Chap1

库仑定律$\vec{F}=Q'\vec{E}=\frac{QQ'\vec{r}}{4\pi\epsilon_0 r^3}$
电场叠加性$\vec{E}=\sum_{i=1}\frac{Q_i\vec{r}_i}{4\pi\epsilon_0 r_i^3}=\int_V\frac{\rho(\vec{x}')\vec{r}dV'}{4\pi\epsilon_0 r^3}$
高斯定理&电场散度$\oint_S\vec{E}\cdot d\vec{S}=\frac{1}{\epsilon_0}\sum_i Q_i=\frac{1}{\epsilon_0}\int_V\rho dV$ or $\nabla\cdot\vec{E}=\frac{\rho}{\epsilon_0}$
静电场旋度$\nabla\times\vec{E}=0$
电荷守恒定律$\oint_S\vec{J}\cdot d\vec{S}=-\int_V\frac{\partial\rho}{\partial t}dV$ or $\nabla\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0$
毕奥-萨伐尔定律$\vec{B}(\vec{x})=\frac{\mu_0}{4\pi}\int_V\frac{\vec{J}(\vec{x}')\times\vec{r}}{r^3}dV'=\frac{\mu_0}{4\pi}\oint_L\frac{Id\vec{l}\times\vec{r}}{r^3}$
磁场环量&旋度$\oint\vec{B}\cdot d\vec{l}=\mu_0 I=\mu_0\int_S\vec{J}\cdot d\vec{S}$ or $\nabla\times\vec{B}=\mu_0\vec{J}$
电磁感应定律$\mathscr{E}=\oint_L\vec{E}\cdot d\vec{l}=-\frac{d}{dt}\int_S\vec{B}\cdot d\vec{S}$ or $\nabla\times\vec{E}=-\frac{\partial B}{\partial t}$
位移电流$\vec{J}_D=\epsilon_0\frac{\partial\vec{E}}{\partial t}$ so $\nabla\times\vec{B}=\mu_0(\vec{J}+\vec{J}_D)$
麦克斯韦方程组$\begin{cases}\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\\\nabla\times\vec{B}=\mu_0(\vec{J}+\epsilon_0\frac{\partial\vec{E}}{\partial t})\\\nabla\cdot\vec{E}=\frac{\rho}{\epsilon_0}\\\nabla\cdot\vec{B}=0\end{cases}$
洛伦兹力密度$\vec{f}=\rho\vec{E}+\vec{J}\times\vec{B}$对点电荷$\vec{F}=q\vec{E}+q\vec{v}\times\vec{B}$
介质的极化电极化强度矢量$\vec{P}=\frac{\sum_i\vec{P}_i}{\Delta V}$束缚电荷密度$\int_V\rho_P dV=-\oint_S\vec{P}\cdot d\vec{S}$ or $\rho_P=-\nabla\cdot\vec{P}$介质分界面束缚电荷面密度$\sigma_P=-\vec{e}_n\cdot(\vec{P}_2-\vec{P}_1)$高斯定理改写为$\epsilon_0\nabla\cdot\vec{E}=\rho_f+\rho_P$ or $\nabla\cdot(\epsilon_0\vec{E}+\vec{P})=\rho_f$ def电位移矢量$\vec{D}=\epsilon_0\vec{E}+\vec{P}$ so $\nabla\vec{D}=\rho_f$对各向同性线性介质$P=\chi_e\epsilon_0\vec{E}$ so $\vec{D}=\epsilon\vec{E}$其中χ_e–极化率$\epsilon=\epsilon_r\epsilon_0$–电容率$\epsilon_r=1+\chi_e$–相对电容率
介质的磁化分子电流磁矩$\vec{m}=i\vec{a}$其中i–分子电流$\vec{a}$–分子电流环绕面积磁化强度$\vec{M}=\frac{\sum_i\vec{m}_i}{\Delta V}$磁化电流$I_M=\oint_L\vec{M}\cdot d\vec{l}$磁化电流密度$\vec{J}_M=\nabla\times\vec{M}$当电场变化时,介质的极化强度矢量$\vec{P}=\frac{\sum_i e_i\vec{x}_i}{\Delta V}$($\Delta V$中每个带电粒子的位置为$\vec{x}_i$,电荷为$e_i$)发生变化,产生极化电流密度$\vec{J}_P=\frac{\partial\vec{P}}{\partial t}=\frac{\sum_i e_i\vec{v}_i}{\Delta V}$ so 磁场的旋度改写为$\frac{1}{\mu_0}\nabla\times\vec{B}=\vec{J}_f+\vec{J}_M+\vec{J}_P+\epsilon_0\frac{\partial E}{\partial t}$ or $\nabla\times(\frac{\vec{B}}{\mu_0}-\vec{M})=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}$ def磁场强度$\nabla\times\vec{H}=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}$对各向同性非铁磁物质$\vec{M}=\chi_M\vec{H}$ so $\vec{B}=\mu\vec{H}$其中对各向同性线性介质χ_M–磁化率$\mu=\mu_r\mu_0$–磁导率$\mu_r=1+\chi_M$
介质中的麦克斯韦方程组$\begin{cases}\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\\\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}\\\nabla\cdot\vec{D}=\rho\\\nabla\cdot B=0\end{cases}$其中$\vec{D}=\epsilon\vec{E}$,$\vec{B}=\mu\vec{H}$欧姆定律$J=\sigma\vec{E}$自此开始略去下角标$f$
对于各向异性介质$D_i=\sum_{j=1}^3\epsilon_{ij}E_j$强磁场下非线性$D_i=\sum_j\epsilon_{ij}E_j+\sum_{j,k}\epsilon_{ijk}E_jE_k+\sum_{jkl}\epsilon_{ijkl}E_jE_kE_l+\cdots$而$\vec{B}$与$\vec{H}$的关系依赖于磁化过程,一般用磁化曲线和磁滞回线表示
麦克斯韦方程积分形式$\begin{cases}\oint_L\vec{E}\cdot d\vec{l}=-\frac{d}{dt}\int_S\vec{B}\cdot d\vec{S}\\\oint_L\vec{H}\cdot d\vec{l}=I_f+\frac{d}{dt}\int_S\vec{D}\cdot d\vec{S}\\\oint_S\vec{D}\cdot d\vec{S}=Q_f\\\oint_S\cdot d\vec{S}=0\end{cases}$
边界处法向分量$D_{2n}+D_{1n}=\sigma$,$B_{2n}=B_{1n}$切向分量$\vec{e}_n\times(\vec{H}_{2t}-\vec{H}_{1t})=\vec{\alpha}_f$其中$\alpha$–自由电流线密度$\vec{e}_n\times(\vec{E}_2-\vec{E}_1)=0$
能量守恒定律$-\oint_S\vec{S}\cdot\vec{\sigma}=\int_V\vec{f}\cdot\vec{v}dV+\frac{d}{dt}\int_V wdV$ or $\nabla\cdot\vec{S}+\frac{\partial w}{\partial t}=-\vec{f}\cdot\vec{v}$其中$w$–能量密度$\vec{S}$–能流密度(坡印廷矢量)$\vec{f}$–场对电荷作用力密度$\vec{v}$–电荷运动速度& $\frac{\partial w}{\partial t}=\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}+\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}$真空中$w=\frac{1}{2}(\epsilon_0E^2+\frac{1}{\mu_0}B^2)$线性介质中$w=\frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})$

