

# PHYS1303 000000II000000

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TOTAL POINTS

**45 / 56**

## QUESTION 1

1 000 10 / 10

✓ - 0 pts Correct

- 1 pts Please use the correct form of the final result as  $e^{\theta}$  unified diff form
- 3 pts Please simplify to the simplest form
- 6 pts The wrong answer of the second part
- 10 pts Please select the page of question / Not answer / Not right at all

## QUESTION 2

2 000 12 / 12

✓ - 0 pts Correct

- 2 pts There 's a slight error in the simplification
- 4 pts Please simplify to the simplest form
- 12 pts Please select the page of question / Not answer / Not right at all
- 4 pts one of three part is totally wrong

## QUESTION 3

3 000 9 / 12

- 0 pts Correct
- 4 pts The equation is wrong
- ✓ - 3 pts The boundary condition is wrong
- 6 pts No boundary condition
- 6 pts No process of proof
- 12 pts blank

## QUESTION 4

4 000 10 / 10

✓ - 0 pts Correct

- 6 pts Lack or error of differential equation
- 4 pts Lack or error of boundary conditions
- 2 pts Simplify mistakes/Gravity is overtaken./Lack of computational process

- 10 pts Please select the page of question / Not answer / Not right at all

## QUESTION 5

5 000 4 / 12

- 0 pts Correct
- 12 pts Please select the page of question / Not answer
- ✓ - 4 pts (1)is wrong
- 4 pts (2)is wrong
- ✓ - 4 pts (3)is wrong
- 2 pts Half wrong in (2)
- 2 pts Half wrong in (3)

# 数学物理方法II第一次作业

陈稼霖

45875852

2019.3.1

## 1

在极坐标系中写出速度 $\frac{d\vec{r}}{dt}$ 和加速度 $\frac{d\vec{v}}{dt}$ ，结果用 $\vec{e}_r$ 、 $\vec{e}_\theta$ 表示。

解：在极坐标系中速度为

$$\begin{aligned}\frac{d\vec{r}}{dt} &= \frac{d(r\vec{e}_r)}{dt} \\ &= \frac{dr}{dt}\vec{e}_r + r\frac{d\vec{e}_r}{dt} \\ &= \frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta\end{aligned}$$

加速度为

$$\begin{aligned}\frac{d\vec{v}}{dt} &= \frac{d(\frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta)}{dt} \\ &= \frac{d^2r}{dt^2}\vec{e}_r + \frac{dr}{dt}\frac{d\vec{e}_r}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + r\frac{d^2\theta}{dt^2}\vec{e}_\theta + r\frac{d\theta}{dt}\frac{d\vec{e}_\theta}{dt} \\ &= \frac{d^2r}{dt^2}\vec{e}_r + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + r\frac{d^2\theta}{dt^2}\vec{e}_\theta - r(\frac{d\theta}{dt})^2\vec{e}_r \\ &= [\frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2]\vec{e}_r + (2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2})\vec{e}_\theta\end{aligned}$$

## 2

设矢量 $\vec{r}$ 为某个源点 $\vec{x}'$ 指向场点 $\vec{x}$ 的矢量，而 $r$ 为源点到场点的距离。

$$\vec{r} = \vec{x} - \vec{x}' = \vec{e}_x(x - x') + \vec{e}_y(y - y') + \vec{e}_z(z - z')$$

请求出

(1)  $\nabla \frac{1}{r}$

解：

$$\begin{aligned}\nabla \frac{1}{r} &= \frac{\partial}{\partial x} \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \vec{e}_x + \frac{\partial}{\partial y} \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \vec{e}_y \\ &\quad + \frac{\partial}{\partial z} \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \vec{e}_z\end{aligned}$$

1 ☐ ☐ ☐ 10 / 10

✓ - 0 pts Correct

- 1 pts Please use the correct form of the final result as  $e^{\theta}$  or unified diff form
- 3 pts Please simplify to the simplest form
- 6 pts The wrong answer of the second part
- 10 pts Please select the page of question / Not answer / Not right at all

