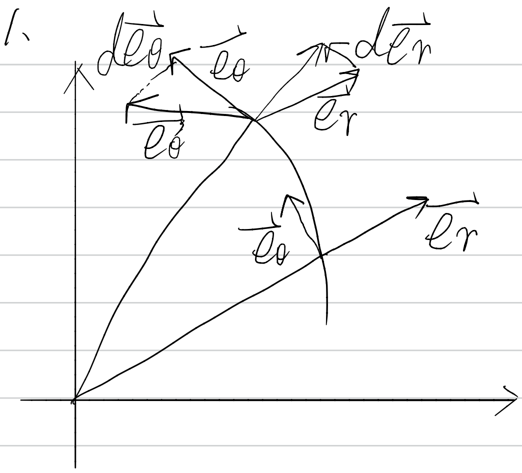
1、



$$\vec{r} = r\cdot\vec{e_r}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\cdot\vec{e_r}) = \dot{r}\vec{e_r} + r\cdot\dot{\vec{e_r}}$$

$$= \frac{dr}{dt}\vec{e_r} + r\cdot\frac{d\vec{e_r}}{dt}$$

$$\frac{d\vec{e_r}}{dt} = \frac{d\vec{e_r}}{d\theta}\cdot\frac{d\theta}{dt} = \dot{\theta}\vec{e_\theta}$$

$$\frac{d\vec{e_\theta}}{dt} = \frac{\vec{e_\theta}}{d\theta}\frac{d\theta}{dt} = -\dot{\theta}\vec{e_r}$$

$$\Rightarrow \vec{v} = \dot{r}\vec{e_r} + r\dot{\theta}\vec{e_\theta} = \frac{d\vec{r}}{dt}$$

$$\frac{d\vec{v}}{dt} = \ddot{r}\vec{e_r} + \dot{r}\dot{\vec{e_r}} + \dot{r}\dot{\theta}\vec{e_\theta} + r\ddot{\theta}\vec{e_\theta} + r\dot{\theta}\dot{\vec{e_\theta}}$$

$$= (\ddot{r} - r\dot{\theta}^2)\vec{e_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e_\theta}$$

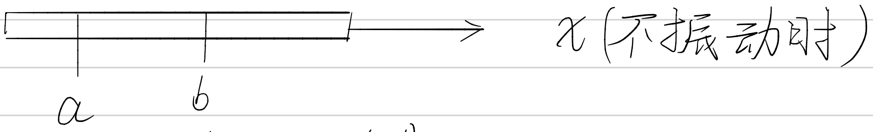
2.(1) $$\nabla\frac{1}{r} = -\frac{1}{2}\frac{-2x+2x'}{A^{3/2}}\vec{e_x} - \frac{1}{2}\frac{-2y+2y'}{A^{3/2}}\vec{e_y} - \frac{1}{2}\frac{-2z+2z'}{A^{3/2}}\vec{e_z}$$

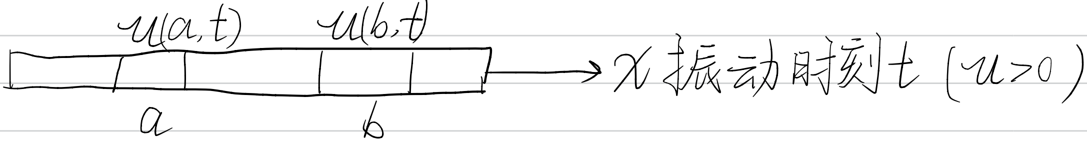
$$= -\frac{\vec{r}}{r}$$

(2) $$\nabla\times\frac{\vec{r}}{r^3} = (\nabla r^{-3})\vec{r} + r^{-3}\nabla\times\vec{r} = 0$$

(3) $$\nabla\times\vec{r} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x-x' & y-y' & z-z' \end{vmatrix} = 0$$

3、





$$\lim_{x\to 0}\frac{u(x+\Delta x,t)-u(x,t)}{\Delta x}=\frac{\partial u(x,t)}{\partial x}$$

$$\sigma(x,t)=Y\frac{\partial u(x,t)}{\partial x}$$

$$m\frac{\partial^2 u(x,t)}{\partial t^2}=S\sigma(x+\Delta x,t)-S\sigma(x,t)+f_0(x,t)\Delta x$$

$$=SY\frac{\partial u(x+\Delta x,t)}{\partial x}-SY\frac{\partial u(x,t)}{\partial x}+f_0(x,t)\Delta x$$

$$\frac{\partial^2(u,t)}{\partial t^2}=SY\Delta x\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)+f_0(x,t)\cdot\Delta x$$

$$\frac{Y}{\rho}\frac{\partial^2 u(x,t)}{\partial x^2}+\frac{f_0(x,t)}{\rho S}$$

$$\frac{\partial^2(u,t)}{\partial t^2}=a^2\frac{\partial^2 u(x,t)}{\partial x^2}+f(x,t)$$

$$a=\sqrt{\frac{Y}{\rho}}\qquad f(x,t)=\frac{1}{\rho S}f_0(x,t)$$

$$\begin{cases}\dfrac{\partial^2(u,t)}{\partial t^2}=a^2\dfrac{\partial^2 u(x,t)}{\partial x^2}+f(x,t)\\ a=\sqrt{\dfrac{Y}{\rho}},\quad f(x,t)=\dfrac{1}{\rho S}f_0(x,t)\\ (1)\ u(0,t)=0\qquad (2)\ \dfrac{\partial^2 u(x,t)}{\partial x^2}\Big|_{x=0}=a(t)\end{cases}$$