Chap2

电势$\vec{E}=-\nabla\phi$
泊松方程(各向同性线性介质)$\nabla^2\phi=-\frac{\rho}{\epsilon}$
边界条件$\phi_1=\phi_2$,$\epsilon_2\frac{\partial\phi_2}{\partial n}-\epsilon_1\frac{\partial\phi_1}{\partial n}=-\sigma$
线性介质中静电场总能量$W=\frac{1}{2}\int_\infty\vec{E}\cdot\vec{D}dV=\frac{1}{2}\int_V\rho\phi dV=\frac{1}{8\pi\epsilon}\int dV\int dV'\frac{\rho(\vec{x}\rho(\vec{x}'))}{r}$
唯一性定理设区域V内给定自由电荷分布$\rho(\vec{x})$,在V的边界S上给定(1)电势$\phi|_S$或(2)电势的法线方向偏导数$\frac{\partial\phi}{\partial n}|_S$,则$V$内的电场唯一地确定有导体存在时的唯一性定理设区域$V$内有一些导体,给定导体之外的电荷分布$\rho$,在$V$的边界$S$上给定(1)电势$\phi|_S$或(2)电势的法线方向偏导数$\frac{\partial\phi}{\partial n}|_S$并且给定(1)每个导体上的电势$\phi_i$或(2)每个导体上的总电荷,则$V$内的电场唯一地确定
若区域V内部自由电荷密度$\rho=0$,泊松方程化为拉普拉斯方程$\nabla^2\phi=0$在直角坐标系中分离变量$\phi(x,y,z)=X(x)Y(y)Z(z)$从而有$\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}+\frac{1}{Z}\frac{d^2Z}{dz^2}=0$设$\frac{1}{X}\frac{d^2X}{dx^2}=-\alpha^2$,$\frac{1}{Y}\frac{d^2Y}{dy^2}=-\beta^2$,$\frac{1}{Z}\frac{d^2Z}{dz^2}=\gamma^2$其中$\gamma^2=\alpha^2+\beta^2$通解为$X(x)=Re(A_\alpha e^{i\alpha x}+B_\alpha e^{-i\alpha x})$,$Y(y)=Re(A_\beta e^{i\beta y}+B_\beta e^{-i\beta y})$,$Z(z)=Re(A_\gamma e^{i\gamma z}+B_\gamma e^{-i\gamma z})$,$\phi(x,y,z)=Re[(A_\alpha e^{i\alpha x}+B_\alpha e^{-i\alpha x})(A_\beta e^{i\beta y}+B_\beta e^{-i\beta y})(A_\gamma e^{i\gamma z}+B_\gamma e^{-i\gamma z})]$,$\phi(x,y,z)=\sum_{l,m,n}(C_{xl}\cos\alpha_l x+D_{xl}\sin\alpha_l x)\cdot(C_{ym}\cos\beta_m y+D_{ym}\sin\beta_m y)\cdot(C_{zn}\cos\gamma_n z+D_{zn}\sin\gamma_n z)$在柱坐标系中的通解$\phi(r,\theta)=\sum_{n=1}^\infty[r^n(A_n\cos n\theta+B_n\sin n\theta)+r^{-n}(C_n\cos n\theta+D_n\sin n\theta)]$若轴对称,则$\phi(r)=A+B\ln r$在球坐标系中的通解$\phi(R,\theta,\phi)=\sum_{n,m}(a_{nm}R^n+\frac{b_nm}{R^{n+1}})P_n^m(\cos\theta)\cos m\phi+\sum_{n,m}(c_{nm}R^n+\frac{d_{nm}}{R^{n+1}})P_n^m(\cos\theta)\sin m\phi$若有对称轴且以之为极轴,则$\phi=\sum_n(a_nR^n+\frac{b_n}{R^{n+1}})P_n(\cos\theta)$
接地无限大平面导体板附近有一点电荷Q,镜像电荷$Q'=-Q$位于点电荷关于导体板对称的位置,电势$\phi=\frac{1}{4\pi\epsilon_0}(\frac{Q}{r}-\frac{Q}{r'})=\frac{1}{4\pi\epsilon_0}[\frac{Q}{\sqrt{x^2+y^2+(z-a)^2}}-\frac{Q}{\sqrt{x^2+y^2+(z+a)^2}}]$;真空中有一半径为$R_0$的接地导体球,距球心为$a(>R_0)$处有一点电荷$Q$,镜像电荷$Q'=-\frac{R_0}{a}Q$位于距球心$b=\frac{R_0^2}{a}$处
