麦克斯韦方程组(时域):$\nabla\times\boldsymbol{E}(\boldsymbol{r},t)=-\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t$(法拉第电磁感应定律①),$\nabla\times\boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{J}(\boldsymbol{r},t)+\partial\boldsymbol{D}(\boldsymbol{r},t)/\partial t$(安培定律②),$\nabla\cdot\boldsymbol{B}(\boldsymbol{r},t)=0$(磁高斯定律,**不存在磁单极子**③),$\nabla\cdot\boldsymbol{D}(\boldsymbol{r},t)=\rho(\boldsymbol{r},t)$(电高斯/库仑定律④),其中$\boldsymbol{E}$-电场强度(V/m),$\boldsymbol{H}$-磁场强度(A/m),$\boldsymbol{D}$-电位移矢量/电通量密度(C/m^2),$\partial\boldsymbol{D}/\partial t$-位移电流,$\boldsymbol{B}$-磁感应强度/磁通量密度(T,Wb/m^2);无源条件下(下同),自由电流密度$\boldsymbol{J}=0$,电荷密度$\rho=0$

麦氏方程组(频域,无源):$\nabla\times\boldsymbol{E}(\boldsymbol{r},\omega)=-j\omega\boldsymbol{B}(\boldsymbol{r},\omega)$(①),$\nabla\times\boldsymbol{H}(\boldsymbol{r},\omega)=j\omega\boldsymbol{D}(\boldsymbol{r},\omega)$(②),$\nabla\cdot\boldsymbol{B}(\boldsymbol{r},\omega)=0$(③),$\nabla\cdot\boldsymbol{E}(\boldsymbol{r},\omega)=0$(④)

本构关系:$\boldsymbol{D}=\epsilon_0\boldsymbol{E}+\boldsymbol{P}\approx$(弱场)$\epsilon_0(1+\chi)\boldsymbol{E}=\epsilon_0\epsilon_r\boldsymbol{E}=\epsilon\boldsymbol{E}$,$\boldsymbol{B}=\mu\boldsymbol{H}=\mu_0\mu_r\boldsymbol{H}\approx$(非磁介质)$\mu_0\boldsymbol{H}$,其中$\epsilon$-介电常数,真空⋯$\epsilon_0=8.85\times10^{-12}F/m\approx(36\pi)^{-1}\times10^{-9}$F/m,$\epsilon_r$-相对⋯,$\chi$-电极化率,弱场下,电极化强度$\boldsymbol{P}=\chi\boldsymbol{E}$,$\mu$-磁导率,真空⋯$\mu_0=4\pi\times10^{-7}$H/m,对非磁介质(下同),相对⋯$\mu_r=1$

边界条件:平行界面有$\boldsymbol{E}_{1t}=\boldsymbol{E}_{2t}$,$\boldsymbol{H}_{1t}=\boldsymbol{H}_{2t}$,垂直界面有$D_{1n}=D_{2n}$,$B_{1n}=B_{2n}$

亥姆霍兹方程:$\nabla^2\boldsymbol{E}+k^2\boldsymbol{E}=0$,$\nabla^2\boldsymbol{H}+k^2\boldsymbol{H}=0$,其中波矢$\boldsymbol{k}=\omega^2\mu\epsilon\hat{k}=\frac{\omega}{v}\hat{k}$,波速$v=1/\sqrt{\mu\epsilon}=1/\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}\approx c/n$,真空光速$c=1/\sqrt{\epsilon_0\mu_0}$,折射率$n=\sqrt{\mu_r\epsilon_r}\approx\sqrt{\epsilon_r}$,有平面波(等相位面为平面)解$\boldsymbol{E}=\boldsymbol{E}_0e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}$,$\boldsymbol{H}=\boldsymbol{H}_0e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}$;证:$\nabla\times$①$\Rightarrow\nabla(\nabla\cdot\boldsymbol{E})-\nabla^2\boldsymbol{E}=-j\omega\nabla\times(\mu\boldsymbol{H})$(①''),④$\Rightarrow\nabla\cdot(\epsilon\boldsymbol{E})=\cancel{(\nabla\epsilon)\cdot\boldsymbol{E}}0$(均匀介质)$+\epsilon\nabla\cdot\boldsymbol{E}=0\Rightarrow\nabla\cdot\boldsymbol{E}=0$,和②入①''毕,$\nabla\times$②$\Rightarrow\nabla(\nabla\cdot\boldsymbol{H})-\nabla^2\boldsymbol{H}=j\omega\nabla\times(\epsilon\boldsymbol{E})$(②''),③$\Rightarrow\nabla\cdot(\mu\boldsymbol{H})=\cancel{(\nabla\mu)\cdot\boldsymbol{H}}0$(均匀介质)$+\mu\nabla\times\boldsymbol{H}=0\Rightarrow\nabla\times\boldsymbol{H}=0$,和①入②''毕

电场,磁场&波矢的关系:$\boldsymbol{k}\times\boldsymbol{E}_0=\omega\mu\boldsymbol{H}_0$,$\boldsymbol{k}\times\boldsymbol{H}_0=-\omega\epsilon\boldsymbol{E}_0$,$\boldsymbol{E}_0=\sqrt{\mu/\epsilon}\boldsymbol{H}_0\times\hat{k}=\eta\boldsymbol{H}_0\times\hat{k}$,$\boldsymbol{H}_0=\frac{1}{\eta}\hat{k}\times\boldsymbol{E}_0$,其中阻抗$\eta=\sqrt{\mu/\epsilon}=\eta_0/n$,真空阻抗$\eta_0=\sqrt{\mu_0/\epsilon_0}$;证:$\nabla\times[\boldsymbol{E}_0e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}]=-j\boldsymbol{k}e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}\times\boldsymbol{E}_0+\cancel{e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}\nabla\times\boldsymbol{E}_0}0$(平面波)$=-j\omega\mu\boldsymbol{H}_0e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}$,$\nabla\times[\boldsymbol{H}_0e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}]=-j\boldsymbol{k}e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}\times\boldsymbol{H}_0+\cancel{e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}\nabla\times\boldsymbol{H}_0}0$(平面波)$=j\omega\epsilon\boldsymbol{E}_0e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}$

波印廷矢量(能流):$\boldsymbol{S}=\frac12\mathrm{Re}[\boldsymbol{E}\times\boldsymbol{H}^*]=\frac{1}{2\eta}|E_0|^2\hat{k}=\frac{\eta}{2}|H_0|^2\hat{k}$

偏振:电场的振动方向,$\boldsymbol{E}=\hat{x}E_x+\hat{y}E_y=\hat{x}E_{x0}\cos(kz-\omega t+\phi_x)+\hat{y}E_{y0}\cos(kz-\omega t+\phi_y)$;若$\phi_x=\phi_y+n\pi$,$\boldsymbol{E}=(\hat{x}E_{x0}\pm\hat{y}E_{y0})\cos(kz-\omega t+\phi_x)$,线偏;若$\Delta\phi=\phi_y-\phi_x=-\pi/2+2n\pi$,右旋(IEEE标准:逆传播方向看);若$\Delta\phi=\pi/2+2n\pi$,左旋;$(\frac{E_x}{E_{x0}})^2+(\frac{E_y}{E_{y0}})^2-2\frac{E_x}{E_{x0}}\frac{E_y}{E_{y0}}\cos\Delta\phi=\sin^2\Delta\phi$,其中长轴与$x$轴夹角$\alpha=\arctan 2E_{x0}E_{y0}/(E_{x0}^2-E_{y0}^2)$;若$\alpha=0$,$\Delta\phi=\pm\frac{\pi}{2}$,$(E_x/E_{x0})^2+(E_y/E_{y0})^2=1$,正椭偏,若还$E_{x0}=E_{y0}$,圆偏;若$\Delta\phi=n\pi$,$E_y=\pm E_{y0}E_x/E_{x0}$,线偏;**偏振分解**:$\boldsymbol{E}=\frac{E_x+jE_y}{\sqrt2}\hat{R}+\frac{E_x-jE_y}{\sqrt2}\hat{L}$,其中右旋分量$\hat{R}=(\hat{x}-j\hat{y})/\sqrt2$,左旋分量$\hat{L}=(\hat{x}+j\hat{y})/\sqrt2$

TE模($\boldsymbol{E}\parallel$界面)在介质界面上的反/折射:入射$\boldsymbol{E}_{\rm in}=\hat{y}E_{\rm in0}e^{-jn_1\boldsymbol{k}_{\rm in}\cdot\boldsymbol{r}}$,$\boldsymbol{H}_{\rm in}=\hat{k}\times\hat{y}\frac{n_1}{\eta_0}E_{\rm in,0}e^{-jn_1\boldsymbol{k}_{\rm in}\cdot\boldsymbol{r}}$,反射$\boldsymbol{E}_{\rm rf}=\hat{y}E_{\rm rf0}e^{-jn_1\boldsymbol{k}_{\rm rf}\cdot\boldsymbol{r}}$,$\boldsymbol{H}_{\rm rf}=\hat{k}_{\rm rf}\times\hat{y}\frac{n_1}{\eta_0}E_{\rm rf0}e^{-jn_1\boldsymbol{k}_{\rm rf}\cdot\boldsymbol{r}}$,透射$\boldsymbol{E}_{\rm tr}=\hat{y}E_{\rm tr0}e^{-jn_2\boldsymbol{k}_{\rm tr}\cdot\boldsymbol{r}}$,$\boldsymbol{H}_{\rm tr}=\hat{k}_{\rm tr}\times\hat{y}\frac{n_2}{\eta_0}E_{\rm tr0}e^{-jn_2\boldsymbol{k}_{\rm tr}\cdot\boldsymbol{r}}$,其中$\boldsymbol{k}_{\rm in}=(\hat{x}\cos\phi_1+\hat{z}\sin\phi_1)k$,$\boldsymbol{k}_{\rm rf}=(-\hat{x}\cos\phi_{\rm rf}+\hat{z}\sin\phi_{\rm rf})k$,$\boldsymbol{k}_{\rm tr}=(\hat{x}\cos\phi_2+\hat{z}\sin\phi_2)k$,$\boldsymbol{r}=\hat{x}x+\hat{y}y+\hat{z}z$,$\boldsymbol{k}_{\rm in}\cdot\boldsymbol{r}=kx\cos\phi_1+kz\sin\phi_1$,$\boldsymbol{k}_{\rm rf}\cdot\boldsymbol{r}=-kx\cos\phi_{\rm rf}+kz\sin\phi_{\rm rf}$,$\boldsymbol{k}_{\rm tr}\cdot\boldsymbol{r}=kx\cos\phi_2+kz\sin\phi_2$,在界面上,$x=0$,$\boldsymbol{k}_{\rm in}\cdot\boldsymbol{r}=kz\sin\phi_1$,$\boldsymbol{k}_{\rm rf}\cdot\boldsymbol{r}=-kz\sin\phi_{\rm rf}$,$\boldsymbol{k}_{\rm tr}\cdot\boldsymbol{r}=kz\sin\phi_2$,边界条件:$E_{\rm in0}e^{-jn_1kz\sin\phi_1}+E_{\rm rf0}e^{-jn_1kz\sin\phi_{\rm rf}}=E_{\rm tr0}e^{-jn_2kz\sin\phi_2}$,$n_1\cos\phi_1E_{\rm in0}e^{-jn_1kz\sin\phi_1}-n_2\cos\phi_{\rm rf}E_{\rm rf0}e^{-jn_1kz\sin\phi_{\rm rf}}=n_2\cos\phi_2E_{\rm tr0}e^{-jn_2kz\sin\phi_2}$,∵反/折射与$z$无关,∴$\phi_1=\phi_{\rm rf}$,$n\sin\phi_1=n_2\sin\phi_2$(**Snell定律**),$E_{\rm in0}=E_{\rm rf0}=E_{\rm tr0}$,$n_1\cos\phi_1E_{\rm in0}-n_2\cos\phi_{\rm rf}E_{\rm rf0}=n_2\cos\phi_2E_{\rm tr0}$;**反射系数**:$\Gamma_\perp=\frac{E_{\rm rf0}}{E_{\rm in0}}=\frac{n_1\cos\phi_1-n_2\cos\phi_2}{n_1\cos\phi_1+n_2\cos\phi_2}=\frac{n_1\cos\phi_1-\sqrt{n_2^2-n_1^2\sin^2\phi_1}}{n_1\cos\phi_1+\sqrt{n_2^2-n_1^2\sin^2\phi_1}}$(**Fresnel方程**);反射率:$R_\perp=|\Gamma_\perp|^2$;若⊥入射,$\Gamma_\perp=\frac{n_1-n_2}{n_1+n_2}$;若光疏⊥入光密,$\Gamma_\perp<0$,入/反射相位差π;若光密入光疏,$\phi_1>\phi_c=\arcsin\frac{n_2}{n_1}$,则**全反射**,$\phi_2$为复数,∵能量有限,∴$\cos\phi_2=-j\sqrt{(n_1/n_2)^2\sin^2\phi-1}$,$\Gamma_\perp=\frac{n_1\cos\phi_1+j\sqrt{n_1^2\sin^2\phi_1-n_2^2}}{n_1\cos\phi_1-j\sqrt{n_1\sin^2\phi_1-n_2^2}}=e^{j2\Phi_\perp}$,$|\Gamma_\perp|=1$,$R=1$,$\Phi_\perp=\arctan\frac{\sqrt{n_1^2\sin^2\phi_1-n_2^2}}{n_1\cos\phi_1}$

TM模($\boldsymbol{H}\parallel$界面)在介质界面上的反/折射:输入$\boldsymbol{H}_{\rm in}=\hat{y}H_{\rm in0}e^{-jn_1\boldsymbol{k}_{\rm in0}\cdot\boldsymbol{r}}$,$\boldsymbol{E}_{\rm in}=\frac{\eta_0}{n_1}\boldsymbol{H}_{\rm in}\times\hat{k}_{\rm in}$,反射$\boldsymbol{H}_{\rm rf}=\hat{y}H_{\rm rf0}e^{-jn_1\boldsymbol{k}_{\rm rf}\cdot\boldsymbol{r}}$,$\boldsymbol{E}_{\rm rf}=\frac{\eta_0}{n_1}\boldsymbol{H}_{\rm rf}\times\hat{k}_{\rm rf}$,折射$\boldsymbol{H}_{\rm tr}=\hat{y}H_{\rm tr0}e^{-jn_2\boldsymbol{k}_{\rm tr}\cdot\boldsymbol{r}}$,$\boldsymbol{E}_{\rm tr}=\frac{\eta_0}{n_2}\boldsymbol{B}_{\rm tr}\times\hat{k}_{\rm tr}$,边界条件:$H_{\rm in0}+H_{\rm rf0}=H_{\rm tr0}$,$\frac{1}{n_1}\cos\phi_{\rm in}-\frac{1}{n_1}\cos\phi_{\rm rf}=\frac{1}{n_2}\cos\phi_{\rm tr}$;**反射系数**:$\Gamma_\parallel=\frac{H_{\rm rf0}}{H_{\rm in0}}=\frac{n_2\cos\phi_1-n_1\cos\phi_2}{n_2\sin\phi_1+n_1\cos\phi_2}=\frac{n_2^2\cos\phi_1-n_1\sqrt{n_2^2-n_1^2\sin^2\phi_1}}{n_2^2\cos\phi_1+n_1\sqrt{n_2^2-n_1^2\sin^2\phi_1}}$;**布儒斯特角**:若$\phi_1=\phi_B=\arctan\frac{n_2}{n_1}$,$\Gamma_\perp=0$,TM全折射,反射仅含TE;若$\phi_1>\phi_c$,$\cos\phi_2=-j\sqrt{(\frac{n_1}{n_2})^2\sin^2\phi_1-1}$,$\Gamma_\parallel=\frac{n_2^2\cos\phi_1+jn_1\sqrt{n_1^2\sin^2\phi_1-n_2^2}}{n_2^2\cos\phi_1-jn_1\sqrt{n_1^2\sin^2\phi_1-n_2^2}}=e^{j2\phi_\parallel}$,$|\Gamma_\parallel|=1$,$\Phi_\parallel=\arctan\frac{n_1\sqrt{n_1^2\sin^2\phi_1-n_2^2}}{n_2^2\cos\phi_1}$

