Group Theory
Homework Assignment 11
Spring, 2020

Consider the permutation group S_4.

1. Consider some of the properties of S_4.
   (a) What are the classes in S_4?
   (b) What are the inequivalent irreducible representations of S_4?
   (c) Write down all the Young tableaux in each irreducible representation of S_4. What is the dimension of each irreducible representation of S_4?

2. Consider the Young operators in the irreducible representation $[3,1]$ of S_4.
   (a) Write down all the Young tableaux and the corresponding Young operators $\mathcal{Y}_\mu^{[3,1]}$'s in the irreducible representation $[3,1]$ of S_4.
   (b) Argue that all the Young operators in the irreducible representation $[3,1]$ of S_4 are orthogonal.

3. Write down all the standard basis vectors in the irreducible representation $[3,1]$ of S_4.

4. Write down all the $Q_{\nu k}$'s in the irreducible representation $[3,1]$ of S_4. Find all the $\mathcal{Y}'$'s using $\mathcal{Y}' = Q_{\nu k}\mathcal{Y}_{\nu k}Q_{\nu k}^{-1}$. Find all the $\mathcal{Y}_\mu(S)$'s in the irreducible representation $[3,1]$ of S_4 for $S = \begin{pmatrix}1 & 2 & 3 & 4\end{pmatrix}$ using $\mathcal{Y}_\mu(S) = S\mathcal{Y}_\mu^{[3,1]}S^{-1}$.

5. Construct the table for $A^\mu_{\nu k}(S)$ for $S = \begin{pmatrix}1 & 2 & 3 & 4\end{pmatrix}$ with $\mathcal{Y}_\mu(S)$ labeling the columns and $\sum_k \delta_k \mathcal{Y}_{\nu k}$ labeling the rows. Write down the representation matrix of $S = \begin{pmatrix}1 & 2 & 3 & 4\end{pmatrix}$.