\nabla\mathbf{B}_e$

　外场对磁偶极子力矩$\mathbf{L} = \mathbf{m}\times\mathbf{B}_e$

Chap4电磁波的传播

真空中的波动方程 取麦氏方程组第一式旋度并利用第二、三式得$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$

　取第二式旋度并利用第一、四式得$\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} = 0$　　其中$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

时谐电磁波$\mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x})e^{-i\omega t}, \mathbf{B}(\mathbf{x},t) = \mathbf{B}(\mathbf{x})e^{-i\omega t}$

　代入麦氏方程组得$\begin{cases}\nabla\times\mathbf{E} = i\omega\mu\mathbf{H} & \nabla\cdot\mathbf{E} = 0\\ \nabla\times\mathbf{H} = -i\omega\varepsilon\mathbf{E} & \nabla\cdot\mathbf{H} = 0\end{cases}$

　可化为$\begin{cases}\nabla^2\mathbf{E}+k^2\mathbf{E} = 0\\ \nabla\cdot\mathbf{E} = 0\\ \mathbf{B} = -\frac{i}{\omega}\nabla\times\mathbf{E}\end{cases}$ 或 $\begin{cases}\nabla^2\mathbf{B}+k^2\mathbf{B} = 0\\ \nabla\cdot\mathbf{B} = 0\\ \mathbf{E} = \frac{i}{k\sqrt{\mu\varepsilon}}\nabla\times\mathbf{B}\end{cases}$　　其中$k = \omega\sqrt{\mu\varepsilon}$

线性均匀绝缘介质中单色波相速度$v = \frac{c}{n}$　　其中折射率$n = \sqrt{\mu_r\varepsilon_r}$

电磁波为横波，电场方向(偏振方向)、磁场方向、传播方向两两正交，$\mathbf{k}\cdot\mathbf{E} = \mathbf{k}\cdot\mathbf{B} = \mathbf{E}\cdot\mathbf{B} = 0$　　**电磁场振幅比**$|\frac{E}{B}| = \frac{1}{\sqrt{\mu\varepsilon}} = v$

电磁场能量密度(线性均匀介质)$w = \frac{1}{2}(\varepsilon E^2 + \frac{1}{\mu}B^2) = (\because$ 电磁场能量相等$)\varepsilon E^2 = \frac{1}{\mu}B^2$

能流密度$\mathbf{S} = \mathbf{E}\times\mathbf{H} = \sqrt{\frac{\varepsilon}{\mu}}\mathbf{E}\times(\mathbf{e}_k\times\mathbf{E}) = \sqrt{\frac{\varepsilon}{\mu}}E^2\mathbf{e}_k = \frac{1}{\sqrt{\mu\varepsilon}}w\mathbf{e}_k = vw\mathbf{e}_k$

能量密度平均值$\bar{w} = \frac{1}{2}\varepsilon E_0^2 = \frac{1}{2\mu}B_0^2$

能流密度平均值$\bar{S} = \frac{1}{2}\mathrm{Re}(\mathbf{E}^*\times\mathbf{H}) = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^2\mathbf{e}_k$

电磁波的反射与折射 在介质界面上仅需考虑$\mathbf{e}_n\times(\mathbf{E}_2-\mathbf{E}_1) = 0, \mathbf{e}_n\times(\mathbf{H}_2-\mathbf{H}_1) = \boldsymbol{\alpha}$

反射和折射定律$\theta = \theta'$　　$\frac{\sin\theta}{\sin\theta''} = \frac{v_1}{v_2} = \frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_1}} = \frac{n_2}{n_1} = n_{21}$

菲涅尔公式 对$\mathbf{E}\perp$入射面(s偏振) $E+E' = E'', H'\cos\theta - H'\cos\theta' = H''\cos\theta''$

　　$\Rightarrow$ 非铁磁性介质$\sqrt{\varepsilon_1}(E-E')\cos\theta = \sqrt{\varepsilon_2}E''\cos\theta''$结合折射定律得

　$\begin{cases}\frac{E'}{E} = \frac{\sqrt{\varepsilon_1}\cos\theta - \sqrt{\varepsilon_2}\cos\theta''}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''} = -\frac{\sin(\theta-\theta'')}{\sin(\theta+\theta'')}\\ \frac{E''}{E} = \frac{2\sqrt{\varepsilon_1}\cos\theta}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''} = \frac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')}\end{cases}$

　对$\mathbf{E}\parallel$入射面(p偏振) $E\cos\theta - E'\cos\theta = E''\cos\theta'', H+H' = H''$

　　$\Rightarrow\sqrt{\varepsilon_1}(E+E') = \sqrt{\varepsilon_2}E''$结合折射定律得

　　$\frac{E'}{E} = \frac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')}, \frac{E''}{E} = \frac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')\cos(\theta-\theta'')}$