波导:默认沿z传输,$\boldsymbol{E}(\boldsymbol{r},\omega)=[\boldsymbol{e}_t(x,y)+\hat{z}e_z(x,y)]e^{-j\beta z}$,$\boldsymbol{H}(\boldsymbol{r},\omega)=[\boldsymbol{h}_t(x,y)+\hat{z}e_z(x,y)]e^{-j\beta z}$,其中$\beta$-传播常数;①$\Rightarrow(\nabla_t,-j\beta\hat{z})\times[\boldsymbol{e}_t(x,y)+\hat{z}e_z(x,y)]e^{-j\beta z}=-j\omega\mu_0[\boldsymbol{h}_t(x,y)+\hat{z}h_z(x,y)]\Rightarrow\nabla_t\times\boldsymbol{e}_t(x,y)+\nabla_t\times[\hat{z}e_z(x,y)]-j\beta\hat{z}\times\boldsymbol{e}_t(x,y)-\cancel{j\beta\hat{z}\times\hat{z}e_z(x,y)}0=-j\omega\mu_0[\boldsymbol{h}_t(x,y)+\hat{z}h_z(x,y)]\Rightarrow\nabla_t\times\boldsymbol{e}_t(x,y)=-j\omega\mu_0h_z(x,y)$(⑥),$\nabla_t\times[\hat{z}e_z(x,y)]-j\beta\hat{z}\times\boldsymbol{e}_t(x,y)=-j\omega\mu_0\boldsymbol{h}_t(x,y)$,其中∵$\nabla_t\times[\hat{z}e_z(x,y)]=\nabla_te_z(x,y)\times\hat{z}+\cancel{e_z(x,y)\nabla_t\times\hat{z}}0$,∴$-\hat{z}\times\nabla_te_z(x,y)-j\beta\hat{z}\times\boldsymbol{e}_t(x,y)=-j\omega\mu_0\boldsymbol{h}_t(x,y)$(⑤),同理②$\Rightarrow-\hat{z}\times\nabla_th_z(x,y)-j\beta\hat{z}\times\boldsymbol{h}_t(x,y)=j\omega\epsilon_0n^2(x,y)\boldsymbol{e}_t(x,y)$(⑦),$\nabla_t\times\boldsymbol{h}_t(x,y)=j\omega\epsilon_0n^2(x,y)e_z(x,y)\hat{z}$(⑧),∵$\hat{z}\times(\hat{z}\times\boldsymbol{F})=-\boldsymbol{F}$,∴$\hat{z}\times$⑤$\Rightarrow\nabla_te_z(x,y)+j\beta\boldsymbol{e}_t(x,y)=-j\omega\mu_0\hat{z}\times\boldsymbol{h}_t(x,y)$,⑦入$\Rightarrow\nabla_te_z(x,y)+j\beta\boldsymbol{e}_t(x,y)=j\omega\mu_0\frac{1}{j\beta}[\hat{z}\times\nabla_th_z(x,y)+j\omega\epsilon_0n^2(x,y)e_t(x,y)]=\frac{\omega\mu_0}{\beta}\hat{z}\times\nabla_th_z(x,y)+\frac{\omega\mu_0}{\beta}j\omega\epsilon_0n^2(x,y)\boldsymbol{e}_t(x,y)\Rightarrow\boldsymbol{e}_t(x,y)=\frac{j[\beta\nabla_te_z(x,y)-\omega\mu_0\hat{z}\times\nabla_th_z(x,y)]}{\beta^2-\omega^2\mu_0\epsilon_0n^2(x,y)}$(⑤'),同理⑤入$\hat{z}\times$⑦$\Rightarrow\boldsymbol{h}_t(x,y)=\frac{j[\beta\nabla_th_z(x,y)+\omega\epsilon_0n^2(x,y)\hat{z}\times\nabla_te_z(x,y)]}{\beta^2-\omega^2\mu_0\epsilon_0n^2(x,y)}$(⑦'),式左均横向分量,右均纵向分量

平板波导:不失一般性,沿y无限延展,芯层折射率n_f>衬底n_s>包层n_c,$n(x,y)=n(x)$,$\frac{\partial}{\partial y}=0$,$\nabla_t=(\frac{\partial}{\partial x},0)$,⑥$\Rightarrow\hat{x}\frac{\rm d}{{\rm d}x}\times[e_x(x)\hat{x}+e_y(x)\hat{y}]=-j\omega\mu_0h_z(x)\hat{z}\Rightarrow\frac{{\rm d}e_y}{{\rm d}x}=-j\omega\mu_0h_z(x)$(⑥''),⑤$\Rightarrow-j\beta\hat{z}\times[e_x(x)\hat{x}+e_y(x)\hat{y}]-\hat{z}\times\frac{{\rm d}e_z}{{\rm d}x}\hat{x}=-j\beta\hat{y}e_x(x)+j\beta\hat{x}e_y(x)-\hat{y}\frac{{\rm d}e_z}{{\rm d}x}=-j\omega\mu_0[h_x(x)\hat{x}+h_y(x)\hat{y}]\Rightarrow-j\beta e_x(x)-\frac{{\rm d}e_z}{{\rm d}x}=-j\omega\mu_0h_y(x)$,$j\beta e_y(x)=-j\omega\mu_0h_x(x)$(⑤''),同理⑦$\Rightarrow j\beta h_y(x)=j\omega\epsilon_0n^2(x)e_x(x)$,$-j\beta h_x(x)-\frac{{\rm d}h_z}{{\rm d}x}=-j\omega\epsilon_0n^2(x)e_y(x)$(⑦''),⑧$\Rightarrow\frac{{\rm d}h_y(x)}{{\rm d}x}=j\omega\epsilon_0n^2(x)e_z(x)$(⑧'');**TE模**:有$e_y,h_x,h_z$分量,⑥''⑧''入⑦'$\Rightarrow-j\beta(-\frac{\beta}{\omega\mu_0})e_y(x)-\frac{j}{\omega\mu_0}\frac{{\rm d}^2e_y}{{\rm d}x^2}=-j\omega\epsilon_0n^2(x)e_y(x)\Rightarrow\frac{{\rm d}^2e_y}{{\rm d}x^2}+[\omega^2\mu_0\epsilon_0n^2(x)-\beta^2]e_y(x)=\frac{{\rm d}^2e_y}{{\rm d}x^2}+[k^2n^2(x)-\beta^2]e_y(x)=0$(**TE特征/色散方程**);**TM模**:有h_y,e_x,e_z分量,同理有**特征方程**$\frac{\rm d}{{\rm d}x}[\frac{1}{n^2(x)}\frac{{\rm d}h_y}{{\rm d}x}]+[k^2-\frac{\beta^2}{n^2(x)}]h_y(x)=0$

TE模:$e_y(y)=\begin{cases}E_ce^{-\gamma_cx}, & x>0\\ E_f\cos(k_fx+\phi)=E_c[\cos k_fh-\frac{\gamma_c}{k_f}\sin k_fx], & -h\le x\le0\\ E_se^{\gamma_s(x+h)}=E_c[\cos k_fh+\frac{\gamma_c}{\kappa_f}\sin k_fh]e^{\gamma_s(x+h)}, & x<-h\end{cases}$,中$\gamma_c=\sqrt{\beta^2-k^2n_c^2}$,$k_f=\sqrt{k^2n_f^2-\beta^2}$,$\gamma_s=\sqrt{\beta^2-k^2n_s^2}$,∵$n_c<n_s<n_f$,∴$k^2n_c^2<k^2n_s^2<\beta^2<k^2n_f^2$

TE特征方程:$k_fh=\arctan\frac{\gamma_r}{k_f}+\arctan\frac{\gamma_s}{k_f}+m\pi$,其中$m$-模式序号

TM模:$h_y(x)=\begin{cases}H_ce^{-\gamma_cx}, & x>0\\ H_f\cos(k_fx+\phi)=H_c[\cos k_fx-\frac{n_f^2\gamma_c}{n_c^2k_f}\sin k_fx], & -h\le x\le\\ H_se^{\gamma_s(x+h)}=H_c[\cos k_fh+\frac{n_f^2\gamma_c}{n_c^2k_f}\sin k_fh]e^{\gamma_s(x+h)}, & x<-h\end{cases}$

TM特征方程:$k_fh=\arctan(\frac{n_f^2}{n_c^2}\frac{\gamma_c}{k_f})+\arctan(\frac{n_f^2}{n_s^2}\frac{\gamma_s}{k_f})+m'\pi$,

归一化系数:**非对称度量**:$a=\frac{n_s^2-n_c^2}{n_f^2-n_s^2}$,表征波导上下非对称性,若包层与衬底同,则$a=0$,**归一化频率/厚度**:$V=kh\sqrt{n_f^2-n_s^2}$,**可导因子**:$b=\frac{N^2-n_s^2}{n_f^2-n_s^2}$,其中**有效折射率**$N=\frac{\beta}{k}$,$c=\frac{n_s^2}{n_f^2}$,$d=\frac{n_c^2}{n_f^2}=c-a(1-c)$,通常$n_c<n_s<N<n_f$,∴$0<b<1$,$d<c<1$;$k_fh=kh\sqrt{n_f^2-N^2}=V\sqrt{1-b}$,$\gamma_sh=kh\sqrt{N^2-n_s^2}=V\sqrt b$,$\gamma_ch=kh\sqrt{N^2-n_c^2}=V\sqrt{a+b}$

归一化TE:$e_y(x)=\begin{cases}E_c\exp(-V\sqrt{a+b}x/h), & x\ge0\\ E_c[\cos(\frac{V\sqrt{1-bx}}{h})-\sqrt{\frac{a+b}{1-b}}\sin(\frac{V\sqrt{1-bx}}{h})], & -h\le x<0\\ E_c[\cos(V\sqrt{1-b})+\sqrt{\frac{a+b}{1-b}}\sin(V\sqrt{1-b})]e^{V\sqrt b[1+(x/h)]}, & x<-h\end{cases}$

归一化TM:$h_y(x)=\begin{cases}H_ce^{-V\sqrt{a+b}x/h}, & x>0\\ H_c[\cos\frac{V\sqrt{1-bx}}{h}-\frac1d\sqrt{\frac{a+b}{1-b}}\sin\frac{V\sqrt{1-bx}}{h}], & -h\le x\le0\\ H_c[\cos V\sqrt{1-b}+\frac1d\sqrt{\frac{a+b}{1-b}}\sin V\sqrt{1-b}]e^{V\sqrt b[1+x/h]}, & x<-h\end{cases}$

归一化TE特征方程:$V\sqrt{1-b}=\arctan\sqrt{\frac{a+b}{1-b}}+\arctan\sqrt{\frac{b}{1-b}}+m\pi$

归一化TM特征方程:$V\sqrt{1-b}=\arctan\frac1d\sqrt{\frac{a+b}{1-b}}+\arctan\frac1c\sqrt{\frac{b}{1-b}}+m'\pi$

截止频率/厚度:模式允许存在的最小频率/厚度,$b=0$入特征方程,对**TE**有$V_m=m\pi+\arctan\sqrt a\Rightarrow h=\frac{m\pi+\arctan\sqrt a}{2\pi\sqrt{n_f^2-n_s^2}}\lambda$,若$a=0$,$V_m=m\pi$,$h=\frac{m\lambda}{2\sqrt{n_f^2-n_s^2}}$,对**TM**有$V_{m'}=m'\pi+\arctan\frac{\sqrt a}{d}$,当$a=0$,$V_{m'}=m'\pi$,$h=\frac{m'\lambda}{2\sqrt{n_f^2-n_s^2}}$;若$V\gg1$,总模式数$\approx2(1+V/\pi)$

$b-V$图特征:$V\uparrow\Rightarrow b\uparrow$,对应一个$V$或有一个或多个模式($b$);$h$,$(n_f^2-n_s^2)\uparrow$或$\lambda\downarrow$,则$V\uparrow$,模式数↑;低阶模$\beta>$高阶模;若$a=0$,基模$b-V$曲线过原点

模式计算步骤:已知波导结构(h,n_c,n_f,n_s)和模式波长λ,算a,c,d,V,由$b-V$图得b,N,β,模场

TE能流:$\boldsymbol{S}=\frac12\mathrm{Re}[\boldsymbol{E}\times\boldsymbol{H}^*]=\frac12\mathrm{Re}[e_y\hat{y}\times(h_x\hat{x}+h_z\hat{z})^*]=\frac12\mathrm{Re}[-e_yh_x^*\hat{z}+e_yh_z^*\hat{x}]=\frac12\mathrm{Re}[e_y\frac{\beta e_y^*}{\omega\mu_0}\hat{z}]-\cancel{\frac12\mathrm{Re}[e_y\frac{j}{\omega\mu_0}\frac{{\rm d}e_y^*}{{\rm d}x}\hat{x}]}0=\frac{\beta|e_y|^2}{2\omega\mu_0}\hat{z}$,**TE单位y上功率**:$P=\int_{-\infty}^{+\infty}\boldsymbol{S}\cdot({\rm d}\boldsymbol{x}\times\hat{y})=\frac{\beta}{2\omega\mu_0}[\int_{-\infty}^{-h}E_s^2e^{2\gamma_s(x+h)}\,{\rm d}x+\int_{-h}^{0}E_f^2\cos^2(k_fx+\phi)\,{\rm d}x+\int_0^{+\infty}E_c^2e^{-2\gamma x}\,{\rm d}x]=\frac{\beta}{4\omega\mu_0}[\frac{E_s^2}{\gamma_s}+E_f^2(h+\frac{\sin\phi-\sin2(-k_fx+\phi)}{2k_f})+\frac{E_c^2}{\gamma_c}]$,由边界条件,$E_f\cos\phi=E_c$,$k_fE_f\sin\phi=\gamma_cE_c\Rightarrow\sin2\phi=\frac{2E_c^2\gamma_c}{E_f^2k_f^2}$,同理$\sin(2k_fh+\phi)=-\frac{2E_s^2\gamma_s}{E_f^2k_f}$,$P=\frac{\beta}{4\omega\mu_0}[E_f^2h+E_s^2(\frac{1}{\gamma_s}+\frac{\gamma_s}{k_f^2})+E_c^2(\frac{1}{\gamma_c}+\frac{\gamma_c}{k_f^2})]$,∵$\sin^2\phi+\cos^2\phi=\frac{E_c^2}{E_f^2}(1+\frac{\gamma_c^2}{k_f^2})=1\Rightarrow E_c^2(\frac{1}{\gamma_c}+\frac{\gamma_c}{k_f^2})=\frac{E_f^2}{\gamma_c}$,同理$E_s^2(\frac{1}{\gamma_s}+\frac{\gamma_s}{k_f^2})=\frac{E_f^2}{\gamma_s}$,∴$P=\frac{\beta}{4\omega\mu_0}E_f[h+\frac{1}{\gamma_s}+\frac{1}{\gamma_c}]=\frac{\beta}{4\omega\mu_0}E_f^2h_{\rm eff}$,其中**等效模场厚度**$h_{\rm eff}=h+\frac{1}{\gamma_s}+\frac{1}{\gamma_c}$,**归一化模场厚度**:$H=k_fh_{\rm eff}\sqrt{n_f^2-n_s^2}=h_f(h+\frac{1}{\gamma_s}+\frac{1}{\gamma_c})\sqrt{n_f^2-n_s^2}=V+\frac{1}{\sqrt{a+b}}+\frac{1}{\sqrt b}$;**TE芯层束缚因子**:$\Gamma_f=\frac{\text{芯层传输功率}}{\text{总传输功率}}=\frac{E_f^2(h+\frac{E_c^2}{E_f^2}\frac{\gamma_c}{k_f^2}+\frac{E_s^2}{E_f^2}\frac{\gamma_s}{k_f^2})}{E_f^2(h+\frac{1}{\gamma_s}+\frac{1}{\gamma_c})}$,由边界条件,$\frac{E_c^2}{E_f^2}=\frac{k_f^2}{k_f^2+\gamma_c^2}$,$\frac{E_s^2}{E_f^2}=\frac{k_f^2}{k_f^2+\gamma_s^2}$,∴$\Gamma_f=\frac{h+\frac{\gamma_c}{k_f^2+\gamma_c^2}+\frac{\gamma_s}{k_f^2+\gamma_s^2}}{h+\frac{1}{\gamma_c}+\frac{1}{\gamma_s}}=\frac{V+\sqrt b+\frac{\sqrt{a+b}}{1+a}}{V+\frac{1}{\sqrt b}+\frac{1}{\sqrt{a+b}}}$,同理**衬底束缚因子**$\Gamma_s=\frac{\text{衬底传输功率}}{\text{总传输功率}}=\frac{1-b}{\sqrt b[V+\frac{1}{\sqrt b}+\frac{1}{\sqrt{a+b}}]}$,**包层束缚因子**$\Gamma_c=\frac{\text{包层传输功率}}{\text{总传输功率}}=\frac{1-b}{(1+a)\sqrt{a+b}[V+\frac1b+\frac{1}{\sqrt{a+b}}]}$