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加速度为

$$\begin{aligned}\frac{d\vec{v}}{dt} &= \frac{d(\frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta)}{dt} \\ &= \frac{d^2r}{dt^2}\vec{e}_r + \frac{dr}{dt}\frac{d\vec{e}_r}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + r\frac{d^2\theta}{dt^2}\vec{e}_\theta + r\frac{d\theta}{dt}\frac{d\vec{e}_\theta}{dt} \\ &= \frac{d^2r}{dt^2}\vec{e}_r + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + r\frac{d^2\theta}{dt^2}\vec{e}_\theta - r(\frac{d\theta}{dt})^2\vec{e}_r \\ &= [\frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2]\vec{e}_r + (2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2})\vec{e}_\theta\end{aligned}$$

## 2

设矢量 $\vec{r}$ 为某个源点 $\vec{x}'$ 指向场点 $\vec{x}$ 的矢量，而 $r$ 为源点到场点的距离。

$$\vec{r} = \vec{x} - \vec{x}' = \vec{e}_x(x - x') + \vec{e}_y(y - y') + \vec{e}_z(z - z')$$

请求出

(1)  $\nabla \frac{1}{r}$

解：

$$\begin{aligned}\nabla \frac{1}{r} &= \frac{\partial}{\partial x} \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \vec{e}_x + \frac{\partial}{\partial y} \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \vec{e}_y \\ &\quad + \frac{\partial}{\partial z} \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \vec{e}_z\end{aligned}$$

$$\begin{aligned}
&= -\frac{x-x'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}} \vec{e}_x - \frac{y-y'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}} \vec{e}_y \\
&\quad - \frac{z-z'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}} \vec{e}_z \\
&= -\frac{(x-x')\vec{e}_x + (y-y')\vec{e}_y + (z-z')\vec{e}_z}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}} \\
&= -\frac{\vec{r}}{r^3} \\
(2) \quad \nabla \times \frac{\vec{r}}{r^3}
\end{aligned}$$

解:

$$\nabla \times \frac{\vec{r}}{r^3} = (\nabla \frac{1}{r^3}) \times \vec{r} + \frac{1}{r^3} \nabla \times \vec{r}$$

式中

$$\begin{aligned}
\nabla \frac{1}{r^3} &= \frac{\partial \frac{1}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}}{\partial x} \vec{e}_x + \frac{\partial \frac{1}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}}{\partial y} \vec{e}_y \\
&\quad + \frac{\partial \frac{1}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}}{\partial z} \vec{e}_z \\
&= -\frac{3(x-x')}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{5}{2}}} \vec{e}_x - \frac{3(y-y')}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{5}{2}}} \vec{e}_y \\
&\quad - \frac{3(z-z')}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{5}{2}}} \vec{e}_z \\
&= -3 \frac{(x-x')\vec{e}_x + (y-y')\vec{e}_y + (z-z')\vec{e}_z}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{5}{2}}} \\
&= -\frac{3\vec{r}}{r^5}
\end{aligned}$$

且

$$\begin{aligned}
\nabla \times \vec{r} &= \left( \frac{\partial(z-z')}{\partial y} - \frac{\partial(y-y')}{\partial z} \right) \vec{e}_x + \left( \frac{\partial(x-x')}{\partial z} - \frac{\partial(z-z')}{\partial x} \right) \vec{e}_y + \left( \frac{\partial(y-y')}{\partial x} - \frac{\partial(x-x')}{\partial y} \right) \vec{e}_z \\
&= \vec{0}
\end{aligned}$$

故

$$\begin{aligned}
\nabla \times \frac{\vec{r}}{r^3} &= -3 \frac{\vec{r}}{r^5} \times \vec{r} + \vec{0} \\
&= \vec{0}
\end{aligned}$$

(3)  $\nabla \times \vec{r}$

解:

$$\begin{aligned}
\nabla \times \vec{r} &= \left( \frac{\partial(z-z')}{\partial y} - \frac{\partial(y-y')}{\partial z} \right) \vec{e}_x + \left( \frac{\partial(x-x')}{\partial z} - \frac{\partial(z-z')}{\partial x} \right) \vec{e}_y + \left( \frac{\partial(y-y')}{\partial x} - \frac{\partial(x-x')}{\partial y} \right) \vec{e}_z \\
&= \vec{0}
\end{aligned}$$