反射系数$R_s = \frac{E_s'^2}{E_s^2} = \frac{\sin^2(\theta-\theta'')}{\sin^2(\theta+\theta'')}, R_p = \frac{E_p'^2}{E_p^2} = \frac{\tan^2(\theta-\theta'')}{\tan^2(\theta+\theta'')}$

当正入射$\theta = 0$，$R_s = R_p = (\frac{n_2-n_1}{n_2+n_1})^2$　　无损耗时，$R+T = 1$

折射系数$T_s = \frac{E_s''^2\sqrt{\varepsilon_2}\cos\theta''}{E_s^2\sqrt{\varepsilon_1}\cos\theta} = \frac{\sin 2\theta\sin 2\theta''}{\sin^2(\theta+\theta'')}, T_p = \frac{E_p''^2\sqrt{\varepsilon_2}\cos\theta''}{E_p^2\sqrt{\varepsilon_1}\cos\theta} = \frac{4\sin 2\theta\sin 2\theta''}{(\sin 2\theta+\sin 2\theta'')^2}$

布儒斯特角满足$\theta+\theta'' = 90^\circ, \tan\theta_B = \frac{n_2}{n_1}$，此时$p$偏振无反射

当光密入射光疏介质$n_{21} < 1$，θ大于临界角$\sin\theta_c = n_{21}$，全反射，折射波电场$\mathbf{E}'' = \mathbf{E}_0''e^{-\kappa z}e^{i(k_x''x-\omega t)}$ 其中$k_x'' = k_x = k\sin\theta, k_z'' = \sqrt{k''^2 - k_x''^2} = ik\sqrt{\sin^2\theta - n_{21}^2} = i\kappa$

　此时$\frac{E'}{E} = \frac{\cos\theta - i\sqrt{\sin^2\theta - n_{21}^2}}{\cos\theta + i\sqrt{\sin^2\theta - n_{21}^2}} = e^{-2i\phi}, \tan\phi = \frac{\sqrt{\sin^2\theta - n_{21}^2}}{\cos\theta}$折射波平均能流密度$\bar{S}_x'' = \frac{1}{2}\mathrm{Re}(E_s''^*H_z'') = \frac{1}{2}\sqrt{\frac{\varepsilon_2}{\mu_2}}|E_0''|^2e^{-2\kappa z}\frac{\sin\theta}{n_{21}}, \bar{S}_z'' = -\frac{1}{2}\mathrm{Re}(E_s''^*H_x'') = 0$

导体内$\frac{\partial\rho}{\partial t} = -\nabla\cdot\mathbf{J} = -\frac{\sigma}{\varepsilon}\rho \Rightarrow \rho = \rho_0e^{-\frac{\sigma}{\varepsilon}t}$,特征时间$\tau = \frac{\varepsilon}{\sigma}$,若$\omega \ll \tau^{-1}$,视为良导体

良导体内麦氏方程组$\begin{cases}\nabla\times\mathbf{E} = i\omega\mu\mathbf{H}\\ \nabla\times\mathbf{H} = -i\omega\varepsilon\mathbf{E}+\sigma\mathbf{E}\\ \nabla\cdot\mathbf{E} = 0\\ \nabla\cdot\mathbf{H} = 0\end{cases}$　引入复电容率$\varepsilon' = \varepsilon + i\frac{\sigma}{\omega}$，有$\nabla\times\mathbf{H} = -i\omega\varepsilon'\mathbf{E}$，且各反、折射规律与前同

反射和折射定律 $\theta=\theta'$, $\frac{\sin\theta}{\sin\theta''}=\frac{\hat{n}_2}{n_1}$, $\hat{n}_2=c\sqrt{\mu_0\varepsilon'}=n+i\kappa$

其中 $n^2=\frac{c^2}{2}[\sqrt{\varepsilon^2+(\frac{\sigma}{\omega})^2}+\varepsilon]$, $\kappa^2=\frac{c^2}{2}[\sqrt{\varepsilon^2+(\frac{\sigma}{\omega})^2}-\varepsilon]$

实折射角 $\mathrm{Re}(\theta'')=\tan^{-1}[\frac{n_1\sin\theta}{q(n\cos\gamma-\kappa\sin\gamma)}]$，衰减深度 $2\frac{\omega}{c}q(\kappa\cos\gamma+n\sin\gamma)$

其中 $q^2\cos2\gamma=1-\frac{n^2-\kappa^2}{(n^2+\kappa^2)^2}(n_1\sin\theta)^2$, $q^2\sin2\gamma=\frac{2n\kappa}{(n^2+\kappa^2)^2}(n_1\sin\theta)^2$

反射系数 $R_s=|\frac{\sin(\theta-\theta'')}{\sin(\theta-\theta'')}|^2$, $R_p=|\frac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')}|^2$ 当垂直入射 $R_s=R_p=|\frac{\hat{n}_2-n_1}{\hat{n}_2+n_1}|^2=\frac{(n-n_1)^2+\kappa^2}{(n+n_1)^2+\kappa^2}$ 折射波电场 $\boldsymbol{E}(\boldsymbol{x},t)=\boldsymbol{E}_0e^{-\frac{\omega}{c}\kappa z+i\frac{\omega}{c}nz-i\omega t}$