TM能流:$\boldsymbol{S}=\frac{\beta|h_y|^2}{2\omega\epsilon_0n(x)^2}\hat{z}$,**单位y上功率**:$P=\frac{\beta}{4\omega\epsilon_0}[\frac{H_s^2}{\gamma_sn_s^2}+\frac{H_f^2}{n_f^2}(h+\frac{\sin2\phi'-\sin2(-k_fh+\phi')}{2k_f})+\frac{H_c^2}{\gamma_cn_c^2}]=\frac{\beta}{4\omega\epsilon_0}\frac{H_f^2}{n_f^2}[h+\frac{1}{\gamma_sq_s}+\frac{1}{\gamma_cq_c}]=\frac{\beta}{4\omega\epsilon_0}\frac{H_f^2}{n_f^2}h_{\rm eff}$,其中$q_s=\frac{N^2}{n_s^2}+\frac{N^2}{n_f^2}-1$,$q_c=\frac{N^2}{n_c^2}+\frac{N^2}{n_f^2}-1$,**等效模场厚度**:$h_{\rm eff}=h+\frac{1}{\gamma_sq_s}+\frac{1}{\gamma_cq_c}$

几何光学角度看,**导模**:波导界面上全反射;**辐射模**:波导界面上有折射

相速度:等相位面移速,$v_p=\frac{\omega}{\beta}=\frac{\omega}{kN}=\frac{c}{N}$,其中$N$-等效折射率,高阶模相速大;**群速度**:波包移速,本质是介质对非单色光的色散,$v_g=\frac{{\rm d}\omega}{{\rm d}\beta}=\frac{c\,{\rm d}k}{{\rm d}\beta}=\frac{c}{n_g}$,其中群折射率$n_g=\frac{{\rm d}\beta}{{\rm d}k}$;**相/群速关系**:$\frac{c^2}{v_pv_g}=\frac{c^2}{\frac{\omega}{\beta}\frac{{\rm d}\omega}{{\rm d}\beta}}=\frac{\beta{\rm d}\beta}{k{\rm d}k}=N\frac{{\rm d}(kN)}{{\rm d}k}=N[N+k\frac{{\rm d}N}{{\rm d}k}]=N^2+\frac{k}{V}\frac{{\rm d}N^2}{{\rm d}k}$,由$V$定义有$\frac{{\rm d}k}{{\rm d}V}=\frac{1}{h\sqrt{n_f^2-n_s^2}}=\frac{k}{V}$,$\frac{{\rm d}N^2}{{\rm d}k}=\frac{{\rm d}N^2/{\rm d}V}{{\rm d}k/{\rm d}V}=\frac{{\rm d}[b(n_f^2-n_s^2)]/{\rm d}V}{k/V}=$(忽略材料色散)$\frac{(n_f^2-n_s^2)\,{\rm d}b/{\rm d}V}{k/V}$,∴$\frac{c^2}{v_pv_g}=(n_f^2-n_c^2)b+n_s^2+\frac{k}{2}(n_f^2-n_s^2)\frac{{\rm d}b}{{\rm d}V}\frac{V}{k}=n_f^2(b+\frac{V}{2}\frac{{\rm d}n}{{\rm d}V})+n_s^2(1-b-\frac{V}{2}\frac{{\rm d}b}{{\rm d}V})$,其中利用特征方程,对**TE模**,$\frac{{\rm d}b}{{\rm d}V}=\frac{2(1-b)}{V+\frac{1}{\sqrt b}+\frac{1}{\sqrt{a+b}}}\Rightarrow\frac{c^2}{v_pv_g}=n_f^2\Gamma_f+n_s^2\Gamma_s+n_c^2\Gamma_c$,对良好束缚(well-guided)的波导,能量主要束缚在芯层,$\Gamma_f\approx1$,$\Gamma_s\approx\Gamma_c\approx0\Rightarrow\frac{c^2}{v_pv_g}\approx n_f^2$,低阶模群速度大

波导传输损耗:$\alpha_{\mathrm{dB}} = -10\lg(P_{\mathrm{out}}/P_{\mathrm{in}})$;来源:1)光与介质中电子(主要),原子,分子相互作用致吸收损耗,化为热,声,2)波导结构缺陷,包括几何上的不规则,材料缺陷和不均匀,(对玻璃等无定型材料)团簇大小和组分的涨落,致散射损耗,表现为反向传播,跳模,辐射模

复电极化率:$\nabla \times \boldsymbol{H} = (j\omega\epsilon + \sigma)\boldsymbol{E} = j\omega\epsilon_0\tilde{\epsilon}_r\boldsymbol{E} \Rightarrow \tilde{\epsilon}_r = \frac{\epsilon}{\epsilon_0} - j\frac{\omega}{\epsilon} = \epsilon_r - j\epsilon_i$

由**Drude(/自由电子)模型**(适用含大量无束缚载流子的介质):$\tilde{\epsilon}_r = 1 - \frac{\omega_p^2}{\omega^2+\omega_c^2} - j\frac{\omega_c\omega_p^2}{\omega(\omega^2+\omega_c^2)}$,其中$\omega_c$-碰撞频率,$\omega_p$-等离子体频率;证:载流子受电场力和(碰撞致)阻尼力,$qE(t) - m\omega_c\dot{x} = m\ddot{x}$,其中$q$-载流子电荷,$m$-质量,$x$-位移,对单色光,电场$E(t) = E_0e^{j\omega t}$,猜$x(t) = x_0e^{j\omega t}$,回代得$x_0 = \frac{qE_0}{jm\omega\omega_c - m\omega^2} \Rightarrow x(t) = \frac{qE(t)}{m(j\omega\omega_c-\omega^2)}$,电偶极矩$p(t) = qx = \frac{q^2E(t)}{m(j\omega\omega_c-\omega^2)}$,电极化强度$P(t) = Np = \frac{Nq^2E(t)}{m(j\omega\omega_c-\omega^2)}$,电位移矢量$D(t) = \epsilon_0E + P = \epsilon_0[1 + \frac{Nq^2}{\epsilon_0 m(j\omega\omega_c-\omega^2)}]E(t) = \epsilon_0\tilde{\epsilon}_rE(t)$,其中$\tilde{\epsilon}_r = 1 - \frac{Nq^2}{\epsilon_0 m(\omega^2-j\omega\omega_c)} = 1 - \frac{\omega_p^2}{\omega^2-j\omega\omega_c}$毕,其中$\omega_p = \sqrt{\frac{Nq^2}{\epsilon_0 m}}$通常在紫外波段;对金属,自由电子罕碰撞,$\omega_c \approx 0$,$\epsilon_i \approx 0$,$\tilde{\epsilon}_r \approx 1 - (\frac{\omega_p}{\omega})^2$

由**Lorenz模型**(适用电荷受核束缚的介质):$\tilde{\epsilon}_r = 1 - \frac{\omega_p(\omega^2-\omega_0^2)}{(\omega^2-\omega_0^2)+\omega^2\omega_c^2} - j\frac{\omega_p^2\omega_c\omega}{(\omega^2-\omega_0^2)+\omega^2\omega_c^2}$,其中$\omega$-谐振频率;证:载流子受电场力,阻尼力和回复力,$qE(t) - m\omega_c\dot{x} - m\omega_0^2x(t) = m\ddot{x}$,同理$x(t) = \frac{qE(t)}{m(\omega_0^2-\omega^2+j\omega\omega_c)}$,$\tilde{\epsilon}_r = 1 + \frac{Nqx}{E} = 1 - \frac{Nq^2}{m\epsilon_0(\omega^2-\omega_0^2-j\omega\omega_c)}$毕;若$\omega = \omega_0$,共振,吸收最强;若$\omega$远离$\omega_0$,$\frac{\mathrm{d}n}{\mathrm{d}\omega} > 0$,正(常)色散;若$\omega$接近$\omega_0$,$\frac{\mathrm{d}n}{\mathrm{d}\omega} < 0$,反(常)色散

复折射率:$\tilde{n} = \sqrt{\tilde{\epsilon}_r} = n - j\kappa$,其中$n = (\frac{\epsilon_r+\sqrt{\epsilon_r^2+\epsilon_i^2}}{2})^{1/2}$,$\kappa = (\frac{-\epsilon_r+\sqrt{\epsilon_r^2+\epsilon_i^2}}{2})^{1/2}$,通常(半导体,绝缘体等)$\kappa \ll n$,对金属$\kappa \gg n$;**复波矢**:$\tilde{k} = k\tilde{n} = nk - j\kappa k \Rightarrow |E| \propto |e^{j\omega t-j\tilde{k}x}| = e^{-\kappa kx}$;**衰减系数**$\alpha = \kappa k$,**衰减长度(集肤深度)**:$\alpha^{-1} = (\kappa k)^{-1}$,对平面波导,$\tilde{n}_c = n_c - j\kappa_c$,$\tilde{n}_f = n_f - j\kappa_f$,$\tilde{n}_s = n_s - j\kappa_s$,对TE模,$\alpha_{\mathrm{TE}} = k[\kappa_sn_s\int_{-\infty}^{-h}|e_y(x)|^2\,\mathrm{d}x + \kappa_fn_f\int_{-h}^{0}|e_y(x)|^2\,\mathrm{d}x + \kappa_cn_c\int_0^{+\infty}|e_y(x)|^2\,\mathrm{d}x]/[N\int_{-\infty}^{+\infty}|e_y(x)|^2\,\mathrm{d}x]$

金属包层平板波导:$\because$完美导体内无电场,由边界条件$e_y(0) = 0\forall$TE;TM有少量$h_y(x)$渗入金属,损耗>TE;TM_0能量大量集中于与金属交界面附近,称**表面波**;$\tilde{\beta} = \beta - j\alpha$,对良好束缚波导,b $\approx 1 \Rightarrow \beta \approx n_fk$,$\tilde{k}_f = \sqrt{k^2\tilde{n}_f^2 - \tilde{\beta}^2} \approx 0$,$\tilde{\gamma}_c = \sqrt{\tilde{\beta}^2 - k^2\tilde{n}_c^2}$,$|\tilde{\gamma}_c| \gg |\tilde{k}_f|$,$\arctan\frac{\tilde{\gamma}_c}{\tilde{k}_f} \approx \frac{\pi}{2} - \arctan\frac{\tilde{k}_f}{\tilde{\gamma}_c} \approx \frac{\pi}{2} - \frac{\tilde{k}_f}{\tilde{\gamma}_c}$,$\tilde{\gamma}_s = \sqrt{\tilde{\beta}^2 - k^2\tilde{n}_s^2}$,$|\tilde{\gamma}_s| \gg |\tilde{k}_f|$,$\arctan\frac{\tilde{\gamma}_s}{\tilde{k}_f} \approx \frac{\pi}{2} - \frac{\tilde{k}_f}{\tilde{\gamma}_s}$,**TE特征方程**:$\tilde{k}_fh \approx (m+1)\pi - \frac{\tilde{k}_f}{\tilde{\gamma}_c} - \frac{\tilde{k}_f}{\tilde{\gamma}_s} \Rightarrow \tilde{k}_f = \frac{(m+1)\pi}{h}(1 + \frac{1}{\tilde{\gamma}_ch} + \frac{1}{\tilde{\gamma}_sh})^{-1} \Rightarrow \tilde{\beta}_{\mathrm{TE}m} = \sqrt{k^2\tilde{n}_f^2 - \tilde{k}_f^2} \approx k\tilde{n}_f(1 - \frac{\tilde{k}_f^2}{2k^2\tilde{n}_f^2}) \approx k\tilde{n}_f - \frac{(m+1)^2\pi^2}{2k\tilde{n}_fh^2}(1+\frac{1}{\tilde{\gamma}_sh}+\frac{1}{\tilde{\gamma}_ch})^{-2}$,若芯层无损,$\kappa_f = 0$,$\tilde{n}_f = n_f$,$\frac{\tilde{\beta}_{\mathrm{TE}m}}{k} \approx n_f - \frac{(m+1)^2\pi}{2n_f(kh)^2}(1+\frac{1}{kh\sqrt{n_f^2-\tilde{n}_c^2}} + \frac{1}{kh\sqrt{n_f^2-\tilde{n}_s^2}})$,$\frac{\alpha_{\mathrm{TE}m}}{k} \approx \frac{(m+1)^2\pi^2}{2n_f(kh)^2}\operatorname{Im}[\frac{1}{kh\sqrt{n_f^2-\tilde{n}_c^2}} + \frac{1}{kh\sqrt{n_f^2-\tilde{n}_s^2}}]^{-2}$,$\because$通常$|\epsilon_r| \gg \epsilon_i$,$\therefore \frac{\alpha_{\mathrm{TE}m}}{k} \approx \frac{(m+1)^2\pi^2}{2n_f(kh)^2}\operatorname{Im}[-2(\frac{1}{\sqrt{n_f^2-\epsilon_{cr}+j\epsilon_{ci}}} + \frac{1}{\sqrt{n_f^2-\epsilon_{sr}+j\epsilon_{si}}})] \approx \frac{(m+1)^2\pi^2}{2n_f(kh)^2}[\frac{\epsilon_{ci}}{(n_f^2-\epsilon_{cr})^{3/2}} + \frac{\epsilon_{si}}{(n_f^2-\epsilon_{sr})^{3/2}}]$;**TM**同理$\frac{\tilde{\beta}_{\mathrm{TM}m'}}{k} \approx n_f - \frac{(m'+1)^2\pi^2}{2n_f(kh)^2}[1 + \frac{\tilde{n}_c^2}{n_f}\frac{1}{kh\sqrt{n_f^2-\tilde{n}_c^2}} + \frac{\tilde{n}_s}{n_f^2}\frac{1}{kh\sqrt{n_f^2-\tilde{n}_s^2}}]^{-2}$,$\frac{\alpha_{\mathrm{TM}m'}}{k} \approx \frac{(m'+1)^2\pi^2}{2n_f(kh)^2}[\frac{\epsilon_{ci}(2n_f^2-\epsilon_{cr})}{n_f^2(n_f^2-\epsilon_{cr})^{3/2}} + \frac{\epsilon_{si}(2n_f^2-\epsilon_{sr})}{n_f^2(n_f^2-\epsilon_{sr})^{3/2}}]$;$m\uparrow$,$h\uparrow$,则$\alpha\downarrow$;$\because \frac{2n_f^2-\epsilon_{cr/sr}}{n_f^2} > 1$,$\therefore$同阶TE损耗<TM;对包层\衬底均金属的$\mathrm{TM}_0$,$n_s^2 = n_c^2 = \epsilon_1$,$n_f^2 = \epsilon_2$,由麦氏方程,$\tilde{\beta} = k\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}} \Rightarrow N^2 = \frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2} \Rightarrow \operatorname{Re}(\frac{N^2}{n_f^2}) = \operatorname{Re}(\frac{\epsilon_1}{\epsilon_1+\epsilon_2}) > 1$,或由特征方程,$\tilde{k}_fh = 2\arctan\frac{n_f^2}{\tilde{n}_s^2}\frac{\tilde{\gamma}_s}{\tilde{k}_f} + m'\pi$,其中$\tilde{k}_f = j\sqrt{\tilde{\beta}^2 - k^2n_f^2}$,$j\sqrt{\tilde{\beta}^2 - k^2n_f^2}h = m'\pi - j2\operatorname{arctanh}\frac{n_f^2}{\tilde{n}_s^2}\frac{\sqrt{\tilde{\beta}^2-k^2\tilde{n}_s^2}}{\sqrt{k^2n^2-\tilde{\beta}^2}}$,金属$|\epsilon_{sr}| \gg \epsilon_{si}$,$\therefore \tilde{n}_s^2 = \epsilon_{sr} - j\epsilon_{si} \approx \operatorname{Re}[\tilde{n}_s^2] < 0 \Rightarrow j\sqrt{\tilde{\beta}^2 - k^2n_f^2}h = m'\pi - j\operatorname{arctanh}\frac{n_f^2}{\operatorname{Re}[\tilde{n}_s^2]}\frac{\sqrt{\tilde{\beta}^2-k^2\operatorname{Re}[\tilde{n}_s^2]}}{\sqrt{\tilde{\beta}^2-k^2n_f^2}}$,对$m' \neq 0$,式左纯虚,$\therefore \beta$必非纯实,对$m' = 0$,$\tanh\frac{\sqrt{\tilde{\beta}^2-k^2n_f^2}h}{2} = -\frac{n_f^2}{\tilde{n}_s^2}\frac{\sqrt{\tilde{\beta}^2-k^2\tilde{n}_s^2}}{\sqrt{\tilde{\beta}^2-k^2n_f^2}}$,良好束缚时$\tilde{k}_fh \to \infty$,$\therefore -\frac{n_f^2}{\tilde{n}_s^2}\frac{\sqrt{\tilde{\beta}^2-k^2\tilde{n}_s^2}}{\sqrt{\tilde{\beta}^2-k^2n_f^2}} \approx 1 \Rightarrow \frac{\tilde{\beta}}{k} \approx \sqrt{\frac{n_f^2\tilde{n}_s^2}{n_f^2+\tilde{n}_s^2}}$,沿芯层与金属交界面附近传播,并非由折射率束缚,称表面等离子基元波

