随机模拟方法与应用导论作业九

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12.5 (Waiting until a hit, continued).

In Exercise 12.4, we saw that the posterior density for Ian Kinsler's hitting probability is a beta density with shape parameters $a = n + 44$ and $b = s + 102$. (The values of n and s are found from the data in Exercise 12.3.)

a. Using the `rbeta` function, simulate 1000 values from the posterior density of p.
b. Use the `hist` function on the simulated sample to display the posterior density.
c. Using the simulated draws, approximate the mean and standard deviation of the posterior density.
d. Using the simulated draws, construct a 95% Bayesian interval estimate. Compare the interval with the exact 95% interval estimate using the `qbeta` function.

a. 根据题12.4，p的后验密度为

$$g(p|\text{data}) \propto p^{n+44-1}(1-p)^{s+102-1}$$

其中n为样本容量，$s = \sum y$是间隔之和。

根据题12.3，样本为

```
0 2 0 4 1 0 2 0 1 0 0 1 1 3 1 0 0 0 1
6 0 9 0 4 1 9 1 0 3 4 5 5 1 0 2 4 0 4
0 3 2 1 0 1 3 7 0 3 1 2 14 4 0 1 6 1 10
1 2 0 1 0 4 5 0 7 3 1 2 1 2 1 2 2 4 3
3 1 1 2 1 2 7 0 3 1 2 2 2 2 0 3 4 1 1
0 0 1 1 1 11 2 2 1 3 1 0 1 2 1 1 1 0 0
2 0 10 1 2 2 1 1 3 1 1 0 0 1 0 1 0 1 1
0 1 0 0 0 2 1 4 5 5 0 0 0 0 2 0 8 5 2
11 8 0 7 1 3 1
```

故n和s分别为

```
y = c(0,2,0,4,1,0,2,0,1,0,0,1,1,3,1,0,0,0,1,
      6,0,9,0,4,1,9,1,0,3,4,5,5,1,0,2,4,0,4,
      0,3,2,1,0,1,3,7,0,3,1,2,14,4,0,1,6,1,10,
      1,2,0,1,0,4,5,0,7,3,1,2,1,2,1,2,2,4,3,
      3,1,1,2,1,2,7,0,3,1,2,2,2,2,0,3,4,1,1,
      0,0,1,1,1,11,2,2,1,3,1,0,1,2,1,1,1,0,0,
```

```
      2,0,10,1,2,2,1,1,3,1,1,0,0,1,0,1,0,1,1,
      0,1,0,0,0,2,1,4,5,5,0,0,0,0,2,0,8,5,2,
      11,8,0,7,1,3,1)
n = length(y)
n
```

```
## [1] 159
```

```
s = sum(y)
s
```

```
## [1] 336
```

从而p的后验密度为

$$g(p|\text{data}) \propto p^{202}(1-p)^{432}$$

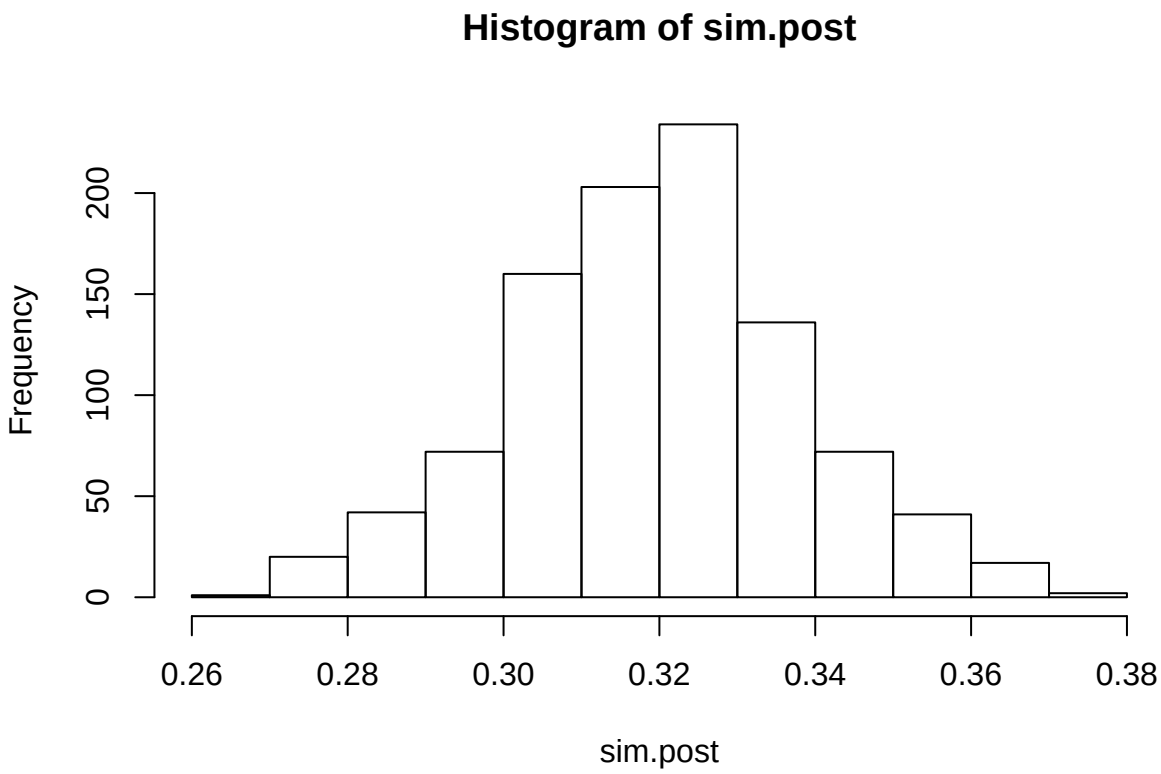
因此，后验密度是一个参数$a = 203$，参数$b = 433$的beta分布的密度函数。

用函数`rbeta`随机产生该密度函数对应的1000个`p`。

```
sim.post = rbeta(1000,203,433)
```

b. 用函数`hist`绘制所得随机数的直方图。

```
hist(sim.post)
```



c. 用所得随机数估算后验密度的平均值和标准差。

```
mean(sim.post)
```

```
## [1] 0.3198943
```

```
sd(sim.post)
```

```
## [1] 0.01857025
```

d. 用所得随机数构建95%贝叶斯区间估计，其上下界分别为

```
quantile(sim.post,c(0.025,0.975))
```

```
##       2.5%      97.5%
## 0.2827188 0.3569525
```

用函数qbeta同样可以构建95%贝叶斯区间估计，其上下界分别为

```
qbeta(c(0.025,0.975),203,433)
```

```
## [1] 0.2835356 0.3559063
```

可以看出，通过两种方法构建的区间相近。