射频谐振腔 在导体表面仅需考虑 $\boldsymbol{e}_n\times\boldsymbol{E}=0,\boldsymbol{e}_n\times\boldsymbol{H}=\boldsymbol{\alpha}$ 再由 $\nabla\cdot\boldsymbol{E}=0$ 得 $\frac{\partial E_n}{\partial n}=0$

矩形谐振腔 分离变量得 $\boldsymbol{E}$ 或 $\boldsymbol{H}$ 的任一正交分量 $u(x,y,z)=$

$(C_1\cos k_xx+D_1\sin k_xx)(C_2\cos k_yy+D_2\sin k_yy)(C_3\cos k_zz+D_3\sin k_zz)$

由边界条件得 $\begin{cases}E_x=A_1\cos k_xx\sin k_yy\sin k_zz\\E_y=A_2\sin k_xx\cos k_yy\sin k_zz\\E_z=A_3\sin k_xx\sin k_yy\cos k_zz\end{cases}$ 其中 $\begin{cases}k_x=\frac{m\pi}{L_1}\\k_y=\frac{n\pi}{L_2}\ m,n,p=0,1,\cdots\\k_z=\frac{p\pi}{L_3}\\k_xA_1+k_yA_2+k_zA_3=0\end{cases}$

本征频率 $\omega_{mnp}=\frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{L_1})^2+(\frac{n}{L_2})^2+(\frac{p}{L_3})^2}$

矩形**波导** 分离变量 $u(x,y,z)=X(x)Y(y)e^{ik_zz}$, $\frac{d^2X}{dx^2}+k_x^2X=0$, $\frac{d^2Y}{dy^2}+k_y^2Y=0$

由边界条件得 $\begin{cases}E_x=A_1\cos k_xx\sin k_yye^{ik_zz}\\E_y=A_2\sin k_xx\cos k_yye^{ik_zz}\\E_z=A_3\sin k_xx\sin k_yye^{ik_zz}\end{cases}$ 其中 $\begin{cases}k_x=\frac{m\pi}{a}\\k_y=\frac{n\pi}{b}\ m,n=0,1,\cdots\\k_x^2+k_y^2+k_z^2=k^2\\k_xA_1+k_yA_2-ik_zA_3=0\end{cases}$

(m,n) 型波的**截止频率** $\omega_{c,mn}=\frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2}$，超过则 k_z 为虚数，振幅沿 z 衰减

等离子体内外电荷分布 $\rho_e(\boldsymbol{x})$ 产生电势 $\varphi(\boldsymbol{x})=\int\frac{\rho_e(\boldsymbol{x})}{4\pi\varepsilon_0|\boldsymbol{x}-\boldsymbol{x}'|}e^{-|\boldsymbol{x}-\boldsymbol{x}'|}dV'$

在屏蔽长度 $\lambda=\sqrt{\frac{kT\varepsilon_0}{n_0e^2}}$ 外可忽略，其中 n_0 为在 $\varphi(\boldsymbol{x})=0$ 热平衡下电子密度

等离子体振荡 $\frac{\partial n}{\partial t}+\nabla\cdot(n\boldsymbol{v})=0$, $m\frac{d\boldsymbol{v}}{dt}=m(\frac{\partial\boldsymbol{v}}{\partial t}+\boldsymbol{v}\nabla\cdot\boldsymbol{v})=-e\boldsymbol{E}$, $\nabla\cdot\boldsymbol{E}=-\frac{(n-n_0)e}{\varepsilon_0}$

设 $n'=n-n_0$ 和 $\boldsymbol{v}$ 为一阶小量后有 $\frac{\partial n'}{\partial t}+n_0\nabla\cdot\boldsymbol{v}=0$, $\frac{\partial\boldsymbol{v}}{\partial t}=-\frac{e}{m}\boldsymbol{E}$, $\nabla\cdot\boldsymbol{E}=-\frac{e}{\varepsilon}n'$

$n'(t)=n'(0)e^{i\omega_pt}$，其中振荡频率 $\omega_p=\sqrt{\frac{n_0e^2}{m_e\varepsilon_0}}$, m 为电子质量(忽略阻尼)

电磁波在等离子体中的传播 $\frac{\partial\boldsymbol{v}}{\partial t}=-\frac{e}{m}(\boldsymbol{E}_i+\boldsymbol{E}_e)$，因 $\nabla\boldsymbol{E}_e=0$，内场 $\boldsymbol{E}_i$ 引起的振荡同前

外场下 $\frac{\partial\boldsymbol{J}}{\partial t}=\frac{n_0e^2}{m}\boldsymbol{E}_e$，设 $\boldsymbol{E}(\boldsymbol{x},t)=\boldsymbol{E}(\boldsymbol{x})e^{-i\omega t}$，欧姆定律 $\boldsymbol{J}(\omega)=\sigma(\omega)\boldsymbol{E}_e$