3D波导:模式命名:$E_{p,q}^{x/y}$,其中x/y-主要电场分量方向,$p-1$,$q-1$-x,y方向电场分布零点数

⑤$\Rightarrow -j\beta\hat{z}\times[e_x(x,y)\hat{x} + e_y(x,y)\hat{y}] - \hat{z}\times[\frac{\partial e_x(x,y)}{\partial x}\hat{x} + \frac{\partial e_y(x,y)}{\partial y}\hat{y}]$,⑥$\Rightarrow \hat{z}[\frac{\partial e_y(x,y)}{\partial x} - \frac{\partial e_x(x,y)}{\partial y}] = -j\omega\mu_0h_z(x,y)\hat{z}$

弱导条件(weakly guiding,$n_f \approx n_s$,3D波导通常用衬底掺杂实现,折射率变化很小,故适用,与良好束缚不冲突)下,$k_f^2 = k^2n_f^2 - \beta^2 = k^2n_f(n_f+n_s)(1-b)\Delta \approx 2k^2n_f^2(1-b)\Delta \Rightarrow \frac{k_f}{kn_f} = \sqrt{2}\sqrt{(1-b)\Delta} < \sqrt{2\Delta} \sim o(\delta)$,其中$\Delta = \frac{n_f-n_s}{n_f}$,$o(\delta)$-一阶小量,$k_f^2 = k_x^2 + k_x^2 \Rightarrow \frac{k_{x/y}}{kn_f} \sim \delta$;对良好束缚的$E^y$模,$|H_x| \sim \frac{n}{\eta_0}|E_y| \sim o(1)$,$|H_z| \sim \frac{n}{\eta_0}|E_z| \sim o(\delta)$,$|H_z| \sim \frac{n}{\eta_0}|E_x| \sim o(\delta^2)$,$\frac{n}{\eta_0}E_x = o(\delta^2)$,$\frac{n}{\eta_0}E_y = -\frac{\beta}{kn}H_x + o(\delta^2) = -\frac{kn}{\beta}H_x + o(\delta^2)$,$\frac{n}{\eta_0}E_z = \frac{j}{kn}\frac{\partial H_x}{\partial y} + o(\delta^2)$,$H_y = o(\delta^2)$,$H_z = -\frac{j}{\beta}\frac{\partial H_x}{\partial x} + o(\delta^2)$;证:初始有$|E_y| \sim 1$,故$H_y$可忽略,③'$\Rightarrow \frac{\partial H_x}{\partial x} + \frac{\partial H_z}{\partial z} = \frac{\partial H_x}{\partial x} - j\beta H_z = 0 \Rightarrow |H_z| \sim |\frac{1}{\beta}\frac{\partial H_x}{\partial x}| \sim |\frac{k_x}{\beta}H_z| \sim |\frac{k_x}{nk}H_x| \sim \delta$,②'$\Rightarrow \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon_0n^2E_y \Rightarrow \frac{n}{\eta_0}E_y = \frac{j}{kn}\frac{\partial H_z}{\partial x} - \frac{\beta}{kn}H_x$,其中$|\frac{1}{kn}\frac{\partial H_z}{\partial x}| \sim |\frac{k_x}{kn}H_z| \sim \delta^2 \Rightarrow |H_x| \sim |\frac{\beta}{kn}H_x| \sim |\frac{n}{\eta_0}E_y| \sim o(1)$,$H_z \approx -\frac{j}{\beta}\frac{\partial H_x}{\partial x}$入$j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon_0n^2E_y \Rightarrow \frac{n}{\eta_0}E_y \approx \frac{1}{kn\beta}(\frac{\partial^2H_x}{\partial x} - \beta^2H_x)$,$\nabla_t^2H_x + (k^2n^2 - \beta)H_x = 0$入$\Rightarrow \frac{n}{\eta_0}E_y \approx -\frac{1}{kn\beta}(\frac{\partial^2H_z}{\partial y^2} + k^2n^2H_x)$,其中$|\frac{1}{kn\beta}\frac{\partial^2H_z}{\partial y^2}| \sim |\frac{k_y^2}{k^2n^2}H_x| \sim o(\delta)^2 \Rightarrow \frac{n}{\eta_0}E_y \approx -\frac{kn}{\beta}H_x$,②'$\Rightarrow j\omega\epsilon_0n^2E_y \approx \frac{\partial H_y}{\partial y} \Rightarrow \frac{n}{\eta_0}E_x \approx -\frac{j}{kn}\frac{\partial H_z}{\partial y} \Rightarrow |\frac{n}{\eta_0}E_x| \sim o(\delta^2)$,②'$\Rightarrow j\omega\epsilon_0n^2E_z \approx \frac{\partial H_x}{\partial y} \Rightarrow \frac{n}{\eta_0}E_z \approx \frac{j}{kn}\frac{\partial H_x}{\partial y} \Rightarrow |\frac{n}{\eta_0}E_z| \sim |\frac{k_y}{kn}H_x| \sim o(\delta)$,①'$\Rightarrow -j\omega\mu_0H_y = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \Rightarrow H_y = \frac{\beta}{\omega\mu_0}E_x - \frac{j}{\omega\mu_0}\frac{\partial E_z}{\partial x} \approx \frac{n}{\eta_0}E_x - \frac{j}{kn}\frac{n}{\eta_0}\frac{\partial E_z}{\partial x} \Rightarrow |H_y| \sim o(\delta^2)$;对良好束缚的$E^x$模,$|H_y| \sim \frac{n}{\eta_0}|E_x| \sim o(1)$,$|H|_z \sim \frac{n}{\eta_0}|E_z| \sim o(\delta)$,$|H|_x \sim \frac{n}{\eta_0}|E_y| \sim o(\delta^2)$,$\frac{n}{\eta_0}E_x = \frac{\beta}{kn}H_y + \delta(\delta^2) = \frac{kn}{\beta}H_y + o(\delta^2)$,$\frac{n}{\eta_0}E_y = o(\delta^2)$,$\frac{n}{\eta_0}E_y = o(\delta^2)$,$\frac{n}{\eta_0}E_z = -\frac{j}{kn}\frac{\partial H_y}{\partial x} + o(\delta^2)$,$H_x = o(\delta^2)$,$H_z = -\frac{-j}{\beta}\frac{\partial H_y}{\partial y} + o(\delta^2)$

Marcatili方法:将3D波导$n(x,y) = n_1(\mathrm{R1}: |x| \leq \frac{w}{2}, |y| \leq \frac{h}{2}), n_2(\mathrm{R2}: |x| \leq \frac{w}{2}, y > \frac{h}{2}), n_3(\mathrm{R3}: x > \frac{w}{2}, |y| \leq \frac{h}{2}), n_4(\mathrm{R4}: |x| \leq \frac{w}{2}, y < \frac{h}{2}), n_5(\mathrm{R5}: x < -\frac{w}{2}, |y| \leq \frac{h}{2})$拆解为横向平板波导H,$n(y) = n_1(|y| \leq \frac{h}{2}), n_2(y > \frac{h}{2}), n_4(y < -\frac{h}{2})$和纵向平板波导W,$n(x) = n_1(|x| \leq \frac{w}{2}), n_3(x > \frac{w}{2}), n_5(x < -\frac{w}{2})$分别求解;对$E^y$模,R1有$H_{x1} = C_1\cos(k_{x1}x + \phi_{x1})\cos(k_{y1} + \phi_{y1})e^{-j\beta z}$,R2有$H_{x2} = C_2\cos(k_{x2}x + \phi_{x2})e^{-jk_{y2}y}e^{-j\beta z}$,R3有$H_{x3} = C_3e^{-jk_3x}\cos(k_{y3}y + \phi_{y3})e^{-j\beta z}$,R4有$H_{x4} = C_4\cos(k_{x4}x + \phi_{x4})e^{jk_{y4}y}e^{-j\beta z}$,R5有$H_{x5} = C_5e^{jk_{x5}x}\cos(k_{y5}y + \phi_{y5})e^{-j\beta z}$,其余4角能量少,故可忽略,其中$k_{xj}^2 + k_{yj} = \beta^2 = k^2n_j^2$,在$y = \pm\frac{h}{2}$,$H_{x1} = H_{x2/4}$,$\Rightarrow k_{x1} = k_{x2} = k_{x4} = k_x$,$\phi_{x1} = \phi_{x2} = \phi_{x4} = \phi_x$,$\frac{n}{\eta_0}E_z \approx \frac{j}{kn}\frac{\partial H_x}{\partial y} \Rightarrow \frac{1}{n^2}\frac{\partial H_x}{\partial y}$连续,$H_z \approx \frac{-j}{\beta}\frac{\partial H_x}{\partial x} \Rightarrow \frac{\partial H_x}{\partial x}$连续,在$x = \pm\frac{w}{2}$,$\mu_0H_{x1} = \mu_0H_{x3/5} \Rightarrow k_{y1} = k_{y3} = k_{y5}$,$\phi_{y1} = \phi_{y3} = \phi_{y5} = \phi_y$,$\frac{n}{\eta_0}E_y \approx -\frac{kn}{\beta}H_x \Rightarrow H_x$连续,$H_z \approx \frac{-j}{\beta}\frac{\partial H_x}{\partial x} \Rightarrow \frac{\partial H_x}{\partial x}$连续,$\frac{n}{\eta_0}E_z \approx \frac{j}{kn}\frac{\partial H_x}{\partial y} \Rightarrow E_{z1} - E_{z3} \approx \frac{j\eta_0}{k}\frac{1}{n_1^2}\frac{\partial}{\partial y}(H_{x1} - H_{x3}) - \frac{j\eta_0}{n_3}\frac{n_1^2-n_3^2}{n_1^2}o(\delta)\frac{1}{kn_3}\frac{\partial H_{x3}}{\partial y}o(\delta) \Rightarrow H_x$连续(已有),在$y = h/2$,$C_1\cos(k_y\frac{h}{2} + \phi_y) = C_2e^{-jk_{y2}h/2}$,$-\frac{k_y}{n_1^2}C_1\sin(k_y\frac{h}{2} + \phi_y) = -\frac{jk_{y2}}{n_2^2}C_2e^{-jk_{y2}h/2}$,两式相除$\Rightarrow \tan(k_y\frac{h}{2} + \phi_y) = \frac{jk_{y2}n_1^2}{k_yn_2^2}$,由$k_{xj}^2 + k_{yj}^2 + \beta^2 = k^2n_j^2$,$j = 1,2$相减$\Rightarrow jk_{y2} = \sqrt{k^2(n_1^2 - n_2^2) - k_y^2}$,回代$\Rightarrow \tan(k_y\frac{h}{2} + \phi_y) = \frac{n_1^2\sqrt{k^2(n_1^2-n_2^2)-n_y^2}}{n_2^2k_y} \Rightarrow$特征方程$k_y\frac{h}{2} + \phi_y = q'\pi + \arctan\frac{n_1^2\sqrt{k^2(n_1^2-n_2^2)-n_y^2}}{n_2^2k_y}$,在$y = -\frac{h}{2}$同理有特征方程$k_y\frac{h}{2} - \phi_y = q''\pi + \arctan\frac{n_1^2\sqrt{k^2(n_1^2-n_4^2)-k_y^2}}{n_4^2k_y}$,两特征方程相加消$\phi_y \Rightarrow k_yh = q\pi + \arctan\frac{n_1^2\sqrt{k^2(n_1^2-n_2^2)-k_y^2}}{n_2^2k_y} + \arctan\frac{n_1^2\sqrt{k^2(n_1^2-n_4^2)}}{n_4^2k_y}$,同理在$x = \pm\frac{w}{2}$,$k_xw = p\pi + \arctan\frac{\sqrt{k^2(n_1^2-n_3^2)-k_x^2}}{k_x} + \arctan\frac{\sqrt{k^2(n_1^2-n_5^2)-k_x^2}}{k_x}$,其中$\beta^2 = n_1^2k^2 - k_x^2 - k_y^2$

归一化:不失一般性,$n_1 > n_5 > n_4 > n_2$,$n_5 > n_3$,对H,$V_H = kh\sqrt{n_1^2 - n_4^2}$,$a_H = \frac{n_4^2-n_2^2}{n_1^2-n_4^2}$,$b_H = \frac{\beta_H^2-k^2n_4^2}{k^2(n_1^2-n_4^2)} = \frac{N_H^2-n_4^2}{n_1^2-n_4^2}$,$c_H = \frac{n_4^2}{n_1^2}$,$d_H = c_H - a_H(1-c_H) = \frac{n_2^2}{n_1^2}$;对W,$V_W = kw\sqrt{n_1^2 - n_5^2}$,$a_w = \frac{n_5^2-n_3^2}{n_1^2-n_5^2}$,$b_W = \frac{\beta_W^2-k^2n_5^2}{k^2(n_1^2-n_5^2)}$

计算步骤:分别由H和W的$b-V$曲线得b_H,$b_W \Rightarrow \beta_H$,$\beta_W \Rightarrow k_y^2 = n_1^2k^2 - \beta_H^2$,$k_x^2 = n_1^2 - \beta_W^2 \Rightarrow \beta^2 = n_1k^2 - k_x^2 - k_y^2 - n_1k^2 = k^2(n_4^2 + n_5^2 - n_1^2) + b_Wk^2(n_1^2 - n_5^2) + b_Hk^2(n_1^2 - n_4^2)$,总传播常数$b_M = \frac{\beta^2-k^2n_5^2}{k^2(n_1^2-n_5^2)} = b_W + \frac{n_1^2-b_4^2}{n_1^2-b_5^2}(b_H - 1)$