格林函数$G(\vec{x},\vec{x}')$满足$\nabla^2G(\vec{x},\vec{x}')=-\frac{1}{epsilon_0}\delta(\vec{x}-\vec{x}')$并在包含$\vec{x}'$的某空间区域$V$的边界$S$上满足第一类边界条件$G|_S=0$或第二类边界条件$\frac{\partial G}{\partial n}|_S=\frac{1}{\epsilon_0 S}$无界空间的格林函数$G(\vec{x},\vec{x}')=\frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$上半空间的格林函数(满足第一类边界条件)$G(\vec{x}\vec{x}')=\frac{1}{4\pi\epsilon_0}[\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}-\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z+z')^2}}]$接地导体球外空间的格林函数$G(\vec{x},\vec{x}')=\frac{1}{4\pi\epsilon_0}[\frac{1}{\sqrt{R^2+R'^2-2RR'\cos\alpha}}-\frac{1}{\sqrt{(\frac{RR'}{R_0})^2+R_0^2+2RR'\cos\alpha}}]$第一类边值问题$\phi(\vec{x})=\int_V G(\vec{x}',\vec{x})\rho(\vec{x}')dV'-\epsilon_0\oint_S\phi(\vec{x}')\frac{\partial}{\partial n'}G(\vec{x}',\vec{x})dS'$二类边值问题$\phi(\vec{x})=\int_V G(\vec{x}',\vec{x})\rho(\vec{x}')dV'+\epsilon_0\oint_S G(\vec{x}',\vec{x})\frac{\partial\phi(\vec{x}')}{\partial n'}dS'+\langle\phi\rangle_S$其中$\langle\phi\rangle_S$–电势在界面$S$上的平均值
电荷体系电势多级展开式$\phi(\vec{x})=\frac{1}{4\pi\epsilon_0}\int_V\rho(\vec{x}')[\frac{1}{R}-\vec{x}'\cdot\nabla\frac{1}{R}+\frac{1}{2!}\sum_{i,j}x_i'x_j'\frac{\partial^2}{\partial x_i\partial x_j}\frac{1}{R}+\cdots]=\frac{1}{4\pi\epsilon_0}(\frac{Q}{R}-\vec{p}\cdot\nabla\frac{1}{R}+\frac{1}{6}\sum_{i,j}\frac{\partial^2}{\partial x_i\partial x_j}\frac{1}{R}+\cdots)$,其中$R=\sqrt{x^2+y^2+z^2}$,$Q=\int_V\rho(\vec{x}')dV'$,电偶极矩$\vec{p}=\int_V\rho(\vec{x}')\vec{x}'dV'$,电四极矩$\mathscr{D}=\int_V 3x_i'x_j'\rho(\vec{x}')dV'$
具有电荷分布$\rho(\vec{x})$的体系在电势为ϕ_e的外电场中能量$W=\int\rho\phi_e dV=\int\rho(\vec{x})[\phi_e(0)+\sum_i x_i\frac{\partial}{\partial x_i}\phi_e(0)+\frac{1}{2!}\sum_{i,j}x_ix_j\frac{\partial^2}{\partial x_i\partial x_j}\phi_e(0)+\cdots]dV=Q\phi_e(0)+\sum_i p_i\frac{\partial}{\partial x_i}\phi_e(0)+\frac{1}{6}\sum_{i,j}\mathscr{D}_{ij}\frac{\partial^2}{\partial x_i\partial x_j}\phi_e(0)+\cdots$电偶极子在外场中受力$\vec{F}=\nabla(\vec{p}\cdot\vec{E}_e)=\vec{p}\cdot\nabla\vec{E}_e$受力矩$L_\theta=-\frac{\partial(\vec{p}\cdot\vec{E}_e)}{\partial\theta}=-pE_e\sin\theta$ so $\vec{L}=\vec{p}\times\vec{E}_e$