4、

二维膜方程 ρ_m为膜密度

ρ方向张力 $T(\rho+\Delta\rho)\Delta\varphi \sin\alpha|_{\rho+\Delta\rho} - T\rho\Delta\varphi\sin\alpha|_{\rho} = F_1$

φ方向张力 $T\Delta\rho\sin\gamma|_{\varphi+\Delta\varphi} - T\Delta\rho\sin\gamma|_{\varphi} = F_2$

$\sin\alpha \approx \tan\alpha \approx \frac{\Delta u}{\Delta\rho}$ $\sin\gamma = \tan\gamma \approx \frac{\Delta u}{\rho\Delta\varphi}$

$F_1 + F_2 = \rho_m \rho\Delta\varphi \cdot \Delta\rho \frac{\partial^2 u}{\partial t^2}$

$$\frac{T(\rho+\Delta\rho)\frac{\partial u}{\partial\rho}\Big|_{\Delta\rho+\rho} - T\rho\frac{\partial u}{\partial\rho}\Big|_{\rho}}{\Delta\rho} + \frac{T\frac{\partial u}{\partial\varphi}\Big|_{\varphi+\Delta\varphi} - T\frac{\partial u}{\partial\varphi}\Big|_{\varphi}}{\rho\ \ \Delta\varphi}$$

$$= \rho_m\rho\frac{\partial^2 u}{\partial t^2}$$

$$T\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right) + T\frac{1}{\rho^2}\frac{\partial}{\partial\varphi}\left(\frac{\partial u}{\partial u}\right) - \rho_m\frac{\partial^2 u}{\partial t^2} = 0$$

$$\nabla^2 u - a^2\frac{\partial^2 u}{\partial t^2} = 0 \qquad \nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}$$

$$a^2 = \frac{\rho_m}{T}$$

四周固定，设边界距中心距离为R，$u|_{\rho=R} = 0$

$u|_{\rho=R} = 0 \Rightarrow u|_{x^2+y^2=R} = 0$

方程组为

$$\begin{cases} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = a^2\frac{\partial^2 u}{\partial t^2} \\ a = \sqrt{\frac{\rho_m}{T}} \\ u|_{x^2+y^2=R} = 0 \end{cases}$$

5、(1) $A=1$ $B=2$ $C=5$ $D=1$ $E=2$, $F=G=0$

$y=2x\pm\sqrt{i}x+C$

$\xi=y-2x$ $\eta=\sqrt{i}x$

$a=1$ $b=0$ $c=1$ $d=0$ $e=\sqrt{i}$

标准形式为 $\frac{\partial^2 u}{\partial\xi^2}-\frac{\partial^2 u}{\partial\eta^2}+\frac{\partial u}{\partial\eta}\sqrt{i}=0$

(2) $A=1$ $B=0$ $C=y$ $E=\frac{1}{2}$ $F,G=0$

$(\frac{dy}{dx})^2+y=0 \Rightarrow y^{\frac{1}{2}}\pm ix=C$

$\xi=ix$ $\eta=(y)^{\frac{1}{2}}$

$a=-1$ $b=0$ $c=\frac{1}{4}$ $d=0$ $e=\frac{1}{4}(y)^{-\frac{1}{2}}$

标准形式为 $-4\frac{\partial^2 u}{\partial\xi^2}+\frac{\partial^2 u}{\partial\eta^2}+y^{-\frac{1}{2}}\frac{\partial u}{\partial\eta}=0$

(3) $A=1$ $B=-\cos x$ $C=-3-\sin^2 x$ $E=1$ $G=y$

$y=C+-\sin x\pm 2x \Rightarrow y+\sin x\pm 2x=C$

$\xi=y+\sin x+2x$ $\eta=y+\sin x-2x$

$a=0$ $b=-8$ $c=-8\cos x$ $d=-\cos^2 x$

$e=-\cos^2 x$ $G=y$

$16\frac{\partial^2 u}{\partial\xi\partial\eta}+8\cos x\frac{\partial u}{\partial\eta^2}+\cos^2 x\frac{\partial u}{\partial\xi}+\cos^2 x\frac{\partial u}{\partial\eta}=y$