其中虚数电导率 $\sigma(\omega)=i\frac{n_0e^2}{m\omega}$，有效电容率 $\varepsilon'=\varepsilon-i\frac{\sigma}{\omega}$，波数 $k=\omega\sqrt{\mu_0\varepsilon'}=\frac{\omega}{c}\sqrt{1-(\omega_p/\omega)^2}$，折射率 $n=\sqrt{1-(\omega_p^2/\omega^2)^2}$(忽略阻尼和外磁场作用)

Chap5电磁波的辐射

电磁场 $\boldsymbol{B}=\nabla\times\boldsymbol{A},\boldsymbol{E}=-\nabla\varphi-\frac{\partial\boldsymbol{A}}{\partial t}$ 经规范变换 $\boldsymbol{A}\to\boldsymbol{A}'=\boldsymbol{A}+\nabla\psi,\varphi\to\varphi'=\varphi-\frac{\partial\psi}{\partial t}$ 不变 **规范** 对 $\nabla\boldsymbol{A}$ 的选择 **库伦规范** $\nabla\boldsymbol{A}=0$ **洛伦兹规范** $\nabla\cdot\boldsymbol{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=0$

适用于一般规范的方程组 $\begin{cases}\nabla^2\boldsymbol{A}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2}-\nabla(\nabla\cdot\boldsymbol{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t})=-\mu_0\boldsymbol{J}\\\nabla^2\varphi+\frac{\partial}{\partial t}\nabla\cdot\boldsymbol{A}=-\frac{\rho}{\varepsilon_0}\end{cases}$

库仑规范下化为 $\begin{cases}\nabla^2\boldsymbol{A}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2}-\frac{1}{c^2}\frac{\partial}{\partial t}\nabla\varphi=-\mu_0\boldsymbol{J}\\\nabla^2\varphi=-\frac{\rho}{\varepsilon_0}\end{cases}$

洛伦兹规范下为达朗贝尔方程 $\begin{cases}\nabla^2\boldsymbol{A}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2}=-\mu_0\boldsymbol{J}\\\nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=-\frac{\rho}{\varepsilon_0}\end{cases}$ 其解为

推迟势 $\boldsymbol{A}(\boldsymbol{x},t)=\frac{\mu_0}{4\pi}\int_V\frac{\boldsymbol{J}(\boldsymbol{x}',t-r/c)}{r}dV'$, $\varphi(\boldsymbol{x},t)=\frac{1}{4\pi\varepsilon_0}\int_V\frac{\rho(\boldsymbol{x}',t-r/c)}{r}dV'$

其中 $r=|\boldsymbol{x}-\boldsymbol{x}'|$，若 $\boldsymbol{J}(\boldsymbol{x}',t)=\boldsymbol{J}(\boldsymbol{x}')e^{-i\omega t}$，则

$\boldsymbol{A}(\boldsymbol{x},t)=\boldsymbol{A}(\boldsymbol{x})e^{-i\omega t}$, $\boldsymbol{A}(\boldsymbol{x})=\frac{\mu_0}{4\pi}\int_V\frac{\boldsymbol{J}(\boldsymbol{x}')e^{ikt}}{r}dV'$ 当 $l\ll\lambda,l\ll r$，分3类

(1)**近区** $r\ll\lambda,kr\ll1\Rightarrow e^{ikr}\ll1$ 近似恒定场；(2)**感应区** $r\sim\lambda$,过渡区域；

(3)**远区** $r\gg\lambda\Rightarrow r\approx R-\boldsymbol{e}_R\cdot\boldsymbol{x}'$, $\boldsymbol{A}(\boldsymbol{x})=\frac{\mu_0}{4\pi}\int_V\frac{\boldsymbol{J}(\boldsymbol{x}')e^{ik(\boldsymbol{R}-\boldsymbol{e}_R\cdot\boldsymbol{x}')}}{R-\boldsymbol{e}_R\cdot\boldsymbol{x}'}dV'$, $\boldsymbol{B}=\nabla\times\boldsymbol{A}\approx ik\boldsymbol{e}_R\times\boldsymbol{A}$, $\boldsymbol{E}=c\boldsymbol{B}\times\boldsymbol{e}_R$, $\boldsymbol{R}$ 由坐标原点至场点，近似辐射场

辐射场多级展开式(远区) $\boldsymbol{A}(\boldsymbol{x})=\frac{\mu_0e^{ikR}}{4\pi R}\int_V\boldsymbol{J}(\boldsymbol{x}')[1-ik\boldsymbol{e}_R\cdot\boldsymbol{x}'+\cdots]dV'$

