第七次作业（量子计算第三章）

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第 1 题 得分: _________. 证明两种图态定义的等价性.

证: 要证两种图态定义的等价性, 即证显式定义的图态是隐式定义中算符 K_p $(p \in V)$ 的本征值为 1 的本征态. 显示定义的图态的表达式为

$$|G\rangle = S_C|+\rangle_1 \cdots |+\rangle_n = \prod_{\{a,b\}\in E} CZ_{ab}|+\rangle_1 \cdots |+\rangle_n. \tag{1}$$

隐式定义中算符

$$K_p = \sigma_p^x \prod_{b\in N_p} \sigma_b^z. \tag{2}$$

将算符 K_p 作用于图态 $|G\rangle$ 上有

$$K_p|G\rangle = K_pS_C|+\rangle_1 \cdots |+\rangle_n = K_p \prod_{\{a,b\}} CZ_{ab}|+\rangle_1 \cdots |+\rangle_n. \tag{3}$$

由于 CZ_{ab} 和 $CZ_{\mu\nu}$ 之间是可对易的且 $CZ_{ab} = |0\rangle_a\langle 0| \otimes I_b + |1\rangle_a\langle 1| \otimes \sigma_b^z \Longrightarrow S_C^2 = I$, 从而

$$K_p|G\rangle = IK_p|G\rangle = S_CS_CK_p \prod_{\{a,b\}\in E} CZ_{ab}|+\rangle_1 \cdots |+\rangle_n = S_C \prod_{\{\mu,\nu\}} CZ_{\mu\nu}K_p \prod_{\{a,b\}\in E} CZ_{ab}|+\rangle_1 \cdots |+\rangle_n. \tag{4}$$

利用算符 CZ_{ab} 与 Pauli 算符之间的对易关系:

$$CZ_{ab}\sigma_a^x CZ_{ab} = \sigma_a^x \otimes \sigma_b^z, \tag{5}$$
$$CZ_{ab}\sigma_b^x CZ_{ab} = \sigma_a^z \otimes \sigma_b^x, \tag{6}$$
$$CZ_{ab}\sigma_c^x CZ_{ab} = \sigma_c^x \quad (c \neq a, b), \tag{7}$$
$$CZ_{ab}\sigma_i^c CZ_{ab} = \sigma_i^z \quad \forall i, \tag{8}$$

无妨将 p 视为某个控制比特, 则有 $CZ_{pb}\sigma_p^x CZ_{pb} = \sigma_p^x \otimes \sigma_b^z$, 得到 σ_b^z 与 K_p 中的 σ_b^z 相消, 最终得到

$$K_p|G\rangle = S_C\sigma_p^x|+\rangle_1 \cdots |+\rangle_n = S_C|+\rangle_1 \cdots |+\rangle_n = |G\rangle, \tag{9}$$

故显式定义的图态 $|G\rangle$ 是隐式定义中算符 K_p $(p \in V)$ 的本征值为 1 的本征态, 显式定义和隐式定义等价. □

第 2 题 得分: _________. 证明 11 页中 LC 变换后图态的形式.

证: 要证 $U_a^\tau|G\rangle = e^{-i\frac{\pi}{4}\sigma_a^x}\prod_{k\in N_a} e^{i\frac{\pi}{4}\sigma_k^z}|G\rangle$ 是图 $\tau_a(G)$ 的图态, 即证 $U_a^\tau|G\rangle = e^{-i\frac{\pi}{4}\sigma_a^x}\prod_{k\in N_z} e^{i\frac{\pi}{4}\sigma_k^z}|G\rangle$ 是图 $\tau_a(G)$ 对应的稳定子群 S 的本征值为 1 的共同本征态. 设图 G 中点 p 对应的稳定子为 K_p. 简记 U_a^τ 为 U_a. 由于

$$U_a|G\rangle = U_aK_p|G\rangle = U_aK_pU_a^\dagger U_a|G\rangle = U_aK_pU_a^\dagger|\tau_a(G)\rangle, \tag{10}$$

因此只需证 $U_aK_pU_a^\dagger$ 是图 $\tau_a(G)$ 的稳定子.

