Homework 3

(0.6')1. Which of the following combinations of quantum numbers are allowed for an electron in a one-electron atom? Which are not?

(a) n=2, l=2, m=1, m_s=1/2 (b) n=3, l=1, m=0, m_s=-1/2
(c) n=5, l=1, m=2, m_s=1/2 (d) n=4, l=-1, m=0, m_s=1/2

(a) 不可以 (b) 可以 (c) 不可以 (d) 不可以

每问 0.15 分

(0.6')2. No object can travel faster than the speed of light, so it would appear evident that the uncertainty in the speed of any object is at most 3 × 10^8 m s^{-1}.

(a) What is the minimum uncertainty in the position of an electron, given that we know nothing about its speed except that it is slower than the speed of light?
(b) Repeat the calculation of part (a) for the position of a helium atom

a) Use the Heisenberg indeterminacy principle

$$\Delta x \cdot \Delta(mv) \geq h/4\pi$$

with $m = 9.109 \times 10^{-31}$ kg and $v = 3.0 \times 10^8$ m s^{-1}. The result is $\Delta x \geq 1.93 \times 10^{-13}$ m. This is 0.002 Å.

b) Because the helium atom is much more massive its Δx is much smaller. Repeat the computation with $m = m_{\mathrm{He}} = 6.646 \times 10^{-27}$ kg. The result is 2.65×10^{-17} m.

每问 0.3 分，每处计算错误扣 0.1 分

(0.8')3. Chapter 3 introduced the concept of a double bond between carbon atoms, represented by C=C, with a length near 1.34 Å. The motion of an electron in such a bond can be treated crudely as motion in a one-dimensional box. Calculate the energy of an electron in each of its three lowest allowed states if it is confined to move in a one-dimensional box of length 1.34 Å. Calculate the wavelength of light necessary to excite the electron from its ground state to the first excited state

$E_n=h^2n^2/8mL^2$ $E_1=3.36\times10^{-18}$ J $E_2=1.34\times10^{-17}$ J $E_3=3.02\times10^{-17}$ J
$\lambda=1.98\times10^{-8}$ m

每处计算错误扣 0.1 分

(0.6')4. Photons are emitted in the Lyman series as hydrogen atoms undergo transitions from various excited states to the ground state. If ground-state He^+ are present in the same gas (near stars, for example), can they absorb these photons? Explain.

$E_n(H)=Z_H^2(1/n_f^2-1/n_i^2)Ry=(1-1/n_i^2)Ry$ $E_n(He^+)=4(1-1/n_f^2)Ry$

$4/n_f^2=3+1/n_i^2$ 左式∈(0,1)，右式∈(3,3.25)，故答案是 no

写出 E_n 表达式得 0.2 分，写出 4/nf2=3+1/ni2 得 0.2 分，正确比较两者得 0.2 分

(0.8')5. (a) If the kinetic energy of an electron is known to lie between 1.59×10^{-19} J and 1.61×10^{-19} J, what is the smallest distance within which it can be known to lie?

(b) Repeat the calculation of part (a) for a helium atom instead of an electron.

(a)$E=p^2/2m$, $p_1=5.382\times10^{-25}$ kg m s^{-1}, $p_2=5.416\times10^{-25}$ kg m s^{-1}

$\Delta x\geq h/4\pi\Delta p=1.55\times10^{-8}$ m

(b)$\Delta p=2.9\times10^{-25}$ kg m s^{-1}

$\Delta x\geq h/4\pi\Delta p=1.82\times10^{-10}$ m

每问 0.4 分，每处计算错误扣 0.1 分

(0.6')6. It has been suggested that spacecraft could be powered by the pressure exerted by sunlight striking a sail. The force exerted on a surface is the momentum *p* transferred to the surface per second. Assume that photons of 6000 Å light strike the sail perpendicularly. How many must be reflected per second by 1 cm^2 of surface to produce a pressure of 10^{-6} atm?

10^{-6} 个大气压=0.101 N m^{-2}

$F=PS=1.01\times10^{-5}$ N

该界面不吸收光子，故$\triangle p=p_2-p_1=2h/\lambda=2.208\times10^{-27}$ kg m s^{-1}

设单位时间内与界面碰撞的光子数为 n (个/秒)，则

$F=\triangle p\times n$，$n=4.6\times10^{21}$ 个/秒

知道 10^{-6} 个大气压=0.101 N m^{-2} 得 0.15 分，算出 F 得 0.05 分，计算出$\triangle p$ 得 0.2 分，计算出 n 得 0.2 分