首项–电偶极 $\boldsymbol{p}=\boldsymbol{p}_0e^{ikR}$ 辐射场 $\boldsymbol{A}=\frac{\mu_0e^{ikR}}{4\pi R}\dot{\boldsymbol{p}}$, $\boldsymbol{B}=ik\boldsymbol{e}_R\times\boldsymbol{A}=\frac{e^{ikR}}{4\pi\varepsilon_0c^3R}\ddot{\boldsymbol{p}}\times\boldsymbol{e}_R$

$\boldsymbol{E}=c\boldsymbol{B}\times\boldsymbol{e}_R=\frac{e^{ikR}}{4\pi\varepsilon_0c^2R}(\ddot{\boldsymbol{p}}\times\boldsymbol{e}_R)\times\boldsymbol{e}_R$

平均辐射能流 $\bar{\boldsymbol{S}}=\frac{1}{2}\mathrm{Re}(\boldsymbol{E}^*\times\boldsymbol{H})=\frac{c}{2\mu_0}\mathrm{Re}[(\boldsymbol{B}^*\times\boldsymbol{e}_R)\times\boldsymbol{B}]=\frac{c}{2\mu_0}(\boldsymbol{B}^*\cdot\boldsymbol{B})\boldsymbol{e}_R=\frac{\mu_0\omega^4p_0^2}{32\pi^2cR^2}\sin^2\theta\boldsymbol{e}_R$, $\sin^2\theta$–角分布因子 **平均辐射功率** $\bar{P}=\oint_S\bar{S}\cdot R^2d\Omega\boldsymbol{e}_R=\frac{\mu_0\omega^4p_0^2}{12\pi c}$

第2项=磁偶极+电四极辐射，磁偶极 $\boldsymbol{m}=\boldsymbol{m}_0e^{-i\omega t}$ 辐射场 $\boldsymbol{A}=\frac{ik\mu_0e^{ikR}}{4\pi R}\boldsymbol{e}_R\times\boldsymbol{m}$

$\boldsymbol{B}=ik\boldsymbol{e}_R\times\boldsymbol{A}=\frac{\mu_0e^{ikR}}{4\pi c^2R}(\ddot{\boldsymbol{m}}\times\boldsymbol{e}_R)\times\boldsymbol{e}_R$, $\boldsymbol{E}=c\boldsymbol{B}\times\boldsymbol{e}_R=-\frac{\mu_0e^{ikR}}{4\pi cR}\ddot{\boldsymbol{m}}\times\boldsymbol{e}_R$

平均辐射能流 $\bar{S}=\frac{c}{2\mu_0}(\boldsymbol{B}^*\cdot\boldsymbol{B})\boldsymbol{e}_R=\frac{\mu_0\omega^4m_0^2}{32\pi^2c^3R^2}\sin^2\theta\boldsymbol{e}_R$

平均辐射功率 $\bar{P}=\frac{\mu_0\omega^4m_0^2}{12\pi c^3}$

电四极辐射场 $\boldsymbol{A}=-\frac{ik\mu_0e^{ikR}}{24\pi R}\dot{\boldsymbol{\mathcal{D}}}=\frac{\mu_0e^{ikR}}{24\pi cR}\ddot{\boldsymbol{\mathcal{D}}}$ 其中 $\boldsymbol{\mathcal{D}}=\boldsymbol{e}_R\cdot\overleftrightarrow{\mathcal{D}}$

$\boldsymbol{B}=\frac{\mu_0e^{ikR}}{24\pi c^2R}\dddot{\boldsymbol{\mathcal{D}}}\times\boldsymbol{e}_R$, $\boldsymbol{E}=c\boldsymbol{B}\times\boldsymbol{e}_R$

平均辐射能流 $\frac{c}{2\mu_0}(\boldsymbol{B}^*\cdot\boldsymbol{B})\boldsymbol{e}_R=\frac{\mu_0}{4\pi}\frac{1}{288\pi c^3R^2}(\dddot{\boldsymbol{\mathcal{D}}}\times\boldsymbol{e}_R)^2\boldsymbol{e}_R$

平均辐射功率 $\bar{P}=\frac{\mu_0}{4\pi}\frac{1}{360c^3}\sum_{i,j=1}^3|\dddot{\boldsymbol{\mathcal{D}}}_{ij}|^2$

短天线辐射 $I(z)=I_0(1-\frac{2}{l}|z|),|z|\le\frac{l}{2}\ll\lambda$ 电偶极变化率 $\dot{\boldsymbol{p}}=\int_{-l/2}^{l/2}\boldsymbol{I}(z)dz=\frac{1}{2}I_0\boldsymbol{l}$

功率 $P=\frac{\mu_0I_0^2\omega^2l^2}{48\pi c}=\frac{\pi}{12}\sqrt{\frac{\mu_0}{\varepsilon_0}}I_0^2(\frac{l}{\lambda})^2=\frac{1}{2}R_rI_0^2$ 辐射电阻 $R_r=\frac{\pi}{6}\sqrt{\frac{\mu_0}{\varepsilon_0}}(\frac{l}{\lambda})^2$

