# Group Theory
# Homework Assignment 02
## Spring, 2020

1. Show that the intersection $S$ of two invariant subgroups $S_1$ and $S_2$ of a group $G$ is an invariant subgroup.

2. The multiplication table of a finite group $G$ is given by

| | $E$ | $A$ | $B$ | $C$ | $D$ | $F$ | $I$ | $J$ | $K$ | $L$ | $M$ | $N$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $E$ | $E$ | $A$ | $B$ | $C$ | $D$ | $F$ | $I$ | $J$ | $K$ | $L$ | $M$ | $N$ |
| $A$ | $A$ | $E$ | $F$ | $I$ | $J$ | $B$ | $C$ | $D$ | $M$ | $N$ | $K$ | $L$ |
| $B$ | $B$ | $F$ | $A$ | $K$ | $L$ | $E$ | $M$ | $N$ | $I$ | $J$ | $C$ | $D$ |
| $C$ | $C$ | $I$ | $L$ | $A$ | $K$ | $N$ | $E$ | $M$ | $J$ | $F$ | $D$ | $B$ |
| $D$ | $D$ | $J$ | $K$ | $L$ | $A$ | $M$ | $N$ | $E$ | $F$ | $I$ | $B$ | $C$ |
| $F$ | $F$ | $B$ | $E$ | $M$ | $N$ | $A$ | $K$ | $L$ | $C$ | $D$ | $I$ | $J$ |
| $I$ | $I$ | $C$ | $N$ | $E$ | $M$ | $L$ | $A$ | $K$ | $D$ | $B$ | $J$ | $F$ |
| $J$ | $J$ | $D$ | $M$ | $N$ | $E$ | $K$ | $L$ | $A$ | $B$ | $C$ | $F$ | $I$ |
| $K$ | $K$ | $M$ | $J$ | $F$ | $I$ | $D$ | $B$ | $C$ | $N$ | $E$ | $L$ | $A$ |
| $L$ | $L$ | $N$ | $I$ | $J$ | $F$ | $C$ | $D$ | $B$ | $E$ | $M$ | $A$ | $K$ |
| $M$ | $M$ | $K$ | $D$ | $B$ | $C$ | $J$ | $F$ | $I$ | $L$ | $A$ | $N$ | $E$ |
| $N$ | $N$ | $L$ | $C$ | $D$ | $B$ | $I$ | $J$ | $F$ | $A$ | $K$ | $E$ | $M$ |

   (a) Find the inverse of each element of $G$.

   (b) Find the elements in each class of $G$.

   (c) Find all invariant subgroups of $G$.

3. Consider the group $D_3$.

   (a) List all the classes of $D_3$.

   (b) Find the right and left cosets of the subgroup $S = \{E, A\}$ of $D_3$.

4. For the group $D_3$ and its invariant subgroup $S = \{E, D, F\}$, find the factor group $D_3/S$. Construct the multiplication table for the factor group.

5. Consider $C_6 = \{E, a, a^2, a^3, a^4, a^5\}$ and its two subgroups $S_1 = \{E, a^3\}$ and $S_2 = \{E, a^2, a^4\}$. Show that $C_6 = S_1 \otimes S_2$.