(1) 当 $p = a$ 时, 有

$$U_aK_aU_a^\dagger = \left(e^{-i\frac{\pi}{4}\sigma_a^x}\prod_{k\in N_a} e^{i\frac{\pi}{4}\sigma_k^z}\right)\left(\sigma_a^x\prod_{j\in N_a}\sigma_j^z\right)\left(e^{-i\frac{\pi}{4}\sigma_a^x}\prod_{l\in N_a} e^{i\frac{\pi}{4}\sigma_l^z}\right)^\dagger. \tag{11}$$

利用 $e^{-i\frac{\pi}{4}\sigma_a^x} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}e^{-i\frac{\pi}{4}} & 0\\ 0 & e^{i\frac{\pi}{4}}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\sqrt{-i} & 0\\ 0 & \sqrt{i}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \sqrt{-i\sigma_a^x}$, $e^{i\frac{\pi}{4}\sigma_k^z} = \begin{pmatrix}e^{i\frac{\pi}{4}} & 0\\ 0 & e^{-i\frac{\pi}{4}}\end{pmatrix} = \begin{pmatrix}\sqrt{i} & 0\\ 0 & \sqrt{-i}\end{pmatrix} = \sqrt{i\sigma_k^z}$ 得

$$U_aK_aU_a^\dagger = \sqrt{-i\sigma_a^x}\sigma_a^x\sqrt{i\sigma_a^x}\prod_{b\in N_a}\sqrt{i\sigma_b^z}\sigma_b^z\sqrt{-i\sigma_b^z} = \sigma_a^x\prod_{b\in N_a}\sigma_b^z = K_a, \tag{12}$$

局域操作 τ_a 不改变顶点 a 与 N_a 之间的连接关系, 故 $U_aK_aU_a^\dagger$ 是图 $\tau_a(G)$ 的稳定子.

(2) 当 $p\in N_a$ 时, 算符 K_p 与 U_a 的支集 $\sup(K_a)$ 和 $\sup(U_a)$ 的交集为

$$\sup(K_p)\cap\sup(U_a)=(p\cup N_p)\cap(a\cup N_a)=p\cup a\cup(N_p\cap N_a),\tag{13}$$

且图 $\tau_a(G)$ 在 p 处的稳定子为

$$K_p'=K_p\prod_{b\in N_a-p}\sigma_b^z=\sigma_p^x\prod_{b\in N_p}\sigma_b^z\prod_{b\in N_a-p}\sigma_b^z=\sigma_p^x\prod_{b\in N_p\cup(N_a-p)-N_p\cap N_a}\sigma_b^z,\tag{14}$$

因此

$$\begin{aligned}
U_aK_pU_a^\dagger=&\left(\sqrt{-i\sigma_a^x}\prod_{k\in N_a}\sqrt{i\sigma_k^z}\right)\left(\sigma_p^x\prod_{j\in N_p}\sigma_j^z\right)\left(\sqrt{i\sigma_a^x}\prod_{l\in N_a}\sqrt{-i\sigma_l^z}\right)\\
=&\sqrt{-i\sigma_a^x}\sigma_a^z\sqrt{i\sigma_a^x}\sqrt{i\sigma_p^z}\sigma_p^z\sqrt{-i\sigma_p^z}\prod_{b\in N_p\cap N_a}\sqrt{i\sigma_b^z}\sigma_b^z\sqrt{-i\sigma_b^z}\prod_{c\in N_p-(N_p\cap N_a)-a}\sigma_c^z\\
=&(-\sigma_p^y)(-\sigma_a^y)\prod_{b\in N_p-a}\sigma_b^z\\
=&\sigma_p^y\sigma_a^y\sigma_a^z\prod_{b\in N_p}\sigma_b^z\\
=&\sigma_p^y\sigma_p^z\sigma_a^y\sigma_a^z\prod_{b\in N_a}\sigma_b^z\prod_{c\in N_p\cup(N_a-p)-N_p\cap N_a}\sigma_c^z\\
=&\sigma_a^x\prod_{b\in N_a}\sigma_b^z\sigma_p^x\prod_{c\in N_p\cup(N_a-p)-N_p\cap N_a}\sigma_c^z\\
=&K_aK_p',
\end{aligned}$$

从而

$$U_aK_pU_a^\dagger|\tau_a(G)\rangle=K_aK_p'|\tau_a(G)\rangle=K_a|\tau_a(G)\rangle,\tag{15}$$

由于局域操作 τ_a 不改变顶点 a 与 N_a 之间的连接关系, 故 K_a 是图 $\tau_a(G)$ 的稳定子,

$$U_aK_pU_a^\dagger|\tau_a(G)\rangle=|\tau_a(G)\rangle,\tag{16}$$

进而 $U_aK_pU_a^\dagger$ 是图 $\tau_a(G)$ 的稳定子.