天线辐射 $I(z)=I_0\sin k(\frac{l}{2}-|z|)=($当 $l=\frac{\lambda}{2})I_0\cos kz,|z|\le\frac{l}{2}\sim\lambda$

辐射场 $\boldsymbol{A}(\boldsymbol{x})=\frac{\mu_0}{4\pi}\int_{\lambda/4}^{\lambda/4}\frac{e^{ikr}}{r}I_0\cos kzdz=\frac{\mu_0I_0e^{ikR}}{2\pi kR}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\boldsymbol{e}_z$

能流密度 $\bar{S}=\frac{1}{2}\mathrm{Re}(\boldsymbol{E}^*\times\boldsymbol{H})=\frac{\mu_0cI_0^2}{8\pi^2R^2}\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\boldsymbol{e}_R$，角分布 $\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$

功率 $P=\oint|\bar{\boldsymbol{S}}|R^2d\Omega=\frac{\mu_0cI_0^2}{4\pi}\int_0^\pi\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta}d\theta$

电磁波衍射 任一分量满足 $(\nabla^2+k^2)\varphi=0$ 用格林函数法 $(\nabla^2+k^2)G=-4\pi\delta(\boldsymbol{x}-\boldsymbol{x}')$

$\Rightarrow$**基尔霍夫公式** $\varphi(\boldsymbol{x})=-\frac{1}{4\pi}\oint_S\frac{e^{ikr}}{r}\boldsymbol{e}_n\cdot[\nabla'\varphi(\boldsymbol{x}')+(ik-\frac{1}{r})\frac{\boldsymbol{r}}{r}\varphi(\boldsymbol{x}')]dS'$

夫琅禾费小孔衍射 $\varphi(\boldsymbol{x})=-\frac{i\varphi_0e^{ikR}}{4\pi R}\int_{S_0}e^{i(\boldsymbol{k}_1-\boldsymbol{k}_2)\cdot\boldsymbol{x}'}(\cos\theta_1+\cos\theta_2)dS'$

其中 φ–入射波振幅，R–孔心距场点，S_0–孔面积，$\boldsymbol{x}'$–孔上任一点，$\theta_{1/2}$–孔前/后波矢 $\boldsymbol{k}_{1/2}$ 与孔面法线夹角

电磁场动量密度 $\boldsymbol{g}=\varepsilon_0\boldsymbol{E}\times\boldsymbol{B}=\frac{\boldsymbol{S}}{c^2}=\frac{w}{c}\boldsymbol{e}_k$

动量流密度 $\overleftrightarrow{\mathcal{T}}=-\varepsilon_0\boldsymbol{E}\boldsymbol{E}-\frac{1}{\mu_0}\boldsymbol{B}\boldsymbol{B}+\frac{1}{2}(\varepsilon_0E^2+\frac{1}{\mu_0}B^2)\overleftrightarrow{\mathcal{I}}=($真空中$)cg\boldsymbol{e}_k\boldsymbol{e}_k=w\boldsymbol{e}_k\boldsymbol{e}_k$

动量守恒定律 $\int_V\boldsymbol{f}dV+\frac{d}{dt}\int_V\boldsymbol{g}dV=-\int_V\cdot\overleftrightarrow{\mathcal{T}}dV=-\oint_Sd\boldsymbol{S}\cdot\overleftrightarrow{\mathcal{T}}$ 或 $\boldsymbol{f}+\frac{\partial\boldsymbol{g}}{\partial t}=-\nabla\cdot\overleftrightarrow{\mathcal{T}}$

辐射压强(完全反射) $P=-\boldsymbol{e}_n\cdot\overleftrightarrow{\mathcal{T}}=2\bar{w}_i\cos^2\theta$ 其中 w_i–入射波能量密度，θ–入射角

Chap6狭义相对论

相对论基本原理 1．相对性原理 所有参考系均等价，物理规律对所有惯性系均为相同形式

2．光速不变原理 真空中光速对任一惯性系沿任一方向恒为 c

间隔不变性 惯性系 Σ 中任意两事件 (x_1,y_1,z_1,t_1) 和 (x_2,y_2,z_2,t_2) 的间隔为 $s^2=c^2(t_2-t_1)^2-(x_2-x_1)^2-(y_2-y_1)^2-(z_2-z_1)^2$

惯性系 Σ' 中对应两事件 (x_1',y_1',z_1',t_1') 和 (x_2',y_2',z_2',t_2') 的间隔为 $s'^2=c^2(t_2'-t_1')^2-(x_2'-x_1')^2-(y_2'-y_1')^2-(z_2'-z_1')^2$ 恒有 $s'^2=s^2$