有效折射率法:类似M法将3D波导拆解为平板波导I(纵向,安排I)/I'(横向,安排II)和II(横向)/II'(纵向),先解I/I'得有效折射率$n_{\mathrm{eff}}^{(\prime)}$(通常$n_{\mathrm{eff}} \neq n'_{\mathrm{eff}}$),将$n_{\mathrm{eff}}^{(\prime)}$作II/II'芯层折射率,得II/II'传播常数$\beta$作为总传播常数;解释:对弱导$E_y$模,$H_x = h_x(x,y)e^{-j\beta z}$,入波动方程$(\nabla^2 + k^2n^2)H_x = 0 \Rightarrow [\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2n^2 - \beta^2]h_x = 0$,分离变量$n_{\mathrm{ps}}^2 = n_x^2(x) + n_y^2(y)$,$h_x(x,y) = X(x)Y(y)$,回代$\Rightarrow \frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} + \frac{1}{Y}\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} + [k^2n_x^2(x) + k^2n_y^2(y) - \beta^2] = 0 \Rightarrow$**安排I**:$\frac{1}{Y}\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} + k^2n_y^2 = -\frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} - [k^2n_x^2(x) - \beta^2] \overset{\mathrm{def}}{=} (ke_{\mathrm{eff}})^2 \Rightarrow \frac{1}{Y}\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} + k^2[n_y(y)^2 - n_{\mathrm{eff}}^2] = 0$,$\frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} + k^2[n_x^2(x) + n_{\mathrm{eff}}^2] - \beta^2 = 0$,近似为赝3D波导$n_{\mathrm{ps}}^2 = n_1^2(\mathrm{R1}), n_2^2(\mathrm{R2}), n_3^2 + n_1^2 - n_{\mathrm{eff}}^2(\mathrm{R3}), n_4^2(\mathrm{R4}), n_5^2 + n_1^2 - n_{\mathrm{eff}}^2(\mathrm{R5})$,拆解为横向平板波导$n_y^2(y) = n_1^2(|y| \leq \frac{h}{2}), n_2(y > \frac{h}{2}), n_4(x < -\frac{h}{2})$和纵向平板波导$n_x^2(x) = 0(|x| \leq \frac{w}{2}), n_3^2 - n_{\mathrm{eff}}^2(x > \frac{w}{2}), n_5^2 - n_{\mathrm{eff}}^2(x < -\frac{w}{2})$,在$y = \pm\frac{h}{2}$,$Y$,$\frac{1}{n_y^2}\frac{\mathrm{d}Y}{\mathrm{d}y}$连续,$\Rightarrow kh\sqrt{n_1^2 - n_{\mathrm{eff}}^2} = q\pi + \arctan\frac{n_1^2}{n_2^2}\frac{\sqrt{n_{\mathrm{eff}}^2-n_2^2}}{\sqrt{n_1^2-n_{\mathrm{eff}}^2}} + \arctan\frac{n_1^2}{n_4^2}\frac{\sqrt{n_{\mathrm{eff}}^2-n_4^2}}{\sqrt{n_1^2-n_{\mathrm{eff}}^2}}$,同理在$x = \pm\frac{w}{2}$,$X$,$\frac{\mathrm{d}X}{\mathrm{d}x}$连续,$kw\sqrt{n_{\mathrm{eff}}^2 - N^2} = p\pi + \arctan\frac{\sqrt{N^2-n_3^2}}{\sqrt{n_{\mathrm{eff}}^2-N^2}} + \arctan\frac{\sqrt{N^2-n_5^2}}{\sqrt{n_{\mathrm{eff}}^2-N^2}}$,其中$N$-3D波导总有效折射率,总传播常数$\beta = kN$,或**安排II**,$\frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} + k^2n_x^2(x) = -\frac{1}{Y}\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} - [k^2n_y^2(y) - \beta^2] \overset{\mathrm{def}}{=} (kn'_{\mathrm{eff}})^2 \Rightarrow \frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} + k^2[n_x^2(x) - n_{\mathrm{eff}}'^2] = 0$,$\frac{1}{Y}\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} + k^2[n_y^2(y) + n_{\mathrm{eff}}'^2] - \beta^2 = 0$,近似为赝3D波导$n_{\mathrm{sp}}^2 = n_1(\mathrm{R1}), n_2^2 + n_1^2 - n_{\mathrm{eff}}'^2(\mathrm{R2}), n_3^2(\mathrm{R3}), n_3^2(\mathrm{R3}), n_4^2 + n_1^2 - n_{\mathrm{eff}}'^2(\mathrm{R4}), n_5^2(\mathrm{R5})$,拆解为纵向平板波导$n_x^2(x) = n_1^2(|x| \leq \frac{w}{2}), n_3^2(x > \frac{w}{2}), n_5^2(x < -\frac{w}{2})$和横向平板波导$n_y(y) = 0(|y| \leq \frac{h}{2}), n_2^2 - n_{\mathrm{eff}}'^2(y > \frac{h}{2}), n_4^2 - n_{\mathrm{eff}}'^2(y < -\frac{h}{2})$,同理$\Rightarrow kw\sqrt{n_1^2 - n_{\mathrm{eff}}'^2} = p\pi + \arctan\frac{\sqrt{n_{\mathrm{eff}}'^2-n_3^2}}{\sqrt{n_1^2-n_{\mathrm{eff}}'^2}} + \arctan\frac{\sqrt{n_{\mathrm{eff}}'^2-n_5^2}}{\sqrt{n_1^2-n_{\mathrm{eff}}'^2}}$,$kh\sqrt{n_{\mathrm{eff}}'^2 - N^2} = q\pi + \arctan\frac{n_{\mathrm{eff}'}^2}{n_2^2}\frac{\sqrt{N^2-n_2^2}}{\sqrt{n_{\mathrm{eff}}'^2-N^2}} + \arctan\frac{n_{\mathrm{eff}}'^2}{n_4^2}\frac{\sqrt{N^2-n_4^2}}{\sqrt{n_{\mathrm{eff}}'^2-N^2}}$

计算步骤:对安排I,波导I,由$b_I - V_I$曲线得可导因子b_I,$n_{\mathrm{eff}}^2 = n_4^2 + b_I(n_1^2 - n_2^2)$,对波导II,由$b_{II} - V_{II}$曲线得$b_{II}$,总有效折射率$N^2 = n_5^2 + b_{II}(n_{\mathrm{eff}}^2 - n_5^2) = n_5^2 + b_{II}[n_4^2 - n_5^2 + b_I(n_1^2 - n_4^2)]$,总可导因子$b_{KT} = \frac{N^2-n_5^2}{n_1^2-n_5^2} = b_{II} + \frac{n_1^2-n_4^2}{n_1^2-n_5^2}b_{II}(b_I - 1)$

折射率偏差$\Delta(n^2)$所致β^2偏差:$\delta(\beta^2) = \frac{k^2\iint|E(x,y,z)|^2\Delta[n^2(x,y)]\,\mathrm{d}x\,\mathrm{d}y}{\iint|E(x,y,z)|^2\,\mathrm{d}x\,\mathrm{d}y}$,设$n = n_2(\mathrm{R2345})$,对有效折射率法$\Delta(n^2) = n_1^2 - n_{\mathrm{eff}}^2(>0, \mathrm{R35}), n_2^2 - n_{\mathrm{eff}}^2(4\text{角}), 0(\text{其他})$,R35高估,4角低估折射率,R35处能量多于4角,故总体高估折射率,$\delta(\beta^2) > 0$;对M法,折射率等效为$n_{\mathrm{eq}}^2(x,y) = n'^2(x) + n''^2(y)$,其中$n'^2(x) = \frac{n_1^2}{2}(|x| \leq \frac{w}{2}), n_2^2 - n_1^2/2(x > \frac{w}{2}), n_2^2 - n_1^2/2(x < -\frac{w}{2})$,$y''^2(y) = n_1^2/2(|y| \leq \frac{h}{2}), n_2^2 - n_1^2/2(y > \frac{h}{2}), n_2^2 - n_1^2/2(y < -\frac{h}{2})$,$\Delta(n^2) = n_2^2 - n_1^2(<0, 4\text{角}), 0(\text{其他})$,4角低估折射率,$\delta(\beta^2) < 0$

耦合波理论:讨论波导间相互影响或扰动下的波导;**定向耦合器**:能量来回传递的两平行波导

方法1:视一波导为对另一波导的微扰,弱耦合下扰动小,可认为单个波导总模式为其两独立模的线性叠加,$\boldsymbol{E}(x,y,z) = a_1(z)\boldsymbol{e}_1(x,y)e^{-j\beta_1z} + a_2(z)\boldsymbol{e}_2(x,y)e^{-j\beta_2z}$;若仅有波导1,无2,对波导1,$\{\nabla_t^2 + k^2n^2[1 + \delta n_1(x,y)]^2 - \beta_1^2\}\boldsymbol{e}_1(x,y) = 0$,其中$n$-背景折射率,$\delta n_1(x,y)$-波导1折射率相对背景偏差比,弱导近似下,$\delta n_1(x,y)$,$(kn - \beta) \sim o(\delta) \Rightarrow k^2n^2[1 + \delta n_1(x,y)]^2 - \beta_1^2 = k^2n^2 + \cancel{k^2n^2\delta n_1^2(x,y)} + 2k^2n^2\delta n_1(x,y) - \beta_1^2 \approx$

$(kn+\beta_1)(kn-\beta_1)+2k^2n^2\delta n_1(x,y)\approx 2kn(kn-\beta_1)+2k^2n^2\delta n_1(x,y)\Rightarrow[\nabla_t^2+2k^2n^2\delta n_1(x,y)+2kn(kn-\beta_1)]\boldsymbol{e}_1(x,y)\approx 0$,同理若仅有波导2,$[\nabla_t^2+2k^2n^2\delta n_2(x,y)+2kn(kn-\beta_2)]\boldsymbol{e}_2(x,y)\approx 0$,理论上用边条解两式即得模场,归一化输入场强$\iint_{波导i截面}|\boldsymbol{e}_1(x,y)|^2\,dS=1\forall i=1,2$下,$e_2(x,y)\cdot$前式$-e_1(x,y)\cdot$后式,积分$\Rightarrow \iint[e_2(x,y)\nabla_t^2e_1(x,y)-e_1(x,y)\nabla_t^2e_2(x,y)]\,dS=-2k^2n^2\iint[\delta n_1(x,y)-\delta n_2(x,y)]e_1(x,y)e_2(x,y)\,dS+2kn(\beta_1-\beta_2)\iint e_1(x,y)e_2(x,y)\,dS$,由格林第二定理,式左x分量$=\iint[e_{2x}(x,y)\nabla_t^2e_{1x}(x,y)-e_{1x}(x,y)\nabla_t^2e_{2x}(x,y)]\,dS=\oint_{\mathcal{C}}[e_{2x}(x,y)\nabla_te_{1x}(x,y)-e_{1x}(x,y)\nabla_te_{2x}(x,y)]\hat{n}\,dl$与$\mathcal{C}$具体路径无关,将$\mathcal{C}$拉至无穷远$\Rightarrow=0\Rightarrow$式左$=0\Rightarrow \cancel{2kn}(\beta_1-\beta_2)\iint e_1(x,y)e_2(x,y)\,dS=\cancel{2k^2n^2}\iint[\delta n_1(x,y)-\delta n_2(x,y)]e_1(x,y)e_2(x,y)\,dS\Rightarrow C(\beta_1-\beta_2)=\kappa_1-\kappa_2$(**Marcatili关系**),其中**交叠积分**$C=\iint e_1(x,y)e_2(x,y)\,dS$,**耦合系数**$\kappa_i=kn\iint\delta n_i(x,y)e_1(x,y)e_2(x,y)\,dS$,下标$_i$-耦到波导$i$;若两波导相同,$\beta_1=\beta_2\Rightarrow\kappa_1=\kappa_2$,若波导1小于2,或有$\beta_{1,低阶}\approx\beta_{2,高阶}\Rightarrow\kappa_1\approx\kappa_2$,若两波导相距很远,$C\approx0\Rightarrow\kappa_1=\kappa_2$

方法2:视为复合波导,用麦氏方程解复合模式,最低两阶分别为对称模和反对称模,$\boldsymbol{E}(x,y,z)=e_{s0}\boldsymbol{e}_s(x,y)e^{-j\beta_sz}+a_{a0}\boldsymbol{e}_a(x,y)e^{-j\beta_az}$;对复合模,$\{\nabla_t^2+2k^2n^2[\delta n_1(x,y)+\delta n_2(x,y)]+2kn(kn-\beta)\}=0$,$e(x,y)\cdot$波导1之式$-e_1(x,y)\cdot$上式$\Rightarrow\iint[e(x,y)\nabla_t^2e_1(x,y)-e_1(x,y)\nabla_t^2e(x,y)]\,dS=2k^2n^2\iint\delta n_2(x,y)e(x,y)e_1(x,y)\,dS+2kn(\beta-\beta_1)\iint e(x,y)e_1(x,y)\,dS$,同理格林第二定理$\Rightarrow kn\iint\delta n_2(x,y)e(x,y)e_1(x,y)\,dS=(\beta-\beta_1)\iint e(x,y)e_1(x,y)\,dS$,同理用$e_2$替$e_1\Rightarrow kn\iint\delta n_1(x,y)e(x,y)e_2(x,y)\,dS=(\beta-\beta_2)\iint e(x,y)e_2(x,y)\,dS$,弱耦合下,视复合模为两独立模叠加,$e(x,y)=e_1(x,y)+re_2(x,y)$,回代$\Rightarrow kn\iint\delta n_1(x,y)e_1(x,y)e_2(x,y)\,dS+knr\iint\delta n_1(x,y)e_2^2(x,y)\,dS=(\beta-\beta_2)[\iint e_1(x,y)e_2(x,y)\,dS+r\iint e_2^2(x,y)\,dS]\Rightarrow\kappa_1+r\rho_1=(C+r)(\beta-\beta_2)$,同理$\rho_2+r\kappa_2=(1+rC)(\beta-\beta_1)$,其中**自耦合系数**$\rho_i=kn\iint\delta_i(x,y)e_{3-i}^2(x,y)\,dS$,两式联立$\Rightarrow\frac{\kappa_1+r\rho_1}{C+r}-\frac{\rho_2+r\kappa_2}{1+rC}=\beta_1-\beta_2$(**Marcatili关系**);已知波导结构,即有$\kappa_1,\kappa_2,\rho_1,\rho_2,C$,需算$\beta_1,\beta_2,r$;弱耦合下,交叠很小,$C\ll1$,自耦合$\ll$互耦合,$\rho_i\ll\kappa_i\Rightarrow\frac{\kappa_1+r\rho_1}{r}-(\rho_2+\kappa_2r)\approx\beta_1-\beta_2\Rightarrow\kappa_2r^2+(\beta_1-\beta_2)r-\kappa_1+\cancel{r(\rho_2-\rho_1)}\approx0\Rightarrow r_{s,a}=\frac{1}{\kappa_2}[-(\beta_1-\beta_2)\pm\sqrt{(\beta_1-\beta_2)^2+4\kappa_1\kappa_2}]$,设$\delta=\frac{\Delta\beta}{2}=\frac{\beta_1-\beta_2}{2}$,**失谐常数**$d=\frac{\delta}{\sqrt{\kappa_1\kappa_2}}\Rightarrow\kappa_1-\kappa_2=C\Delta\beta=2Cd\sqrt{\kappa_1\kappa_2}\Rightarrow2Cd=\sqrt{\frac{\kappa_1}{\kappa_2}}-\sqrt{\frac{\kappa_2}{\kappa_1}}\Rightarrow\frac{\kappa_1}{\kappa_2}=[Cd+\sqrt{1+(Cd)^2}]^2\Rightarrow$**对称/反对称模**$r_{s,a}=\frac{2\sqrt{\kappa_1\kappa_2}}{2\kappa_2}[-\frac{\Delta\beta}{2\sqrt{\kappa_1\kappa_2}}\pm\sqrt{1+(\frac{\Delta\beta}{2\sqrt{\kappa_1\kappa_2}})^2}]=\sqrt{\frac{\kappa_1}{\kappa_2}}(-d\pm\sqrt{1+d^2})=[Cd+\sqrt{1+(Cd)^2}](-d\pm\sqrt{1+d^2})$,**(/反)对称模的传播常数**$\beta_{s,a}\approx\frac{\beta_1+\beta_2}{2}\pm\sqrt{\kappa_1\kappa_2(1+d^2)}=\frac{\beta_1+\beta_2}{2}\pm\sigma$,其中$\sigma=\sqrt{\kappa_1\kappa_2+\delta^2}$;弱耦合下对称与反对称模正交,$\iint[e_1(x,y)+r_se_2(x,y)][e_1(x,y)+r_ae_2(x,y)]\,dS=1+r_sr_a+(r_s+r_a)C=1-\frac{\kappa_1}{\kappa_2}-C\frac{\beta_1-\beta_2}{2}=1-\frac{\kappa_1}{\kappa_2}-\frac{\kappa_1-\kappa_2}{\kappa_2}=2(1-\frac{\kappa_1}{\kappa_2})\approx0$;若$\kappa_1=\kappa_2\Rightarrow r_{s,a}=\pm1\Rightarrow e(x,y)=e_1(x,y)\pm e(x,y),\beta_{s,a}=\beta_1\pm\kappa_1$