(3) 当 $p\neq a$ 且 $p\notin N_a$ 时, K_p 与 U_a 的支集 $\sup(K_p)$ 与 $\sup(U_a)$ 之间的交集为 $N_a\cap N_p$, 图 $\tau_a(G)$ 对应的稳定子为 $K_p'=K_p$, 从而

$$\begin{aligned}
U_aK_pU_a^\dagger=&\left(\sqrt{-i\sigma_a^x}\prod_{k\in N_a}\sqrt{i\sigma_k^z}\right)\left(\sigma_p^x\prod_{j\in N_p}\sigma_j^z\right)\left(\sqrt{i\sigma_a^x}\prod_{l\in N_a}\sqrt{-i\sigma_l^z}\right)\\
=&\prod_{b\in N_a\cap N_p}\sqrt{i\sigma_b^z}\sigma_p^x\prod_{c\in N_p}\sigma_c^z\prod_{d\in N_a\cap N_p}\sqrt{-i\sigma_d^z}\\
=&\sigma_p^x\prod_{c\in N_p}\sigma_c^z\\
=&K_p=K_p',
\end{aligned}\tag{17}$$

进而

$$U_aK_pU_a^\dagger|\tau_a(G)\rangle=K_p'|\tau_a(G)\rangle=|\tau_a(G)\rangle,\tag{18}$$

故 $U_aK_pU_a^\dagger$ 是图 $\tau_a(G)$ 的稳定子.

综上, $|\tau_a(G)\rangle$ 是图 $\tau_a(G)$ 的稳定子群的本征值为 1 的共同本征态, 故量子态 $|\tau_a(G)\rangle$ 是图 $\tau_a(G)$ 对应的图态. □

第 3 题 得分: ________. 推导第 12 页中的 P_x 测量的结果.

证: 由于 $|G\rangle$ 为 G 对应的图态, 故 $\langle G|K_p|G\rangle = \langle G|G\rangle = 1$, $(p \in V)$.

稳定子算符集合 $S_a = \{K_p \mid p \in V - a\}$ 可分为两部分:

(1) S_a^1 中算符的支集不含顶点 a;

(2) S_a^2 中算符的支集含顶点 a.

若 $a \in S_a^1$, 即 K_p 的支集中不包含 a, 则 K_p 与 a 上的测量操作 $P_a^{z,\pm} = \frac{1}{2}(I \pm \sigma_a^z)$ 可对易, 因此测量操作 P_a 不影响 K_p 的测量值, 即 $\langle G_a|K_p|G_a\rangle = \langle G|K_p|G\rangle = 1$, 其中 $|G_a\rangle$ 表示测量操作 P_a 后 (去除测量比特 a) 系统的量子态.

若 $p \in S_a^2$, 即 K_p 的支集中包含 a, $p \in N_a$, 即 K_p 作用在 a 上的算符为 σ_a^z. 此时,

- 若 σ_a^z 的测量结果为 1, 则

$$\langle G_a|K_p'|G_a\rangle = \langle G|K_p'\sigma_a^z|G\rangle = \langle G|K_p|G\rangle = 1, \tag{19}$$

- 若 σ_a^z 的测量结果为 -1, 则

$$\langle G_a|K_p'|G_a\rangle = -1. \tag{20}$$

由此, 测量后的量子态 $|G_a\rangle$ 是稳定算符 S_1 和 S_2' (由 K_p' 组成) 的共同本征态:

$$\begin{aligned}
\langle G_a|K_p'|G_a\rangle =&1 \quad (p \in N_a,\ \text{测量结果为 } 1), &(21)\\
\langle G_a|K_p'|G_a\rangle =&-1 \quad (p \in N_a,\ \text{测量结果为 } -1), &(22)\\
\langle G_a|K_p|G_a\rangle =&1 \quad (p \notin N_a). &(23)
\end{aligned}$$