洛伦兹变换 参考系 Σ' 相对 Σ 以 $\boldsymbol{v}$ 运动且两者 x 正向均沿 $\boldsymbol{v}$

则其时空坐标变换为 $\begin{cases}x'=\frac{x-vt}{\sqrt{1-(\frac{v^2}{c^2})^2}}\\y'=y\\z=z'\\t'=\frac{t-\frac{v}{c^2}x}{\sqrt{1-(\frac{v}{c})^2}}\end{cases}$ 速度变换为 $\begin{cases}u_x'=\frac{u_x-v}{1-\frac{vu_x}{c^2}}\\u_y'=\frac{u_y\sqrt{1-(\frac{v}{c})^2}}{1-\frac{vu_x}{c^2}}\\u_z'=\frac{u_z\sqrt{1-(\frac{v}{c})^2}}{1-\frac{vu_x}{c^2}}\end{cases}$

相对论时空结构 1．$s^2=0$ 即 $r=ct$，类光间隔(光锥，两事件可由光波联系)

2．$s^2>0$ 即 $r<ct$，类时间隔(光锥内，两事件可由低于光速的作用来联系 (a)上半光锥–绝对未来；(b)下半–过去)

3．$s^2<0$ 即 $r>ct$ 类空间隔(光锥外，两事件绝无联系) 对于给定两事件此种间隔分类不因参考系改变而改变

洛伦兹变换的四维形式 将三维空间坐标与时间虚数坐标统一为四维坐标

$x_\mu=(\boldsymbol{x},ict)=(x_1,x_2,x_3,x_4)$，洛伦兹变换可表为 $x_\mu'=a_{\mu\nu}x_\nu$，其中

沿 x 特殊洛伦兹变换矩阵 $a=\begin{bmatrix}\gamma&&0&i\beta\gamma\\0&1&0&0\\0&0&1&0\\-i\beta\gamma&0&0&\gamma\end{bmatrix}$ 其中 $\begin{cases}\beta=\frac{v}{c}\\\gamma=\frac{1}{\sqrt{1-(\frac{v}{c})^2}}\end{cases}$

因有间隔不变性 $x_\mu'x_\mu'=x_\mu x_\mu=\mathrm{const}$，洛伦兹变换为正交变换 $a_{\mu\nu}a_{\mu\tau}=\delta_{\nu\tau}$ 即 $a^Ta=I$ $\begin{cases}\textbf{洛伦兹标量(不变量)}&\text{在洛伦兹变换下不变}\\\textbf{四维矢量}&\text{满足}V_\mu=a_{\mu\nu}V_\nu\\\textbf{四维张量}&\text{满足}T'_{\mu\nu}=a_{\mu\lambda}a_{\nu\tau}T_{\lambda\tau}\end{cases}$

间隔 $d(s^2)=-dx_\mu dx_\mu$ 和固有时(物体静止坐标系中测出的时间) $d\tau=\frac{1}{c}ds$ 为洛伦兹标量

四维速度矢量 $U_\mu=\frac{dx_\mu}{d\tau}=\gamma_\mu(u_1,u_2,u_3,ic)$ **四维波矢量** $k_\mu=(\boldsymbol{k},i\frac{\omega}{c})$

特殊洛伦兹变换下相对论多普勒效应和光行差公式 $\begin{cases}\omega'=\omega\gamma(1-\frac{v}{c}\cos\theta)\\\tan\theta'=\frac{\sin\theta}{\gamma(\cos\theta-\frac{v}{c})}\end{cases}$

四维电流密度 $J_\mu=\rho_0U_\mu=(\boldsymbol{J},ic\rho)$ **四维势** $A_\mu=(\boldsymbol{A},i\varphi/c)$

协变矢量算符 $\frac{\partial}{\partial x_\mu}=(\nabla,\frac{1}{ic}\frac{\partial}{\partial t})=\partial_n$ **协变标量算符** $\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\mu}=\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}=\partial_\mu\partial_\mu$

电磁场张量 $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu=\begin{bmatrix}0&B_3&-B_2&-iE_1/c\\-B_3&0&B_1&-iE_2/c\\B_2&-B_1&0&-iE_3/c\\iE_1/c&iE_2/c&iE_3/c&0\end{bmatrix}$

电荷守恒定律 $\partial_\mu J_\mu=0$ **洛伦兹规范** $\partial_\mu A_\mu=0$ **达朗贝尔方程** $\partial_\nu\partial_\nu A_\mu=-\mu_0J_\mu$

麦克斯韦方程 $\partial_\nu F_{\mu\nu}=\mu_0J_\mu,\partial_\lambda F_{\mu\nu}+\partial_\mu F_{\nu\lambda}+\partial_\nu F_{\lambda\mu}=0$

能量动量守恒定律 $f_\mu=\partial_\lambda T_{\mu\lambda}$

电磁场变换关系 $\begin{cases}\boldsymbol{E}_\parallel'=\boldsymbol{E}_\parallel&\boldsymbol{E}_\perp'=\gamma(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})_\perp\\\boldsymbol{B}_\parallel'=\boldsymbol{B}_\parallel&\boldsymbol{B}_\perp'=\gamma(\boldsymbol{B}-\frac{\boldsymbol{v}}{c^2}\times\boldsymbol{E})_\perp\end{cases}$