耦合波方程(CME):$\boldsymbol{E}=a_{s0}[e_1(x,y)+r_se_2(x,y)]e^{-j\beta_sz}+a_{a0}[e_1(x,y)+r_ae_2(x,y)]e^{-j\beta_az}=(a_{s0}e^{-j\beta_sz}+a_{a0}e^{-j\beta_az})e_1(x,y)+(a_{s0}r_se^{-j\beta_sz}+a_{a0}r_ae^{-j\beta_az})e_2(x,y)=a_1(z)e_1(x,y)e^{-j\beta_1z}+a_2(z)e_2(x,y)e^{-j\beta_2z}$,其中$a_1(z)=(a_{s0}e^{-j\sigma z}+a_{a0}e^{j\sigma z})e^{j\delta z}$,$a_2(z)=(a_{s0}r_se^{-j\sigma z}+a_{a0}r_ae^{j\sigma z})e^{-j\delta z}\Rightarrow a_{s0}e^{-j\sigma z}=\frac{r_aa_1(z)e^{-j\delta z}-a_2(z)e^{j\delta z}}{r_a-r_s}$,$a_{a0}e^{j\sigma z}=\frac{r_sa_1(z)e^{-j\delta z}-a_2(z)e^{j\delta z}}{r_s-r_a}$,传输方向上各分量变化速率:$\frac{da_1}{dz}=j\delta a_1(z)+j\sigma(a_{a0}e^{j\sigma z}-a_{s0}e^{-j\sigma z})e^{j\delta z}=j\delta a_1(z)+j\sigma\frac{(r_s+r_a)a_1(z)e^{-j\delta z}-2a_2(z)e^{j\delta z}}{r_s-r_a}e^{j\delta z}$,$\because r_s-r_a=\frac{2\sigma}{\kappa_2}$,$\delta+\sigma\frac{r_s+r_a}{r_s-r_a}=\delta+\sigma\frac{-2\delta/\kappa_2}{2\sigma/\kappa_2}=0$,$\therefore\frac{da_1}{dz}=-j\kappa_2a_2(z)e^{j2\delta z}$,同理$\frac{da_2}{dz}=-j\kappa_1a_1(z)e^{-j2\delta z}$(CME),总能量变化速率:$\frac{d}{dz}(|a_1(z)|^2+|a_2(z)|^2)=\frac{d}{dz}[a_1(z)a_1^*(z)+a_2(z)a_2^*(z)]=-j\kappa_2a_2(z)e^{j2\delta z}a_1^*(z)+a_1(z)[j\kappa_2^*a_2^*(z)e^{-j2\delta z}]-j\kappa_1a_1(z)e^{-j2\delta z}a_2^*(z)+a_2(z)[j\kappa_1^*a_1^*(z)e^{j2\delta z}]=j(\kappa_1^*-\kappa_2)a_1^*(z)a_2(z)e^{j2\delta z}-j(\kappa_1-\kappa_2^*)a_1(z)a_2^*(z)e^{-j2\delta z}$;若$\kappa_1=\kappa_2^*$,$\frac{d}{dz}(|a_1(z)|^2+|a_2(z)|^2)=0$,能量在两波导间来回交换但总量守恒;对$A_1(z)=a_1(z)e^{-j\beta_1z},A_2(z)=a_2(z)e^{-j\beta_2z}$有$\frac{dA_1}{dz}=-j\beta A_1(z)+\frac{da_1}{dz}e^{-j\beta_1z}=-j\beta A_1(z)-j\kappa_2a_2(z)e^{j2\delta z}e^{-j\beta_2z}=-j\beta_1A_1(z)-j\kappa_2A_2(z)$,同理$\frac{dA_2}{dz}=-j\beta_2A_2(z)-j\kappa_1A_1(z)$,即$\frac{d}{dz}\begin{bmatrix}A_1\\A_2\end{bmatrix}=-j\begin{bmatrix}\beta_1&\kappa_2\\\kappa_1&\beta_2\end{bmatrix}\begin{bmatrix}A_1(z)\\A_2(z)\end{bmatrix}$(CME)

传输矩阵法:两波导仅在$0<z<L$处平行耦合,对$R(z)=a_1(z)e^{-j\delta z},S(z)=a_2(z)e^{j\delta z}$有$|R(z)|=|a_1(z)|,|S(z)|=|a_2(z)|$,$\frac{dR}{dz}=-j\delta R(z)-j\kappa_2S(z)$,$\frac{dS}{dz}=j\delta S(z)-j\kappa_1R(z)(CME)\Rightarrow\frac{d^2R}{dz^2}=-j\delta\frac{dR}{dz}-j\kappa_2\frac{dS}{dz}=-j\delta[-j\delta R(z)-j\kappa_2S(z)]-j\kappa_2[j\delta S(z)-j\kappa_1R(z)]\Rightarrow\frac{d^2R}{dz^2}+(\kappa_1\kappa_2+\delta^2)R(z)=\frac{d^2R}{dz^2}+\sigma^2R(z)=0$,同理$\frac{d^2S}{dz^2}+\sigma^2S(z)=0$,有通解$R(z)=C_1\cos\sigma z+C_2\sin\sigma z$,$S(z)=\frac{j}{\kappa_2}[(\sigma C_2+j\delta C_1)\cos\sigma z+(j\delta C_2-\sigma C_1)\sin\sigma z]$,边条$\Rightarrow C_1=R(0),C_2=\frac{R(L)-R(0)\cos\sigma L}{\sin\sigma L}\Rightarrow\begin{bmatrix}R(z)\\S(z)\end{bmatrix}=\begin{bmatrix}\cos\sigma z-j\frac{\delta}{\sigma}\sin\sigma z&-j\frac{\kappa_2}{\sigma}\sin\sigma z\\-j\frac{\kappa_1}{\sigma}\sin\sigma z&\cos\sigma z+j\frac{\delta}{\sigma}\sin\sigma z\end{bmatrix}\begin{bmatrix}R(0)\\S(0)\end{bmatrix}$,其中$2\times2$矩阵-传输矩阵;若$\kappa_1=\kappa_2=\sqrt{\kappa_1\kappa_2}\equiv\kappa$且仅由波导1输入,$R(0)=1,S(0)=0,R(z)=\cos\sigma z-j\frac{\delta}{\sigma}\sin\sigma z,S(z)=-j\frac{\kappa}{\sigma}\sin\sigma z$,$|a_2(z)|^2_{\max}=|S(z)|^2_{\max}=\frac{\kappa^2}{\sigma^2}=\frac{\kappa^2}{\kappa^2+\delta^2}=\frac{1}{1+\delta^2/\kappa^2}$,$|a_1(z)|^2=\cos^2\sigma z+\frac{\delta^2}{\sigma^2}\sin^2\sigma z=1-\frac{\delta^2+\sigma^2}{\sigma^2}\sin^2\sigma z=1-\frac{\kappa^2}{\sigma^2}\sin^2\sigma z$,$|a_1(z)|^2_{\min}=1-\frac{\kappa^2}{\sigma^2}=\frac{\delta^2}{\kappa^2+\delta^2}=\frac{1}{1+\kappa^2/\delta^2}$,$|a_1(z)|^2+|a_2(z)|^2=|S(z)|^2+|R(z)|^2=1$,**耦合长度**$l_c=\frac{\pi}{2\sigma}$,每经$2l_c$,能量交换一来回,若$\delta^2/\kappa^2\uparrow$,失谐越严重,$|a_2(z)|^2_{\max}\downarrow$,$|a_1(z)|^2_{\min}\uparrow$,交换越频繁

3dB耦合器:将一波导的能量平分至两相同波导,$\beta_1=\beta_2$,长$L=(m+\frac12)l_c$,输入$R(0)=1,S(0)=0$,输出$\begin{bmatrix}R(L)\\S(L)\end{bmatrix}=\frac{1}{\sqrt2}\begin{bmatrix}1&-j\\-j&1\end{bmatrix}\begin{bmatrix}R(0)\\S(0)\end{bmatrix}=\frac{1}{\sqrt2}\begin{bmatrix}1\\-j\end{bmatrix}$,$|S(z)|^2=|R(z)|^2=\frac12$

光开关(路由):输入$R(0)=1,S(0)=1$,用热光效应/非线性效应(Pockel效应:$n\sim E$,Kerr效应:$n\sim E^2$)调节$n_f\Rightarrow\beta$以控制输出;**bar态**:输出$R(L)=1,S(0)=0\Rightarrow\sigma L=m\pi\Rightarrow(\frac{L}{\pi})^2(\kappa^2+\delta^2)=m^2$,对应$\frac{\delta L}{\pi}-\frac{\kappa L}{\pi}$图中$\frac14$圆弧;**cross态**:输出$S(L)=0,R(L)=1\Rightarrow\frac{\kappa}{\sigma}=1,\sigma z=\frac{\pi}{2}(2m+1)\Rightarrow(\frac{L}{\pi})^2(\kappa^2+\delta^2)=(2m+1)^2/4,\delta=0\Rightarrow\frac{\kappa L}{\pi}=m+\frac12$,对应$\frac{\kappa L}{\pi}$轴上离散点,工程难实现;改进-**交换$\Delta\beta$耦合器**:长$L/2$,传播常数$\beta_1=\beta+\delta$和$\beta_2=\beta-\delta$的耦合器接同长度,传播常数$\beta_2,\beta_1$的耦合器,前一段传输矩阵$M_1^+\approx\begin{bmatrix}\mathcal{A}_1&-j\mathcal{B}_1\\-j\mathcal{B}_1^*&\mathcal{A}_1^*\end{bmatrix}$,第二段传输矩阵$M_1^-\approx\begin{bmatrix}\mathcal{A}_1^*&-j\mathcal{B}_1\\-j\mathcal{B}_1^*&\mathcal{A}_1\end{bmatrix}$,其中$\mathcal{A}_1=\cos\frac{\sigma L}{2}-j\frac{\delta}{\sigma}\sin\frac{\sigma L}{2}$,$\mathcal{B}_1=\frac{\kappa}{\sigma}\sin\frac{\sigma L}{2}$,总传输矩阵$M_2=M_1^-M_1^+=\begin{bmatrix}\mathcal{A}_2&-j\mathcal{B}_2\\-j\mathcal{B}_2^*&\mathcal{A}_2^*\end{bmatrix}$,其中$\mathcal{A}_2=|\mathcal{A}_1|^2-|\mathcal{B}_1|^2=1-2|\mathcal{B}_1|^2=2|\mathcal{A}_1|^2-1$,$\mathcal{B}_2=2\mathcal{A}_1^*\mathcal{B}_1$;**bar态**:$\mathcal{B}_2=0\Rightarrow\mathcal{A}_1=0\Rightarrow\frac{\sigma L}{2}=\frac{\pi}{2}(2m+1),\delta=0$,工程难实现或$\mathcal{B}_1=0\Rightarrow(\frac{L}{\pi})^2(\delta^2+\kappa^2)=(2m)^2$对应$\frac{\delta L}{\pi}-\frac{\kappa L}{\pi}$图中$\frac14$圆弧;**cross态**:$\mathcal{A}_2=0\Rightarrow\frac{\kappa^2}{\kappa^2+\delta^2}\sin^2\sqrt{\kappa^2+\delta^2}\frac{L}{2}=\frac12$

滤波器:波导1输入,波导2滤出$|a_2(L)|^2=|S(L)|^2=\kappa_1^2L^2(\frac{\sin\sqrt{\kappa^2+\delta^2}L}{\sqrt{\kappa^2+\delta^2}L})^2=\frac{\kappa_1/\kappa_2}{1+(\frac{\delta}{\kappa})^2}\sin^2\sqrt{1+(\frac{\delta}{\kappa})^2}\kappa L$;若$\lambda\uparrow$,能量发散,或两波导靠近,则交叠增强,$\kappa_i\uparrow,l_c\downarrow$;若$\beta_1=\beta_2$,$|a_2(L)|^2=\sin^2\kappa L$;中心波长$\lambda_0$满足$\kappa(\lambda_0)L=(m+\frac12)\pi$,半高波长$\lambda_{1,2}$满足$\kappa(\lambda_1)L=(m+\frac34)\pi$,$\kappa(\lambda_2)L=(m+\frac14)\pi$,$m=0,1,\cdots$,设$\kappa(\lambda)\approx\kappa(\lambda_0)+\frac{d\kappa}{d\lambda}\big|_{\lambda=\lambda_0}(\lambda-\lambda_0)\Rightarrow$带宽:半高宽$\Delta\lambda\equiv\lambda_1-\lambda_2=2(\lambda_1-\lambda_0)\approx\frac{\pi/2}{L\frac{d\kappa}{d\lambda}}$,设$\kappa(\lambda_0)\approx K\lambda_0\Rightarrow\Delta\lambda=\frac{\lambda_0}{2m+1}$,$m\uparrow$,相互作用距离$L\uparrow$,带宽$\Delta\lambda\downarrow$;缺点:带宽不够窄,主,旁瓣等高;改进:波导1折射率大$(\Delta n_1>\Delta n_2)$,波导2尺寸(h,W)大,对$\lambda=\lambda_0,\beta_1=\beta_2\Rightarrow\delta=0,L=(2m+1)l_c\Rightarrow|a_2(L)|^2=\frac{\kappa_1}{\kappa_2}\approx1$,对其他$\lambda,\delta\neq0$,$|a_2(L)|^2$较小,半功率点$\delta_{\mathrm{HP}m}=q_m\sqrt{\kappa_1\kappa_2}$,其中$q_0=\pm0.798,q_1=\pm0.538,q_2=\pm0.429$,$\delta(\lambda)=\frac{\beta_2(\lambda)-\beta_1(\lambda)}{2}=\frac{\pi}{\lambda}[N_2(\lambda)-N_1(\lambda)]\approx\cancel{\delta(\lambda_0)}+\frac{d\delta}{d\lambda}\big|_{\lambda=\lambda_0}(\lambda-\lambda_0)=\frac{\pi}{\lambda}(\frac{dN_2}{d\lambda}-\frac{dN_1}{d\lambda})_{\lambda=\lambda_0}(\lambda-\lambda_0)\Rightarrow$**半功率波长**$\frac{\lambda_{\mathrm{HP}m}-\lambda_0}{\lambda_0}\approx\frac{q_m\sqrt{\kappa_1\kappa_2}}{\pi(\frac{dN_2}{d\lambda}-\frac{dN_1}{d\lambda})_{\lambda=\lambda_0}}\approx\frac{q_m(m+\frac12)}{L(\frac{dN_2}{d\lambda}-\frac{dN_1}{d\lambda})_{\lambda=\lambda_0}}$,$\frac{\Delta\lambda}{\lambda_0}=2\frac{\lambda_{\mathrm{HP}m}-\lambda_0}{\lambda_0}\approx\frac{q_m(2m+1)}{L(\frac{dN_2}{d\lambda}-\frac{dN_1}{d\lambda})_{\lambda=\lambda_0}}$,通常$\frac{\Delta\lambda}{\lambda_0}$可达0.02;改进-**锥形定向耦合滤波器**:两波导间距随位置变化,$g=g(z)\Rightarrow\kappa=\kappa(\lambda,g(z))$,$\beta_i,\delta$无影响,边条:$R(-\frac{L}{2})=1,R(-\frac{L}{2})=0$,设$\rho(z)=-j\frac{S(z)}{R(z)}\Rightarrow|S(z)|^2=\frac{|\rho(z)|^2}{1+|\rho(z)|^2}$,$\frac{d\rho}{dz}=-j\frac{1}{R^2(z)}[\frac{dS}{dz}R(z)-S(z)\frac{dR}{dz}]=-j\frac{1}{R(z)}[j\delta R(z)-j\kappa_1R(z)]+j\frac{S(z)}{R^2(z)}[-j\delta R(z)-j\kappa_2S(z)]=\delta\frac{S(z)}{R(z)}-\kappa_1+\delta\frac{S(z)}{R(z)}+\kappa_2\frac{S^2(z)}{R^2(z)}=j2\delta\rho(z)=[\kappa_1(z)+\kappa_2(z)\rho^2(z)]$;若$\delta=0,\kappa_1(z)=\kappa_2(z)$,则$\frac{1}{1+\rho^2(z)}\frac{d\rho}{dz}=-\kappa_1(z)\Rightarrow\rho(z)=-\tan[\int_{-L/2}^z\kappa_1(z')\,dz']\Rightarrow|S(L/2)|^2=\sin^2[\int_{-L/2}^{L/2}\kappa_1(z')\,dz']$,旁瓣进一步压缩