Chap3

def**矢势**$\vec{B} = \nabla \times \vec{A}$ so $\int_S \vec{B}\cdot d\vec{S} = \oint_L \vec{A}\cdot d\vec{l}$当满足库伦规范$\nabla\cdot\vec{A}=0$时$\nabla^2\vec{A}=-\mu_0\vec{J}$其解为$\vec{A}(\vec{x})=\frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{x}')}{r}dV'$

边界条件$\vec{e}_n \times (\frac{1}{\mu_2}\nabla\times\vec{A}_2 - \frac{1}{\mu_1}\nabla\times\vec{A}_1)$**介质分界面上矢势连续**$\vec{A}_2=\vec{A}_1$

磁场总能量$W=\int_\infty \vec{B}\cdot\vec{H}dV=\int_V \vec{A}\cdot\vec{J}dV$前一积分遍及磁场分布区域,后一积分遍布电流分布区域,电流$\vec{J}$在外场$\vec{A}_e$中的相互作用能量$W_i=\int_V \vec{J}\cdot\vec{A}_e dV$

在$\vec{J}_f=0$的单连通区域内def**磁标势**$\vec{H}=-\nabla\phi_m$def**假想磁荷密度**$\rho_m=-\mu_0\nabla\cdot\vec{M}$ so $\nabla^2\phi_m=-\frac{\rho_m}{\mu_0}$

边界条件$\vec{e}_n\times(-\nabla\phi_2+\nabla\phi_1)=\alpha_f$,$B_{2n}=B_{1n}$若介质线性均匀,且界面上$\alpha_f=0$,则$\phi_2=\phi_1$,$\mu_2\frac{\partial\phi_2}{\partial n}=\mu_1\frac{\partial\phi_1}{\partial n}$ 在磁标势法中,静电场–静磁场