2 ☐ ☐ ☐ 12 / 12

✓ - 0 pts Correct

- 2 pts There 's a slight error in the simplification
- 4 pts Please simplify to the simplest form
- 12 pts Please select the page of question / Not answer / Not right at all
- 4 pts one of three part is totally wrong

### 3

从杆的纵振动问题导出波动方程，其问题设置如下：

均匀细杆在外力作用下沿杆长方向作微小振动，设杆长方向为 $x$ 轴， $u(x, t)$ 为 $x$ 处的截面在 $t$ 时刻沿杆长方向的位移。其中理想化假设如下 i) 振动方向与杆的方向一致。

ii) 均匀细杆：同一横截面上各点的质量密度 $\rho$ ，横截面面积 $S$ 与杨氏模量 $Y$ （应力与应变值比值）都是常数。

iii) 杆有弹性，服从Hooke定律：即应力与相对伸长成正比。

iv) 外力与杆的方向一致，各点单位长度上的外力为 $f_0(x, t)$ ，重力不计。并求出如下情况对应的边界条件：

(1)  $x = 0$ 处固定。

(2)  $x = 0$ 处受 $G(t)$ 的横向外力。

解：对于杆在区间 $[a + u(a, t), b + u(b, t)]$ 上的部分，其左右端分别受到张力

$$\begin{aligned} T_a &= -YS \frac{u(a + \Delta x, t) - u(a, t)}{\Delta x} = -YS \frac{\partial u}{\partial x} \Big|_{x=a} \\ T_b &= YS \frac{u(b + \Delta x, t) - u(b, t)}{\Delta x} = YS \frac{\partial u}{\partial x} \Big|_{x=b} \end{aligned}$$

根据牛顿第二定律得到波动方程

$$\begin{aligned} T_a + T_b + (b - a)f_0 &= \rho(b - a)S \frac{\partial^2 u}{\partial t^2} \\ \Rightarrow YS \frac{\frac{\partial u}{\partial x} \Big|_{x=b} - \frac{\partial u}{\partial x} \Big|_{x=a}}{b - a} + f_0 &= \rho S \frac{\partial^2 u}{\partial t^2} \\ \Rightarrow \frac{\partial^2 u}{\partial t^2} - \frac{Y}{\rho} \frac{\partial^2 u}{\partial x^2} &= \frac{f_0}{\rho S} \end{aligned}$$

(1)  $u|_{x=0} = 0$

(2) 根据牛顿第二定律有

$$\rho \Delta x S \frac{\partial^2 u}{\partial t^2} = G(t) + YS \frac{u(\Delta x, t) - u(0, t)}{\Delta x} + f_0(0, t) \Delta x$$

令 $\Delta x \rightarrow 0$ 得到

$$\frac{\partial u}{\partial x} \Big|_{x=0} = -\frac{g(t)}{YS}$$

### 4

二维波动方程的推出：有一均匀的各向同性的弹性圆膜，四周固定。试列出膜的横振动方程与边界条件(设 $\rho_m$ 为面密度，沿任何方向单位长度张力为 $T$ )。

提示：在极坐标中进行微元分析，进而化为直角坐标下的波动方程。

解：设时刻 $t$ 点 $\mathbf{r}$ 偏离平衡位置 $u(\mathbf{r}, t)$ ，弹性圆膜半径为 $R$ 。原题图中所取膜面积元的质量为

$$\Delta m = \rho_m \cdot \Delta S = \rho_m \cdot \rho \Delta \phi \cdot \Delta \rho$$

3 ☐ ☐ ☐ 9 / 12

- 0 pts Correct
- 4 pts The equation is wrong
- ✓ - 3 pts The boundary condition is wrong
- 6 pts No boundary condition
- 6 pts No process of proof
- 12 pts blank



### 3

从杆的纵振动问题导出波动方程，其问题设置如下：

均匀细杆在外力作用下沿杆长方向作微小振动，设杆长方向为 $x$ 轴， $u(x, t)$ 为 $x$ 处的截面在 $t$ 时刻沿杆长方向的位移。其中理想化假设如下 i) 振动方向与杆的方向一致。