传输矩阵法:$\frac{dA}{dz}=-jQA(z)$,其中传输矩阵$Q=\begin{bmatrix}\beta_1&\kappa_2\\\kappa_1&\beta_2\end{bmatrix}$的特征值$\beta_{s,a}=\frac12[\beta_1+\beta_2\pm\sqrt{\Delta\beta^2+4\kappa_1\kappa_2}]$,特征矢$V_s=\begin{bmatrix}V_{s1}\\V_{s2}\end{bmatrix}$,$V_a=\begin{bmatrix}V_{a1}\\V_{a2}\end{bmatrix}$,设$V=\begin{bmatrix}V_s&V_a\end{bmatrix}=\begin{bmatrix}V_{s1}&V_{a1}\\V_{s2}&V_{a2}\end{bmatrix}$,$\Lambda=\begin{bmatrix}\beta_s&0\\0&\beta_a\end{bmatrix}=V^{-1}QV$,$u(z)=V^{-1}A(z)$,代入$\Rightarrow\frac{d[Vu]}{dz}=-jQVu\Rightarrow\frac{du}{dz}=-jV^{-1}QVu=-j\Lambda u\Rightarrow u(z)=\begin{bmatrix}u_1(0)e^{-j\beta_sz}\\u_2(0)e^{-j\beta_az}\end{bmatrix}$,其中$u(0)=\begin{bmatrix}a_{s0}\\a_{a0}\end{bmatrix}$,$A(z)=Vu(z)=\begin{bmatrix}V_{s1}a_{s0}e^{-j\beta_sz}+V_{a1}a_{a0}e^{-j\beta_az}\\V_{s2}a_{s0}e^{-j\beta_sz}+V_{a2}a_{a0}e^{-j\beta_az}\end{bmatrix}$;若$\beta_1=\beta_2,\beta_s=\beta_1+\kappa,\beta_a=\beta_1-\kappa,V_S=\begin{bmatrix}1\\1\end{bmatrix},V_a=\begin{bmatrix}1\\-1\end{bmatrix}$,$A(z)=\begin{bmatrix}a_{s0}e^{-j\beta_sz}+a_{a0}e^{-j\beta_az}\\a_{s0}e^{-j\beta_sz}-a_{a0}e^{-j\beta_az}\end{bmatrix}$;对同平面平行三波导,$A(z)=\begin{bmatrix}A_1(z)\\A_2(z)\\A_3(z)\end{bmatrix}$,$Q=\begin{bmatrix}\beta_1&\kappa_{12}&\kappa_{13}\\\kappa_{21}&\beta_2&\kappa_{23}\\\kappa_{31}&\kappa_{32}&\beta_3\end{bmatrix}$,其中下标$_{ij}$-波导$j$耦至$i$,若三波导相同$\beta_1=\beta_2=\beta_3\equiv\beta$,仅考虑近邻耦合,忽略次近邻耦合,$\kappa_{12}=\kappa_{21}=\kappa_{23}=\kappa_{32}\equiv\kappa,\kappa_{13}=\kappa_{31}=0$,则$Q=\begin{bmatrix}\beta&\kappa&0\\\kappa&\beta&\kappa\\0&\kappa&\beta\end{bmatrix}$的特征值:$\beta,\beta\pm\sqrt2\kappa$,特征矢:$\frac{1}{\sqrt2}\begin{bmatrix}1\\\sqrt2\\1\end{bmatrix}$,$\frac{1}{\sqrt2}\begin{bmatrix}-1\\\sqrt2\\-1\end{bmatrix}$,$\frac{1}{\sqrt2}\begin{bmatrix}\sqrt2\\0\\-\sqrt2\end{bmatrix}$,$V=\frac{1}{\sqrt2}\begin{bmatrix}1&-1&\sqrt2\\\sqrt2&\sqrt2&0\\1&-1&-\sqrt2\end{bmatrix}$,$V^{-1}=\frac{1}{2\sqrt2}\begin{bmatrix}1&\sqrt2&1\\-1&\sqrt2&-1\\\sqrt2&0&-\sqrt2\end{bmatrix}$,$u(z)=\begin{bmatrix}u_1(0)e^{-j(\beta+\sqrt2\kappa)z}\\u_2(0)e^{-j(\beta-\sqrt2\kappa)z}\\u_3(0)e^{-j\beta z}\end{bmatrix}$,若$A(0)=\begin{bmatrix}0\\1\\0\end{bmatrix}$,$u(0)=\frac12\begin{bmatrix}1\\1\\0\end{bmatrix}$,$A(z)=\begin{bmatrix}-\frac{j}{\sqrt2}\sin\sqrt2\kappa z\\\cos\sqrt2\kappa z\\-\frac{j}{\sqrt2}\sin\sqrt2\kappa z\end{bmatrix}e^{-j\beta z}$,当$\sqrt2\kappa z=(m+\frac12)$,$A_1,A_3$分到能量极大

TE模在介质界面上的反/折射:$(\epsilon,\mu)(z)=(\epsilon_1,\mu_1)(z<0),(\epsilon_2,\mu_2)(z>0)$,入射$(\boldsymbol{E}_1,\boldsymbol{k}_1)$由$zx$平面第三象限向原点$O$,与$z$轴夹角$\theta_1$,反射$(\boldsymbol{E}_1',\boldsymbol{k}_1')O\to$二象限,折射$(\boldsymbol{E}_2,\boldsymbol{k}_2)O\to$一象限,与$z$夹角$\theta_2$,反入射$(\boldsymbol{E}_2',\boldsymbol{k}_2')$四象限$\to O$,与$z$夹角$\pi-\theta_2$,电场$\boldsymbol{E}=\begin{cases}(E_1e^{-j\boldsymbol{k}_1\cdot\boldsymbol{r}}+E_1'e^{-j\boldsymbol{k}_1'\cdot\boldsymbol{r}})e^{i\omega t},&z<0\\(E_2e^{-j\boldsymbol{k}_2\cdot\boldsymbol{r}}+E_2'e^{-j\boldsymbol{k}_2'\cdot\boldsymbol{r}})e^{j\omega t},&z>0\end{cases}$,其中$\boldsymbol{r}=(x,0,z)$,在$x=0$有$E_1e^{-jk_{1x}x}+E_1'e^{-jk_{1x}'x}=E_2e^{-jk_{2x}x}+E_2'e^{-jk_{2x}'x}\forall x\Rightarrow k_{1x}=k_{1x}'=k_{2x}=k_{2x}'=k_x,E_1+E_1'=E_2+E_2'$,①$\Rightarrow\boldsymbol{H}=\frac{\nabla\times\boldsymbol{E}}{-j\omega\mu}=\frac{(-j\boldsymbol{k})\times\boldsymbol{E}}{-j\omega\mu}=\frac{\boldsymbol{k}\times\boldsymbol{E}}{\omega\mu}=\begin{cases}\frac{\boldsymbol{k}_1\times\hat{y}E_1+\boldsymbol{k}_1'\times\hat{y}E_1'}{\omega\mu},&z=0^-\\\frac{\boldsymbol{k}_2\times\hat{y}E_2+\boldsymbol{k}_2'\times\hat{y}E_2'}{\omega\mu},&z=0^+\end{cases}$,其中$\boldsymbol{k}_{1/2}\times\hat{y}=-k_{1/2z}\hat{x}+k_{1/2x}\hat{z}$,$k_{1/2z}'=-k_{1/2z}\Rightarrow H_x=\begin{cases}-\frac{k_{1z}(E_1-E_1')}{\omega\mu_1},&z=0^-\\-\frac{k_{2z}(E_2-E_2')}{\omega\mu_2},&z=0^+\end{cases}\Rightarrow\frac{k_{1z}}{\mu_1}(E_1-E_1')=\frac{k_{2z}}{\mu_2}(E_2-E_2')\Rightarrow\begin{pmatrix}1&1\\\frac{k_{1z}}{\mu_1}&-\frac{k_{1z}}{\mu_1}\end{pmatrix}\begin{pmatrix}E_1\\E_1'\end{pmatrix}=\begin{pmatrix}1&1\\\frac{k_{2z}}{\mu_2}&-\frac{k_{2z}}{\mu_2}\end{pmatrix}\begin{pmatrix}E_2\\E_2'\end{pmatrix}$,其中$\frac{k_{1/2z}}{\mu_{1/2}}=\frac{k_{1/2}\cos\theta_{1/2}}{\mu_{1/2}}=\frac{k_0\sqrt{\mu_{1/2}\epsilon_{1/2}}\cos\theta_{1/2}}{\mu_{1/2}}=k_0\sqrt{\frac{\epsilon_{1/2}}{\mu_{1/2}}}\cos\theta_{1/2}\Rightarrow\begin{pmatrix}1&1\\\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_1&-\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_1\end{pmatrix}\begin{pmatrix}E_1\\E_1'\end{pmatrix}=\begin{pmatrix}1&1\\\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_2&-\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_2\end{pmatrix}\begin{pmatrix}E_2\\E_2'\end{pmatrix}$,**反射系数**$r_{12}=\frac{E_1'}{E_1},r_{21}=\frac{E_2}{E_2'}$,**透射系数**$t_{12}=\frac{E_2}{E_1},t_{21}=\frac{E_1'}{E_2'}$,其中下标$_{m/n}$-$m$入$n$,线性系统中光路可逆性$\Rightarrow E_1=r_{12}E_1'+t_{21}E_2,E_2'=t_{12}E_1'+r_{21}E_2\Rightarrow E_1=r_{12}^2E_1+t_{12}t_{21}E_1$,菲涅尔公式$\Rightarrow r_{12}=-r_{21}\Rightarrow r_{12}^2+t_{12}t_{21}=1$,若$E_2'=0$,在$z=0$有$E_1+E_1'=E_2\Rightarrow E_1+r_{12}E_1=t_{12}E_1\Rightarrow1+r_{12}=t_{12}$,入上矩阵式$\Rightarrow\frac{k_{1z}}{\mu_1}(1-r_{12})=\frac{k_{12}}{\mu_2}t_{12}=\frac{k_{2z}}{\mu_2}(1+r_{12})\Rightarrow r_{12}=\frac{\mu_2k_{1z}-\mu_1k_{2z}}{\mu_2k_{1z}+\mu_1k_{2z}}$,$t_{12}=1+r_{12}=\frac{2\mu_2k_{1z}}{\mu_2k_{1z}+\mu_1k_{2z}}$,若$\mu_1=\mu_2,r_{12}=\frac{k_{1z}-k_{2z}}{k_{1z}+k_{2z}},t_{12}=\frac{2k_{1z}}{k_{1z}+k_{2z}}$

3层介质膜中TE模的传播:$(\epsilon,\mu,n)(z)=(\epsilon_1,\mu_1,n_1)(z<0),(\epsilon_2,\mu_2,n_2)(0<z<d),(\epsilon_3,\mu_3,n_3)(z>d)$,入射$E_i(x,z)=Ae^{-j\boldsymbol{k}_i\cdot\boldsymbol{r}}=Ae^{-j(k_{1x}x+k_{1z}z)}(z<0)$与$z$夹角$\theta_1$,反射$E_r(x,z)=Be^{-j\boldsymbol{k}_r\cdot\boldsymbol{r}}=Be^{-j(k_{1x}x-k_{1z}z)}(z<0)$,透射$E_t(x,z)=Fe^{-j\boldsymbol{k}_t\cdot(\boldsymbol{r}-\boldsymbol{d})}=Fe^{-j[k_{3x}(x-d)+k_{3z}z]}(z>d)$与$z$夹角$\theta_3$,中间层右传$Ce^{-j(k_{2x}x+k_{2z}z)}(0<z<d)$与$z$夹角$\theta_2$,左传$De^{-j(k_{2x}x-k_{2z}z)}(0<z<d)$,边界条件$\Rightarrow k_{1x}=k_{2x}=k_{3x}=k_x,k_{iz}=\sqrt{k_0^2n_i^2-k_x^2}$,电场$E(x,z)=\begin{cases}(Ae^{-jk_{1z}z}+Be^{jk_{1z}z})e^{-jk_xx},&z<0\\(Ce^{-jk_{2z}z}+De^{jk_{2z}z})e^{-jk_xx},&0<z<d\\Fe^{-jk_{3z}(z-d)}e^{-jk_xx},&z>d\end{cases}$,设$\mu_1=\mu_2=\mu_3=\mu$,$H_x=\begin{cases}\frac{k_{1z}}{\omega\mu}(Ae^{-jk_{1z}z}-Be^{jk_{1z}z})e^{-jk_xx},&z<0\\\frac{k_{2z}}{\omega\mu}(Ce^{-jk_{2z}z}-De^{jk_{2z}z})e^{-jk_xx},&0<z<d\\\frac{k_{3z}}{\omega\mu}Fe^{-jk_{3z}(z-d)}e^{-jk_xx},&z>d\end{cases}$,边界条件$\Rightarrow A+B=C+$

D,$k_{1z}(A-B)=k_{2z}(C-D)$,$Ce^{-jk_{2z}d}+De^{jk_{2z}d}=F$,$k_{2z}(Ce^{-jk_{2z}d}-De^{jk_{2z}d})=k_{3z}F\Rightarrow F=A\frac{4k_{1z}k_{2z}e^{-jk_{2z}d}}{(k_{1z}+k_{2z})(k_{2z}+k_{3z})+(k_{1z}-k_{2z})(k_{2z}-k_{3z})e^{-j2k_{2z}d}}$,$B=A\frac{(k_{1z}-k_{2z})(k_{2z}+k_{3z})+(k_{1z}+k_{3z})(k_{2z}-k_{3z})e^{-jk_{2z}d}}{(k_{1z}+k_{2z})(k_{2z}+k_{3z})+(k_{1z}-k_{2z})(k_{2z}-k_{3z})e^{-j2k_{2z}d}}$,$C=\frac{1}{2}F(1+\frac{k_{3z}}{k_{2z}})e^{jk_{2z}d}$,$D=\frac{1}{2}(1-\frac{k_{3z}}{k_{2z}})e^{-jk_{2z}d}$,$k_{iz}=\frac{\omega}{c}n_i\cos\theta_i$,$r_{12}=\frac{k_{1z}-k_{2z}}{k_{1z}+k_{2z}}$,$r_{23}=\frac{k_{2z}-k_{3z}}{k_{2z}+k_{3z}}$,$t_{12}=\frac{2k_{1z}}{k_{1z}+k_{2z}}$,$t_{23}=\frac{2k_{2z}}{k_{2z}+k_{3z}}$,**总透射系数**$t=\frac{F}{A}=\frac{t_{12}t_{23}e^{-j\phi}}{1+r_{12}r_{23}e^{-j2\phi}}$,**总反射系数**$r=\frac{B}{A}=\frac{r_{12}+r_{23}e^{-j2\phi}}{1+r_{12}r_{23}e^{-j2\phi}}$,其中$\phi=k_{2z}d=\frac{2\pi}{\lambda}n_2d\cos\theta_2$;**方法2**:入射$\sim Ae^{-jk_{1z}z}$,反射$\sim rAe^{jk_{1z}z}$,透射$\sim tAe^{-jk_{3z}(z-d)}$,中间层右传$\sim Ce^{-jk_{2z}z}$,左传$\sim De^{jk_{2z}z}$,其中$C=t_{12}A+r_{12}D$,$rA=r_{12}A+t_{21}D$,$tA=r_{23}Ce^{-jk_{2z}d}$,$De^{jk_{2z}d}=r_{23}Ce^{-jk_{2z}d}\Rightarrow r=r_{12}+\frac{t_{12}t_{21}r_{23}e^{-j2\phi}}{1-r_{21}r_{23}e^{-j2\phi}}$,$t=\frac{t_{12}t_{23}e^{-j\phi}}{1-r_{21}r_{23}e^{-j2\phi}}$,$C=\frac{t_{12}A}{1-r_{21}r_{23}e^{-j2\phi}}$,$D=r_{23}e^{-j2\phi}C$;**方法3**(TE/M均适用):$r=t_{12}+\sum_{m=0}^{\infty}t_{12}r_{23}t_{21}e^{-j2\phi}(r_{21}r_{23}e^{-j2\phi})^m=r_{12}+\frac{t_{12}r_{23}t_{21}e^{-j2\phi}}{1-r_{21}r_{23}e^{-j2\phi}}$,由$r_{12}=-r_{21}$,$t_{12}t_{21}-r_{12}r_{21}=1\Rightarrow r=\frac{r_{12}+r_{23}e^{-j2\phi}}{1+r_{12}r_{23}e^{-j2\phi}}$,同理$t=t_{12}t_{23}e^{-j\phi}\sum_{m=0}^{\infty}(r_{23}r_{21}e^{-j2\phi})^m=\frac{t_{12}t_{23}e^{-j\phi}}{1-r_{23}r_{21}e^{-j2\phi}}$,$r(\phi+\pi)=r(\phi)$,$t(\phi+2\pi)=t(\phi)$,$r(0)=r(\pi)=r_{13}$,$t(0)=-t(\pi)=t_{13}$;**总反射率**$R=|r|^2$,**总透射率**$T=\frac{P_{3z}}{P_{1z}}=\frac{n_3\cos\theta_3}{n_1\cos\theta_1}|t|^2$,若$n_1=n_3$,$T=|t|^2$,**总吸收率**(若有)$A=1-R-T$;**隧穿效应**:若$n_1>n_2$,$d\to 0$且$\theta_1>\theta_c=\arcsin\frac{n_2}{n_1}$即$n_1k_0\sin\theta_1>n_2k_0$,$k_{2z}=\sqrt{k_0^2n_2^2-k_x^2}=\sqrt{k_0^2n_2^2-k_0^2n_1^2\sin^2\theta_1}=j|k_{2z}|$,$k_{3z}=\sqrt{n_3k_0^2-k_x^2}$,当$n_3>n_1\sin\theta_1\Rightarrow k_3=k_0n_3>k_0n_1\sin\theta_1=k_x$,$k_{3z}$为实数,光场可传至$z>d$;**增透膜**:对⊥入射,$r_{12}=\frac{k_{1z}-k_{2z}}{k_{1z}+k_{2z}}=\frac{n_1-n_2}{n_1+n_2}$,$r_{23}=\frac{n_2-n_3}{n_2+n_3}$,要$r=0$,则$r_{12}+r_{23}e^{-j2\phi}=\frac{n_1-n_2}{n_1+n_2}+\frac{n_2-n_3}{n_2+n_3}e^{-j2k_0n_2d}=0$,令$e^{-j2k_0n_2d}=-1$即$2k_0n_2d=\frac{4\pi}{\lambda}n_2d=\pi$,此时$d_{\min}=\frac{\lambda}{4n_2}\Rightarrow\frac{n_1-n_2}{n_1+n_2}=\frac{n_2-n_3}{n_2+n_3}\Rightarrow n_2=\sqrt{n_1n_3}$,