当比特 a 上 σ^z 的测量为 -1 时, 对 N_a 上的量子比特做 σ^z 操作就可将测量后的量子态 $|G_a\rangle$ 转化为标准的图态 $|G-a\rangle$:

$$\begin{aligned}
\langle G_a|\prod_{b\in N_a}\sigma_b^z K_p \prod_{c\in N_a}\sigma_c^z|G_a\rangle =&\langle G_a|\prod_{b\in N_a}\sigma_b^z\sigma_p^x\prod_{d\in N_p}\sigma_d^z\prod_{c\in N_a}\sigma_c^z|G_a\rangle = \langle G_a|(-\sigma_p^x)\prod_{d\in N_p}\sigma_d^z|G_a\rangle = \langle G_a|(-K_p)|G_a\rangle\\
=&1 \quad (p \in N_a,\ \text{测量结果为 } -1).
\end{aligned}\tag{24}$$

因此

$$P_a^{z,\pm}|G\rangle = \frac{1}{\sqrt{2}}|z,\pm\rangle_a \otimes U_a^{z,\pm}|G-a\rangle, \tag{25}$$

其中 $U_a^{z,+} = 1$, $U_a^{z,-} = \prod_{k\in N_a}\sigma_k^z$.

下证 $P_a^{y,\pm}|G\rangle = \frac{1}{\sqrt{2}}|y,\pm\rangle_a \otimes U_a^{y,\pm}|\tau_a(G)-a\rangle$: 利用

$$\begin{aligned}
(U_a^\tau(G))^\dagger P_a^{z,\pm} U_a^\tau(G) =& \left(e^{-i\frac{\pi}{4}\sigma_a^x}\prod_{k\in N_a}e^{i\frac{\pi}{4}\sigma_k^z}\right)^\dagger P_a^{z,\pm}\left(e^{-i\frac{\pi}{4}\sigma_a^x}\prod_{k\in N_a}e^{i\frac{\pi}{4}\sigma_k^z}\right)\\
=&e^{i\frac{\pi}{4}\sigma_a^x}P_a^{z,\pm}e^{-i\frac{\pi}{4}\sigma_a^z}\\
=&\sqrt{i\sigma_a^x}\frac{1}{2}(I \pm \sigma_a^z)\sqrt{-i\sigma_a^x}
\end{aligned}$$

$$
\begin{aligned}
=&\frac{1}{2}(I\pm\sqrt{i\sigma_a^x}\sigma_a^z\sqrt{-i\sigma_a^x})\\
=&\frac{1}{2}(I\pm\sigma_a^y)=P_a^{y,\pm},
\end{aligned}
$$

从而

$$
P_a^{y,\pm}|G\rangle=(U_a^\tau(G))^\dagger P_a^{z,\pm}U_a^\tau(G)|G\rangle=(U_a^\tau(G))^\dagger P_a^{z,\pm}|\tau_a(G)\rangle. \tag{26}
$$

利用前面已证的 $P_a^{z,\pm}|G\rangle=\frac{1}{\sqrt{2}}|z,\pm\rangle_a\otimes U_a^{z,\pm}|G-a\rangle$, 可得

(1) 当 P_a^z 的测量结果为 $+1$ 时,

$$
\begin{aligned}
(U_a^\tau(G))^\dagger P_a^{z,+}|\tau_a(G)\rangle=&(U_a^\tau(G))^\dagger\frac{1}{\sqrt{2}}|z,+\rangle_a\otimes|\tau_a(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}\sigma_a^x}|z,+\rangle_a\otimes\prod_{k\in N_a}e^{-i\frac{\pi}{4}\sigma_k^z}|\tau_a(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(|z,+\rangle_a+i|z,-\rangle_a)\otimes\prod_{k\in N_a}e^{-i\frac{\pi}{4}\sigma_k^z}|\tau_a(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}|y,+\rangle_a\otimes\prod_{k\in N_a}e^{-i\frac{\pi}{4}\sigma_k^z}|\tau_a(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}|y,+\rangle_a\otimes\prod_{k\in N_a}\sqrt{-i\sigma_k^z}|\tau_a(G)-a\rangle
\end{aligned} \tag{27}
$$

