



Group Theory

Homework Assignment 10

Spring, 2020

1. Simplify the following permutations into the product of cycles without any common object.
 - (a) $(1\ 2)(2\ 3)(1\ 2)$.
 - (b) $(1\ 2\ 3)(1\ 3\ 4)(3\ 2\ 1)$.
 - (c) $(1\ 2\ 3\ 4)^{-1}$.
 - (d) $(1\ 2\ 4\ 5)(4\ 3\ 2\ 6)$.
 - (e) $(1\ 2\ 3)(4\ 2\ 6)(3\ 4\ 5\ 6)$.
2. Write down all the Young patterns of the permutation group S_6 from the largest to the smallest.
3. Using the hook rule, calculate the number $d_{[3,2,1,1]}(S_7)$ of the standard Young tableaux for the Young pattern $[3, 2, 1, 1]$ of the permutation group S_7 .
4. Write down the Young operator corresponding to the following Young tableau.

1	2
3	4

5. Write down the permutation R_{12} transforming the Young tableau \mathcal{Y}_2 to the Young tableau \mathcal{Y}_1 .

$$\mathcal{Y}_1: \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 & & \\ \hline \end{array} \quad \mathcal{Y}_2: \begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 & & \\ \hline \end{array}$$

Show that $\mathcal{P}_1 R_{12} = R_{12} \mathcal{P}_2$, $\mathcal{Q}_1 R_{12} = R_{12} \mathcal{Q}_2$, and $\mathcal{Y}_1 R_{12} = R_{12} \mathcal{Y}_2$.