1. 在直角坐标系中，$\vec{A} = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$

$$d\vec{S} = \vec{n}\cdot dS$$

$$= dS\cdot\cos(\vec{n},\vec{x})\vec{i} + dS\cdot\cos(\vec{n},\vec{y})\vec{j}$$

$$+ dS\cdot\cos(\vec{n},\vec{z})\vec{k}$$

$$= dzdy\,\vec{i} + dxdz\,\vec{j} + dxdy\,\vec{k}$$

故 $\oint \vec{A}\cdot d\vec{S} = \iint P\,dydz + Q\,dxdz + R\,dxdy$

$$\iiint_{\Omega}\frac{\partial R}{\partial z}dV = \iint_{D_{xy}}dxdy\int_{z_1(x,y)}^{z_2(x,y)}\frac{\partial R}{\partial z}dz$$

$$= \iint_{D_{xy}} R(x,y,z_2(x,y)) - R(x,y,z_1(x,y))\,dxdy$$

且有：
$$\iint_{\Sigma_1} R(x,y,z)\,dxdy = -\iint_{D_{xy}} R(x,y,z_1(x,y))\,dxdy$$
$$\iint_{\Sigma_2} R(x,y,z)\,dxdy = \iint_{D_{xy}} R(x,y,z_2(x,y))\,dxdy$$
$$\iint_{\Sigma_3} R(x,y,z)\,dxdy = 0$$

因此 $\iiint_{\Omega}\frac{\partial R}{\partial z}dV = \oiint_{\Sigma} R(x,y,z)\,dxdy$

同理：
$$\iiint_{\Omega}\frac{\partial P}{\partial x}dV = \oiint_{\Sigma} P(x,y,z)\,dydz$$
$$\iiint_{\Omega}\frac{\partial Q}{\partial y}dV = \oiint_{\Sigma} Q(x,y,z)\,dzdx$$

对两边同时对体积取极限可得：$\mathrm{div}\vec{A} = \lim_{\Delta V\to 0}\frac{\oiint \vec{A}\cdot d\vec{S}}{\Delta V}$

2、在直角坐标系中：$\vec{A} = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$

$$d\vec{l} = dl\cos(t,x)\vec{i} + dl\cos(t,y)\vec{j} + dl\cos(t,z)\vec{k}$$

$$= dx\vec{i} + dy\vec{j} + dz\vec{k}$$

故 $\oint_L \vec{A}\cdot d\vec{l} = \oint_L P\,dx + Q\,dy + R\,dz$

由斯托克斯公式可得：$= \iint_S \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\,dydz + \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\,dzdx + \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dxdy$

其中 $\mathrm{curl}(\vec{A}) = (\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z})i + (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x})j + (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})k$

因此有：$\iint_S \mathrm{curl}(\vec{A})\cdot\hat{n}\,ds = \oint_L \vec{A}\cdot d\vec{l}$

将包围的面积取极限：$\mathrm{curl}(\vec{A})\cdot\hat{n} = \lim_{\Delta s\to 0} \frac{\oint_L \vec{A}\cdot d\vec{l}}{\Delta s}$