Problem 3.1. 腔长为0.5m的氩离子激光器，发射中心频率为$\nu_0 = 5.85 \times 10^{14}$Hz，荧光线宽$\Delta\nu = 6 \times 10^8$Hz。问它可能存在几个纵模，相应的$q$值为多少？（设$\mu = 1$）

Solution: 满足谐振条件的相邻纵模频率之差为

$$\Delta\nu_q = \frac{c}{2\mu L} = \frac{3 \times 10^8\text{m/s}}{2 \times 1 \times 0.5\text{m}} = 3 \times 10^8\text{Hz} \tag{1}$$

发射中心频率对应的纵模序数为

$$q_0 = \frac{\nu_0}{\Delta\nu_q} = \frac{5.58 \times 10^{14}\text{Hz}}{3 \times 10^8\text{Hz}} = 1950000 \tag{2}$$

荧光线宽范围内可能存在的纵模数为

$$n = \frac{\Delta\nu}{\Delta\nu_q} + 1 = \frac{6 \times 10^8\text{Hz}}{3 \times 10^8\text{Hz}} + 1 = 3$$

相应的纵模序数为

$$q_{-1} = q_0 - 1 = 1949999, \quad q_0 = 1950000, \quad q_{+1} = q_0 + 1 = 1950001$$

□

Problem 3.7. 一个共焦腔（对称）的$L = 0.40$m，$\lambda = 0.6328\mu$m，求束腰半径和离腰处56cm处的光束有效截面半径。

Solution: 束腰半径为

$$w_0 = \sqrt{\frac{\lambda L}{2\pi}} = \sqrt{\frac{0.6328 \times 10^{-6}\text{m} \times 0.40\text{m}}{2\pi}} = 2.0 \times 10^{-4}\text{m} = 0.20\text{mm} \tag{3}$$

离腰56cm处的光束有效截面半径为

$$\begin{aligned} w(z = 56\text{cm}) =& w_0\sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2} \\ =& 2.0 \times 10^{-4}\text{m}\sqrt{1 + \left(\frac{0.6328 \times 10^{-6}\text{m} \times 56 \times 10^{-2}m}{\pi(2.0 \times 10^{-4}\text{m})^2}\right)^2} \\ =& 6.0 \times 10^{-3}\text{m} = 6.0\text{mm} \end{aligned} \tag{4}$$

□

Problem 3.11. 试从式(3-88)出发，证明用最佳透射率表示的非均匀增宽激光器的最佳输出功率为

$$P_m = AI_s\frac{t_m^2}{(a - t_m)}$$

Solution: 根据式(3-88)，纵模ν的输出频率为

$$P(\nu) = \frac{1}{2} A t_1 I_s \left[\left(\frac{2LG_D^0(\nu)}{a_1 + t_1} \right)^2 - 1 \right] \tag{5}$$

上式关于透射率求导得

$$\frac{dP}{dt_1} = \frac{1}{2} A I_s \left[\frac{(2LG_D^0(\nu))^2 (a_1 - t_1)}{(a_1 + t_1)^3} - 1 \right] \tag{6}$$

当达到最佳输出时，

$$\left. \frac{dP}{dt_1} \right|_{t_1 = t_m} = 0 \Longrightarrow \left(2LG_D^0(\nu) \right)^2 = \frac{(a_1 + t_m)^3}{(a_1 - t_m)} \tag{7}$$

代入式(3-88)中得到

$$P_m = \frac{1}{2} A I_s t \left[\frac{\frac{(a_1 + t_1)^3}{(a_1 - t_1)}}{(a_1 + t_1)^2} - 1 \right] = A I_s \frac{t_m^2}{(a_1 - t_m)} \tag{8}$$

□