Group Theory
Homework Assignment 08
Spring, 2020

1. Identify the point group that is obtained by combining the two symmetry elements in each case.
   (a) A 2-fold rotation axis and an inversion center.
   (b) Two mirror planes at right angles to each other.
   (c) A 2-fold rotation axis and an intersecting mirror plane.

2. Let a rotation about an axis passing through the origin and perpendicular to the xOy plane through an angle of θ be represented by the matrix R_θ and a reflection in the line passing through the origin and making an angle of $\theta/2$ with the positive x axis be represented by the matrix S_θ. Show that R_θ and S_θ can be expressed as

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad S_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}.$$

3. Continue from the above problem.
   (a) Compute the effect of rotating the vector $2_x + 3e_y$ counterclockwise about the origin through an angle of $\pi/2$ radians.
   (b) Compute the effect of reflecting the vector $e_x + e_y$ through the line $y = 2x$.
   (c) Compute the effect of rotating the vector e_y counterclockwise about the origin through an angle of $\pi/3$ radians and then reflecting through the line $y = 2x$.

4. Continue from the above problem.
   (a) Show that $S_\theta S_\psi$ is a rotation and find the angle of rotation.
   (b) Show that $S_\theta R_\psi S_\theta = R_{-\psi}$.
   (c) Let T_v be a translation through v, $T_v w = w + v$. Show that $T_{R_\theta v} R_\theta = R_\theta T_v$.

5. Consider the point groups C_{2v} and D_{2h}.
   (a) Find all invariant subgroups of C_{2v}.
   (b) Find all invariant subgroups of D_{2h}.