洛伦兹不变量 $\frac{1}{2}F_{\mu\nu}F_{\mu\nu}=B^2-\frac{1}{c^2}E^2=0$, $\frac{i}{8}\varepsilon_{\mu\nu\lambda\tau}F_{\mu\nu}F_{\lambda\tau}=\frac{1}{c}\boldsymbol{B}\cdot\boldsymbol{E}=0$

能量–动量四维矢量 $p_\mu=m_0U_\mu=(\boldsymbol{p},p_4)=(\gamma m_0\boldsymbol{v},ic\gamma m_0)$

动质量 $m=\gamma m_0$ 故**相对论动量** $\boldsymbol{p}=\gamma m_0\boldsymbol{v}=m\boldsymbol{v}$，**相对论能量** $W=\gamma m_0c^2=mc^2$

能量、动量和质量关系式 $W^2=p^2c^2+m_0^2c^4$ **质能关系** $\Delta W=(\Delta M)c^2$

四维力矢量 $K_\mu=\frac{dp_\mu}{d\tau}=(\frac{d\boldsymbol{p}}{d\tau},\frac{i}{c}\frac{dW}{d\tau})=(\gamma\boldsymbol{F},\frac{i}{c}\gamma\boldsymbol{F}\cdot\boldsymbol{v})=(\boldsymbol{K},\frac{i}{c}\boldsymbol{K}\cdot\boldsymbol{v})$

四维洛伦兹力密度 $f_\mu=\rho_0F_{\mu\nu}U_\nu=F_{\mu\nu}J_\nu=(\boldsymbol{f},i\boldsymbol{E}\cdot\boldsymbol{J}/c)$

标量场的梯度无旋 $\nabla\times\nabla\varphi=0$ 矢量场的旋度无源 $\nabla\cdot\nabla\times\boldsymbol{f}=0$

$\nabla(\varphi\psi)=\varphi\nabla\psi+\psi\nabla\varphi$

$\nabla\cdot(\varphi\boldsymbol{f})=(\nabla\varphi)\cdot\boldsymbol{f}+\varphi\nabla\cdot\boldsymbol{f}$

$\nabla\times(\varphi\boldsymbol{f})=(\nabla\varphi)\times\boldsymbol{f}+\varphi\nabla\times\boldsymbol{f}$

$\nabla\cdot(\boldsymbol{f}\times\boldsymbol{g})=(\nabla\times\boldsymbol{f})\cdot\boldsymbol{g}-\boldsymbol{f}\cdot(\nabla\times\boldsymbol{g})$

$\nabla\times(\boldsymbol{f}\times\boldsymbol{g})=(\boldsymbol{g}\cdot\nabla)\boldsymbol{f}+(\nabla\cdot\boldsymbol{g})\boldsymbol{f}-(\boldsymbol{f}\cdot\nabla)\boldsymbol{g}-(\nabla\cdot\boldsymbol{f})\boldsymbol{g}$

$\nabla(\boldsymbol{f}\cdot\boldsymbol{g})=\boldsymbol{f}\times(\nabla\times\boldsymbol{g})+(\boldsymbol{f}\cdot\nabla)\boldsymbol{g}+\boldsymbol{g}\times(\nabla\times\boldsymbol{f})+(\boldsymbol{g}\cdot\nabla)\boldsymbol{f}$

$\nabla\times(\nabla\times\boldsymbol{f})=\nabla(\nabla\cdot\boldsymbol{f})-\nabla^2\boldsymbol{f}$

$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=\boldsymbol{b}(\boldsymbol{a}\cdot\boldsymbol{c})-\boldsymbol{c}(\boldsymbol{a}\cdot\boldsymbol{b})$

高斯定理 $\iint_{\partial V}\boldsymbol{E}\cdot d\boldsymbol{S}=\iiint\nabla\cdot\boldsymbol{E}dV$

格林定理 $\nabla\cdot(u\nabla v)=u\nabla\cdot\nabla v+(\nabla u)\cdot(\nabla v)$

斯多克斯定理 $\oint_{\partial S}\boldsymbol{E}\cdot d\boldsymbol{l}=\iint_S(\nabla\times\boldsymbol{E})\cdot d\boldsymbol{S}$

	直角坐标系	柱坐标系	球坐标系
梯度 ∇f	$\frac{\partial f}{\partial x}\hat{x}+\frac{\partial f}{\partial y}\hat{y}+\frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial\rho}\hat{\rho}+\frac{1}{\rho}\frac{\partial f}{\partial\varphi}\hat{\varphi}+\frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\hat{\varphi}$
散度 $\nabla\cdot\boldsymbol{A}$	$\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi}+\frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$
拉普拉斯算子 ∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$	$\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r})+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})+\frac{1}{r^2\sin\theta^2\theta}\frac{\partial^2}{\partial\varphi^2}$