多层介质膜中TE模的传播:由$z=0$入射等厚不等折射率多层介质膜,在第i个界面($z=(i-1)d$)左边左传$\sim A_i$,右传$\sim B_i$,右边左传$\sim A'_{i+1}$,右传$\sim B'_{i+1}$,$A'_{i+1}=t_{i,i+1}A_i+r_{i+1,i}B'_{i+1}$,$B_i=r_{i,i+1}A_i+t_{i+1,i}B'_{i+1}\Rightarrow\begin{pmatrix}1&-r_{i+1,i}\\0&t_{i+1,i}\end{pmatrix}\begin{pmatrix}A'_{i+1}\\B'_{i+1}\end{pmatrix}=\begin{pmatrix}t_{i,i+1}&0\\-r_{i,i+1}&1\end{pmatrix}\begin{pmatrix}A_i\\B_i\end{pmatrix}\Rightarrow\begin{pmatrix}A_i\\B_i\end{pmatrix}=\begin{pmatrix}t_{i,i+1}&0\\-r_{i,i+1}&1\end{pmatrix}^{-1}\begin{pmatrix}1&-r_{i+1,i}\\0&t_{i+1,i}\end{pmatrix}\begin{pmatrix}A'_{i+1}\\B'_{i+1}\end{pmatrix}$,或对TE/M,$D_{\mathrm{s/p},i}\begin{pmatrix}A_i\\B_i\end{pmatrix}=D_{\mathrm{s/p},i+1}\begin{pmatrix}A'_{i+1}\\B'_{i+1}\end{pmatrix}\Rightarrow\begin{pmatrix}A_i\\B_i\end{pmatrix}=D_{\mathrm{s/p},i}^{-1}D_{\mathrm{s/p},i+1}\begin{pmatrix}A'_{i+1}\\B'_{i+1}\end{pmatrix}$,其中$D_{\mathrm{s},i}=\begin{pmatrix}1&1\\\sqrt{\frac{\epsilon_i}{\mu_i}}\cos\theta_i&-\sqrt{\frac{\epsilon_i}{\mu_i}}\cos\theta_i\end{pmatrix}$,$D_{\mathrm{p},i}=\begin{pmatrix}\cos\theta_i&\cos\theta_i\\\sqrt{\frac{\epsilon_i}{\mu_i}}&-\sqrt{\frac{\epsilon_i}{\mu_i}}\end{pmatrix}$,第$i$层介质($(i-1)d<z<id$)中,$A_i=A'_ie^{-jk_{2z}d}$,$B_i=B'_ie^{jk_{2z}d}\Rightarrow\begin{pmatrix}A'_i\\B'_i\end{pmatrix}=P_i\begin{pmatrix}A_i\\B_i\end{pmatrix}$,其中$P_i=\begin{pmatrix}e^{jk_{iz}d}&0\\0&e^{-jk_{iz}d}\end{pmatrix}$,若无损,$|P_i|=1$,$\begin{pmatrix}A_1\\B_1\end{pmatrix}=D_1^{-1}(D_2P_2D_2^{-1})\cdots(D_nP_nD_n^{-1})D_{n+1}\begin{pmatrix}A'_{n+1}\\B'_{n+1}\end{pmatrix}=D_1^{-1}(\prod_{i=2}^{n}D_iP_iD_i^{-1})D_{n+1}\begin{pmatrix}A'_{n+1}\\B'_{n+1}\end{pmatrix}=M\begin{pmatrix}A'_{n+1}\\B'_{n+1}\end{pmatrix}$,其中传输矩阵$M=\begin{pmatrix}M_{11}&M_{12}\\M_{21}&M_{22}\end{pmatrix}$,$\because$单向输入,$B'_{n+1}=0\Rightarrow A_1=M_{11}A'_{n+1}$,$B_1=M_{21}A'_{n+1}$;**总反射系数**$r=\frac{B_1}{A_1}=\frac{M_{21}}{M_{11}}$,**总透射系数**$t=\frac{A'_{n+1}}{A_1}=\frac{1}{M_{11}}$,**总反射率**$R=|r|^2$,**总透射率**$T=\frac{n_{n+1}\cos\theta_{n+1}}{n_1\cos\theta_1}|t|^2$;若$k_{iz}d_i=m\pi$,$m\in\mathbb{N}\forall i$,$P_i=\pm\begin{pmatrix}1&0\\0&1\end{pmatrix}$,$D_iP_iD_i^{-1}=\pm\begin{pmatrix}1&0\\0&1\end{pmatrix}$,$\begin{pmatrix}A_1\\B_1\end{pmatrix}=\pm D_1^{-1}D_{n+1}\begin{pmatrix}A'_{n+1}\\B'_{n+1}\end{pmatrix}$,若$k_{iz}d_i=(2m+1)\frac{\pi}{2}\forall i$,$P_i=\pm\begin{pmatrix}j&0\\0&-j\end{pmatrix}$

1D光子晶体:入射区折射率n_0,出射区n_s,其间以厚为a,b,折射率为n_1,n_2的介质膜(元胞,厚$\Lambda=a+b$)周期性排列n层,$\begin{pmatrix}M_{11}&M_{12}\\M_{21}&M_{22}\end{pmatrix}=D_0^{-1}(D_1P_1D_1^{-1}D_2P_2D_2)^nD_s$,$P_1=\begin{pmatrix}e^{jk_{1z}a}&0\\0&e^{-jk_{1z}a}\end{pmatrix}$,$P_2=\begin{pmatrix}e^{jk_{2z}b}&0\\0&e^{-jk_{2z}b}\end{pmatrix}$,亥姆霍兹方程通解$E_K(x,z)=E_K(z)e^{-jk_xx}e^{-jKz}$,其中$K$-布洛赫波数,$\because n(z+\Lambda)=n(z)$,$\because n(z+\Lambda)=n(z)$,$E_K(z+\Lambda)=E_K(z)$,$E_K(x,z+\Lambda)=E_K(z+\Lambda)e^{-jk_xx}e^{-jK(z+\Lambda)}=E_K(x,z)e^{-jK\Lambda}$,第$i$个元胞$n_2$中右传$\sim a_i$,左传$\sim b_i$,$n_1$中左传$c_i$,右传$d_i$,$\begin{pmatrix}a_{i-1}\\b_{i-1}\end{pmatrix}=e^{jK\Lambda}\begin{pmatrix}a_n\\b_n\end{pmatrix}=\begin{pmatrix}A&B\\C&D\end{pmatrix}\begin{pmatrix}a_n\\b_n\end{pmatrix}$,其中$e^{jK\Lambda}$为单个元胞传输矩阵$\begin{pmatrix}A&B\\C&D\end{pmatrix}$的本征值$\Rightarrow\begin{vmatrix}e^{jK\Lambda}-A&-B\\-C&e^{jK\Lambda}-D\end{vmatrix}=e^{j2K\Lambda}-(A+D)e^{jK\Lambda}+AD-BC=0\Rightarrow e^{jK\Lambda}=\frac{(A+D)\pm\sqrt{(A+D)^2-4(AD-BC)}}{2}$,若无损,$\begin{vmatrix}A&B\\C&D\end{vmatrix}=1\Rightarrow e^{jK\Lambda}=\frac{1}{2}(A+D)\pm\sqrt{[\frac{1}{2}(A+D)]^2-1}$,本征矢$\begin{pmatrix}a_0\\b_0\end{pmatrix}=\begin{pmatrix}B\\e^{jK\Lambda}-A\end{pmatrix}$,$2\cos K\Lambda=e^{jK\Lambda}+e^{-jK\Lambda}=A+D\Rightarrow K(k_{1x},\omega)=\frac{1}{\Lambda}\arccos\frac{A+D}{2}$,其中对TE,$A=e^{jk_{1z}a}[\cos(k_{2z}b)+\frac{j}{2}(\frac{k_{2z}}{k_{1z}}+\frac{k_{1z}}{k_{2z}})\sin(k_{2z}b)]$,$D=e^{-jk_{1z}a}[\cos(k_{2z}b)-\frac{j}{2}(\frac{k_{2z}}{k_{1z}}+\frac{k_{1z}}{k_{2z}})\sin(k_{2z}b)]$,$E_K(z)e^{-jKz}=(a_0e^{-jk_{1z}(z-n\Lambda)}+b_0e^{jk_{1z}(z-n\Lambda)})e^{jK(z-n\Lambda)}e^{-jKz}$,对TM,$A=e^{jk_{1z}a}[\cos(k_{2z}b)+\frac{j}{2}(\frac{n_2^2k_{1z}}{n_1^2k_{2z}}+\frac{n_1^2k_{2z}}{n_2^2k_{1z}})\sin(k_{2z}b)]$,$D=e^{-jk_{1z}a}[\cos(k_{2z}b)-\frac{j}{2}(\frac{n_1^2k_{2z}}{n_2^2k_{1z}})\sin(k_{2z}b)]$,$k_{iz}=\sqrt{n_i^2k_0^2-k_x^2}$;若$|\frac{A+D}{2}|<1$,$K$为实数,光可持续传输(导带),若$\frac{A+D}{2}>1$,$K$含虚数,光迅速衰减,不可持续传输(禁带);若$\Lambda<\frac{\lambda}{2n_{\mathrm{eff}}}$,可视为单轴均匀介质,对TE,$\cos(K\Lambda)=\frac{1}{2}[(e^{jk_{1z}a}+e^{-jk_{2z}a})\cos(k_{2z}b)+\frac{j}{2}(\frac{k_{2z}}{k_{1z}}+\frac{k_{1z}}{k_{2z}})\sin(k_{2z}b)(e^{jk_{1z}a}-e^{-jk_{1z}a})]=\cos(k_{1z}a)\cos(k_{2z}b)-\frac{1}{2}(\frac{k_{2z}}{k_{1z}}+\frac{k_{1z}}{k_{2z}})\sin(k_{2z}b)\sin(k_{2z}a)$,一阶近似($k_{1z}a\ll 1$,$k_{2z}b\ll 1$,$K\Lambda\ll 1$)$\Rightarrow 1-\frac{1}{2}(K\Lambda)^2=[1-\frac{1}{2}(k_{1z}a)^2][1-\frac{1}{2}(k_{2z}b)^2]-\frac{1}{2}(\frac{k_{2z}}{k_{1z}}+\frac{k_{1z}}{k_{2z}})(k_{2z}b)(k_{1z}a)\Rightarrow K^2\Lambda^2=k_{2z}^2b^2+k_{1z}a^2-\cancel{\frac{1}{2}k_{1z}k_{2z}a^2b^2}+k_{1z}^2ab+k_{1z}^2ab+k_{2z}^2ab\Rightarrow K^2=\frac{1}{\Lambda^2}(a+b)(k_{1z}^2a+k_{2z}^2b)=\frac{1}{\Lambda}(k_{1z}^2a+k_{2z}^2b)=\frac{1}{\Lambda}\{[n_1^2(\frac{\omega}{c})^2-k_x^2]a+[n_2^2(\frac{\omega}{c})^2-k_x^2]b\}=\frac{1}{\Lambda}(\frac{\omega}{c})^2(n_1^2a+n_2^2b)-\frac{k_x^2}{\Lambda}(a+b)\Rightarrow\Lambda(K^2+k_x^2)=(\frac{\omega}{c})^2\Rightarrow(\frac{K}{n_0})+(\frac{k_x}{n_0})^2=(\frac{\omega}{c})^2$,其中$n_0^2=\frac{a}{\Lambda}n_1^2+\frac{b}{\Lambda}n_2^2$,$\epsilon_0=f\epsilon_1+(1-f)\epsilon_2$,$n_1$占空比$f=\frac{a}{\Lambda}$,$\boldsymbol{E}$恒⊥ z,对TM,$1-\frac{1}{2}(K\Lambda)^2=[1-(\frac{1}{2}k_{1z}a)^2][1-(\frac{1}{2}k_{2z}b)^2]-\frac{1}{2}(\frac{n_2^2}{n_1^2}\frac{k_{1z}}{k_{2z}}+\frac{n_1^2}{n_2^2}\frac{k_{2z}}{k_{1z}})(k_{1z}a)(k_{2z}b)\Rightarrow K^2\Lambda^2\approx k_{1z}^2a^2+k_{2z}^2b^2+(\frac{n_2}{n_1})^2ab+(\frac{n_1}{n_2})^2ab=[(\frac{n_1}{n_2})^2a+b][(\frac{n_2}{n_1})^2k_{1z}^2a+k_{2z}^2b]=[(\frac{n_1}{n_2})^2a+b]\{(\frac{n_2}{n_1})^2[(\frac{n_1\omega}{c})^2-k_x^2]a+[(\frac{n_2\omega}{c})^2-k_x^2]b\}\Rightarrow\frac{K^2\Lambda^2}{(\frac{n_1}{n_2})^2a+b}+k_x^2[(\frac{n_2}{n_1})^2a+b]=(\frac{n_2\omega}{c})^2(a+b)\Rightarrow\frac{K^2\Lambda^2}{(n_1^2a+n_2^2b)(a+b)}+\frac{k_x^2[(\frac{n_2}{n_1})^2a+b]}{n_2^2(a+b)}=(\frac{\omega}{c})^2\Rightarrow\frac{K^2}{n_o^2}+\frac{k_x^2}{n_e^2}=(\frac{\omega}{c})^2$,其中$n_o=\frac{1}{\Lambda}(n_1^2a+n_2^2b)$,$n_e^{-2}=\frac{1}{\Lambda}(n_1^{-2}a+n_2^{-2}b)$,$\boldsymbol{E}$有⊥和∥ z分量