(2) 当 P_a^z 的测量结果为 -1 时,

$$
\begin{aligned}
(U_a^\tau(G))^\dagger P_a^{z,-}|\tau_a(G)\rangle=&(U_a^\tau(G))^\dagger\frac{1}{\sqrt{2}}|z,-\rangle_a\otimes\prod_{b\in N_a}\sigma_b^z|\tau_a(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}\sigma_a^x}|z,-\rangle_a\otimes\prod_{k\in N_a}e^{-i\frac{\pi}{4}\sigma_k^z}\prod_{b\in N_a}\sigma_b^z|\tau_a(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}|y,-\rangle_a\otimes\prod_{k\in N_a}\sqrt{i\sigma_k^z}|\tau_a(G)-a\rangle.
\end{aligned}
$$

(忽略整体相位 $e^{i\pi/2}$)

综上, 得

$$
P_a^{y,\pm}|G\rangle=\frac{1}{\sqrt{2}}|y,\pm\rangle\otimes U_a^{y,\pm}|\tau_a(G)-a\rangle, \tag{28}
$$

其中 $U_a^{y,\pm}=\prod_{k\in N_a}\sqrt{\mp i\sigma_k^z}$.

下证 $P_a^{x,\pm}|G\rangle=\frac{1}{\sqrt{2}}|x\rangle_a\otimes U_a^{x,\pm}|\tau_{b_0}(\tau_a\tau_{b_0}(G)-a)\rangle$: 利用

$$
\begin{aligned}
(U_{b_0}^\tau(G))^\dagger P_a^{y,\pm}U_{b_0}^\tau(G)=&\left(e^{-i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{c\in N_{b_0}}e^{i\frac{\pi}{4}\sigma_c^z}\right)^\dagger\frac{1}{2}(I\pm\sigma_a^y)\left(e^{-i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{c\in N_{b_0}}e^{i\frac{\pi}{4}\sigma_c^z}\right)\\
=&\left(e^{i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{c\in N_{b_0}}e^{-i\frac{\pi}{4}\sigma_c^z}\right)\frac{1}{2}(I\pm\sigma_a^y)\left(e^{-i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{c\in N_{b_0}}e^{i\frac{\pi}{4}\sigma_c^z}\right)\\
=&e^{-i\frac{\pi}{4}\sigma_a^z}\frac{1}{2}(I\pm\sigma_a^y)e^{i\frac{\pi}{4}\sigma_a^z}\\
=&\sqrt{-i\sigma_a^z}\frac{1}{2}(I\pm\sigma_a^y)\sqrt{i\sigma_a^z}
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{1}{2}(I\pm\sqrt{-i\sigma_a^z}\sigma_a^y\sqrt{i\sigma_a^z})\\
=&\frac{1}{2}(I\pm\sigma_a^x)=P_a^{z,\pm}\quad b_0\in N_a,
\end{aligned}\tag{29}
$$

从而

$$
\begin{aligned}
P_a^{x,\pm}|G\rangle=&(U_{b_0}^\tau(G))^\dagger P_a^{y,\pm}U_{b_0}^\tau(G)|G\rangle=(U_{b_0}^\tau(G))^\dagger P_a^{y,\pm}|\tau_{b_0}(G)\rangle\\
=&(U_{b_0}^\tau(G))^\dagger(U_a^\tau(G))^\dagger P_a^{z,\pm}U_a^\tau(G)|\tau_{b_0}(G)\rangle\\
=&(U_{b_0}^\tau(G))^\dagger(U_a^\tau(G))^\dagger P_a^{z,\pm}|\tau_a\tau_{b_0}(G)\rangle\\
=&(U_{b_0}^\tau(G))^\dagger(U_a^\tau(G))^\dagger\frac{1}{\sqrt{2}}|z,\pm\rangle_a\otimes U_a^{z,\pm}|\tau_a\tau_{b_0}(G)-a\rangle
\end{aligned}
$$