$\nabla\times\vec{E}=0$–$\nabla\times\vec{H}=0$

$\nabla\cdot\vec{E}=\frac{(\rho_f+\rho_P)}{\epsilon_0}$–$\nabla\cdot\vec{H}=\frac{\rho_m}{\mu_0}$

$\rho_P=-\nabla\cdot\vec{P}$–$\rho_m=-\mu_0\nabla\cdot\vec{M}$

$\vec{D}=\epsilon_0\vec{E}+\vec{P}$–$\vec{B}=\mu_0\vec{H}+\mu_0\vec{M}$

$\vec{E}=-\nabla\phi$–$\vec{H}=-\nabla\phi_m$

$\nabla^2\phi=-\frac{(\rho_f+\rho_P)}{\epsilon_0}$–$\nabla^2\phi_m=-\frac{\rho_m}{\mu_0}$

磁矢势多级展开$\vec{A}(\vec{x})=\frac{\mu_0}{4\pi}\int_V \vec{J}(\vec{x}')[\frac{1}{R}-\vec{x}'\cdot\nabla\frac{1}{R}+\frac{1}{2!}\sum_{i,j}x_i'x_j'\frac{\partial^2}{\partial x_i\partial x_j}\frac{1}{R}+\cdots]dV'$其中$\vec{A}^{(0)}=0$,$\vec{A}^{(1)}=-\frac{\mu_0 I}{4\pi}\frac{\vec{m}\times\vec{R}}{R^3}$**磁矩**$\vec{m}=\frac{1}{2}\int_V \vec{x}'\times\vec{J}(\vec{x}')dV'$对于闭合环路$\vec{m}=I\Delta\vec{S}$

磁偶极矩的磁场和磁矩$\vec{B}^{(1)}=-\mu_0\nabla\phi_m^{(1)}$,$\phi_m^{(1)}=\frac{\vec{m}\cdot\vec{R}}{4\pi R^3}$在外场中的势能$U=-\vec{m}\cdot\vec{B}_e$受力$\vec{F}=-\nabla U=\vec{m}\cdot\nabla\vec{B}_e$受力矩$L=-\frac{\partial}{\partial\theta}U=-mB_e\sin\theta$ so $\vec{L}=\vec{m}\times\vec{B}_e$

矢量微分/哈密顿算子$\nabla=\frac{\partial}{\partial x}\boldsymbol{i}+\frac{\partial}{\partial y}\boldsymbol{j}+\frac{\partial}{\partial z}\boldsymbol{k}$;**梯度**$\nabla u=\frac{\partial u}{\partial x}\boldsymbol{i}+\frac{\partial u}{\partial y}\boldsymbol{j}+\frac{\partial u}{\partial z}\boldsymbol{k}$;**散度**$\nabla\cdot\boldsymbol{E}=\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}$;**旋度**$\nabla\times\boldsymbol{E}=[\boldsymbol{i}\,\boldsymbol{j}\,\boldsymbol{k};\ \frac{\partial}{\partial x}\ \frac{\partial}{\partial y}\ \frac{\partial}{\partial z};\ E_x\ E_y$ $(\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z})\boldsymbol{i}+(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x})\boldsymbol{j}+(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y})\boldsymbol{k}$;**拉普拉斯算子**$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$,作用于函数$\nabla^u=\nabla\cdot(\nabla u)=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\partial^2 u\partial z^2$,作用于矢量$\nabla^2\boldsymbol{E}=(\nabla^2E_x)\boldsymbol{i}+(\nabla^2E_y)\boldsymbol{j}+\partial^2u\partial z^2\boldsymbol{k}$;

标量场的梯度无旋$\nabla\times\nabla\phi=0$,矢量场的旋度无源$\nabla\times\nabla\times\boldsymbol{f}=0$;

$\nabla(\phi\psi)=\phi\nabla\psi+\psi\nabla\phi$

$\nabla\cdot(\phi\boldsymbol{f})=(\nabla\phi)\cdot\boldsymbol{f}+\phi\nabla\cdot\boldsymbol{f}$

$\nabla\times(\phi\boldsymbol{f})=(\nabla\phi)\times\boldsymbol{f}+\phi\nabla\times\boldsymbol{f}$

$\nabla\cdot(\boldsymbol{f}\times\boldsymbol{g})=(\nabla\times\boldsymbol{f})\cdot\boldsymbol{g}-\boldsymbol{f}\cdot(\nabla\times\boldsymbol{g})$