(1) 当 P_a^z 的测量结果为 $+1$ 时,

$$
\begin{aligned}
&(U_{b_0}^\tau(G))^\dagger(U_a^\tau(G))^\dagger\frac{1}{\sqrt{2}}|z,+\rangle_a\otimes|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\left(e^{i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{k\in N_{b_0}}e^{-i\frac{\pi}{4}\sigma_k^z}\right)\left(e^{i\frac{\pi}{4}\sigma_a^x}\prod_{j\in N_a}e^{-i\frac{\pi}{4}\sigma_j^z}\right)\frac{1}{\sqrt{2}}|z,+\rangle_a\otimes|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}e^{-i\frac{\pi}{4}\sigma_a^z}e^{i\frac{\pi}{4}\sigma_a^x}|z,+\rangle_a\otimes e^{i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{k\in N_{b_0}-a}e^{-i\frac{\pi}{4}\sigma_k^z}\prod_{j\in N_a}e^{-i\frac{\pi}{4}\sigma_j^z}|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}\sigma_a^y}|z,+\rangle_a\otimes e^{i\frac{\pi}{4}\sigma_{b_0}^y}\prod_{k\in N_a-(N_{b_0}+b_0)}\sigma_k^z e^{-i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{j\in N_{b_0}}e^{i\frac{\pi}{4}\sigma_j^z}|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}|x,+\rangle_a\otimes\sqrt{i\sigma_{b_0}^y}\prod_{k\in N_a-(N_{b_0}+b_0)}\sigma_k^z U_{b_0}^\tau|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}|x,+\rangle_a\otimes\sqrt{i\sigma_{b_0}^y}\prod_{k\in N_a-(N_{b_0}+b_0)}\sigma_k^z|\tau_{b_0}(\tau_a\tau_{b_0}(G)-a)\rangle.
\end{aligned}\tag{30}
$$

(2) 当 P_a^z 的测量结果为 -1 时,

$$
\begin{aligned}
&(U_{b_0}^\tau(G))^\dagger(U_a^\tau(G))^\dagger\frac{1}{\sqrt{2}}|z,-\rangle_a\otimes\prod_{k\in N_a}\sigma_k^z|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\left(e^{i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{k\in N_{b_0}}e^{-i\frac{\pi}{4}\sigma_k^z}\right)\left(e^{i\frac{\pi}{4}\sigma_a^x}\prod_{j\in N_a}e^{-i\frac{\pi}{4}\sigma_j^z}\right)\frac{1}{\sqrt{2}}|z,-\rangle_a\otimes\prod_{k\in N_a}\sigma_k^z|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}\sigma_a^y}|z,-\rangle_a\otimes e^{-i\frac{\pi}{4}\sigma_{b_0}^y}\prod_{k\in N_a-(N_{b_0}+b_0)}\sigma_k^z e^{-i\frac{\pi}{4}\sigma_{b_0}^x}\prod_{j\in N_{b_0}}e^{i\frac{\pi}{4}\sigma_j^z}|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}|x,-\rangle_a\otimes e^{-i\frac{\pi}{4}\sigma_{b_0}^y}\prod_{k\in N_a-(N_{b_0}+b_0)}\sigma_k^z U_{b_0}^\tau|\tau_a\tau_{b_0}(G)-a\rangle\\
=&\frac{1}{\sqrt{2}}|x,-\rangle\otimes\sqrt{-i\sigma_{b_0}^y}\prod_{k\in N_a-(N_{b_0}+b_0)}\sigma_k^z|\tau_{b_0}(\tau_a\tau_{b_0}(G)-a)\rangle.
\end{aligned}\tag{31}
$$

综上, 得

$$
P_a^{x,\pm}|G\rangle=\frac{1}{\sqrt{2}}|x,\pm\rangle\otimes U_a^{x,\pm}|\tau_{b_0}(\tau_a\tau_{b_0}(G)-a)\rangle,\tag{32}
$$

其中 $U_a^{x,\pm}=\sqrt{\pm i\sigma_{b_0}^y}\prod_{k\in N_a-(N_{b_0}+b_0)}\sigma_k^z$. □

第 4 题 得分: ＿＿＿＿＿＿. 证明第 18 页中的对易过程.

解:

$$He^{i\alpha_4 Z/2}Z^{s_4} \tag{33}$$

□

第 5 题 得分: ＿＿＿＿＿＿. 推导 22-23 页中 CNOT 门构造中的关系式.

解: □