$\nabla\times(\boldsymbol{f}\times\boldsymbol{g})=(\boldsymbol{g}\cdot\nabla)\boldsymbol{f}+(\nabla\cdot\boldsymbol{g})\boldsymbol{f}-(\boldsymbol{f}\cdot\nabla)\boldsymbol{g}-(\nabla\cdot\boldsymbol{f})\boldsymbol{g}$,$\nabla(\boldsymbol{f}\cdot\boldsymbol{g})=\boldsymbol{f}\times(\nabla\times\boldsymbol{g})+(\boldsymbol{f}\cdot\nabla)\boldsymbol{g}+\boldsymbol{g}\times(\nabla\times\boldsymbol{f})+(\boldsymbol{g}\cdot\nabla)\boldsymbol{f}$,$\nabla\times(\nabla\times\boldsymbol{f})=\nabla(\nabla\cdot\boldsymbol{f})-\nabla^2\boldsymbol{f}$

$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=\boldsymbol{b}(\boldsymbol{a}\cdot\boldsymbol{c})-\boldsymbol{c}(\boldsymbol{a}\cdot\boldsymbol{b})$

高斯定理$\iint_{\partial V}\boldsymbol{E}\cdot d\boldsymbol{S}=\iiint\nabla\cdot\boldsymbol{E}dV$

格林定理$\nabla\cdot(u\nabla v)=u\nabla\cdot\nabla v+(\nabla u)\cdot(\nabla v)$

斯多克斯定理$\oint_{\partial S}\boldsymbol{E}\cdot d\boldsymbol{l}=\iint_S(\nabla\times\boldsymbol{E})\cdot d\boldsymbol{S}$

表 1: 坐标变换

	直角坐标系	柱坐标系	球坐标系
直角		$x=\rho\cos\phi, y=\rho\sin\phi, z$	$x=r\sin\theta\cos\phi, y=r\sin\theta\sin\phi, z=r\cos\theta$
柱	$\rho=\sqrt{x^2+y^2}, \phi=\arctan(y/x), z=z$		$\rho=r\sin\theta, \phi, z=r\cos\theta$
球	$r=\sqrt{x^2+y^2+z^2}, \theta=\arccos(z/r), \phi=\arctan(y/x)$	$r=\sqrt{\rho^2+z^2}, \theta=\arctan(\rho/z), \phi$	

表 2: 梯度、散度、旋度和拉普拉斯算子变换

	直角坐标系	柱坐标系	球坐标系
矢量$\boldsymbol{A}$	$A_x\hat{x}+A_y\hat{y}+A_z\hat{z}$	$A_\rho\hat{\rho}+A_\phi\hat{\phi}+A_z\hat{z}$	$A_r\hat{r}+A_\theta\hat{\theta}+A_\phi\hat{\phi}$
梯度∇f	$\frac{\partial f}{\partial x}\hat{x}+\frac{\partial f}{\partial y}\hat{y}+\frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial\rho}\hat{\rho}+\frac{1}{\rho}\frac{\partial f}{\partial\phi}\hat{\phi}+\frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\hat{\phi}$
散度$\nabla\cdot\boldsymbol{A}$	$\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi}+\frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$
旋度$\nabla\times\boldsymbol{A}$	$(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z})\hat{x}+(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x})\hat{y}$ $+(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y})\hat{z}$	$(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi}-\frac{\partial A_\phi}{\partial z})\hat{\rho}+(\frac{\partial A_\rho}{\partial z}-\frac{\partial A_z}{\partial\rho})\hat{\phi}$ $+\frac{1}{\rho}(\frac{\partial(\rho A_\phi)}{\partial\rho}-\frac{\partial A_\rho}{\partial\phi})\hat{z}$	$\frac{1}{r\sin\theta}(\frac{\partial(A_\phi\sin\theta)}{\partial\theta}-\frac{\partial A_\theta}{\partial\phi})\hat{r}+\frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{\partial(rA_\phi)}{\partial r})\hat{\theta}$ $+\frac{1}{r}(\frac{\partial(rA_\theta)}{\partial r}-\frac{\partial A_r}{\partial\theta})\hat{\phi}$
拉普拉斯算子∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$	$\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}+\frac{\partial^2}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r})+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})+\frac{1}{r^2\sin\theta^2\theta}\frac{\partial^2}{\partial\phi^2}$