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解：对于杆在区间 $[a + u(a, t), b + u(b, t)]$ 上的部分，其左右端分别受到张力

$$\begin{aligned} T_a &= -YS \frac{u(a + \Delta x, t) - u(a, t)}{\Delta x} = -YS \frac{\partial u}{\partial x} \Big|_{x=a} \\ T_b &= YS \frac{u(b + \Delta x, t) - u(b, t)}{\Delta x} = YS \frac{\partial u}{\partial x} \Big|_{x=b} \end{aligned}$$

根据牛顿第二定律得到波动方程

$$\begin{aligned} T_a + T_b + (b - a)f_0 &= \rho(b - a)S \frac{\partial^2 u}{\partial t^2} \\ \Rightarrow YS \frac{\frac{\partial u}{\partial x} \Big|_{x=b} - \frac{\partial u}{\partial x} \Big|_{x=a}}{b - a} + f_0 &= \rho S \frac{\partial^2 u}{\partial t^2} \\ \Rightarrow \frac{\partial^2 u}{\partial t^2} - \frac{Y}{\rho} \frac{\partial^2 u}{\partial x^2} &= \frac{f_0}{\rho S} \end{aligned}$$

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令 $\Delta x \rightarrow 0$ 得到

$$\frac{\partial u}{\partial x} \Big|_{x=0} = -\frac{g(t)}{YS}$$

### 4

二维波动方程的推出：有一均匀的各向同性的弹性圆膜，四周固定。试列出膜的横振动方程与边界条件(设 $\rho_m$ 为面密度，沿任何方向单位长度张力为 $T$ )。

提示：在极坐标中进行微元分析，进而化为直角坐标下的波动方程。

解：设时刻 $t$ 点 $\mathbf{r}$ 偏离平衡位置 $u(\mathbf{r}, t)$ ，弹性圆膜半径为 $R$ 。原题图中所取膜面积元的质量为

$$\Delta m = \rho_m \cdot \Delta S = \rho_m \cdot \rho \Delta \phi \cdot \Delta \rho$$

在径向，根据牛二律有

$$\rho_m \cdot \rho \Delta \phi \cdot \Delta \rho \frac{\partial^2 u}{\partial t^2} = -T \rho \Delta \phi \sin \alpha|_{\rho} + T(\rho + \Delta \rho) \Delta \phi \sin \alpha|_{\rho + \Delta \rho} + T \Delta \rho \sin \beta|_{\phi} - T \Delta \rho \sin \beta|_{\phi + \Delta \phi}$$

在小振动下有如下近似

$$\sin \alpha \approx \frac{\partial u}{\partial \rho}, \quad \sin \beta \approx \frac{1}{\rho} \frac{\partial u}{\partial \phi}$$

故原方程可化为

$$\begin{aligned} \rho_m \rho \Delta \phi \Delta \rho \frac{\partial^2 u}{\partial t^2} &= T \Delta \phi \left[ \left( \rho \frac{\partial u}{\partial \rho} \right) \Big|_{\rho + \Delta \rho} - \left( \rho \frac{\partial u}{\partial \rho} \right) \Big|_{\rho} \right] + T \Delta \rho \frac{1}{\rho} \left[ \frac{\partial u}{\partial \phi} \Big|_{\phi + \Delta \phi} - \frac{\partial u}{\partial \phi} \Big|_{\phi} \right] \\ &= T \Delta \phi \frac{\partial}{\partial \rho} \left( \rho \frac{\partial u}{\partial \rho} \right) \Delta \rho + T \Delta \rho \frac{1}{\rho} \frac{\partial}{\partial \phi} \left( \frac{\partial u}{\partial \phi} \right) \Delta \phi \\ &\Rightarrow \frac{\partial^2 u}{\partial t^2} = \frac{T}{\rho_m} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial u}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \phi^2} \right] \end{aligned}$$

由于四周固定，故

$$u|_{\rho=R} = 0$$

转换到直角坐标系，振动方程及边界条件为

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = \frac{T}{\rho_m} \nabla^2 u \\ u|_{\sqrt{x^2+y^2}=R} = 0 \end{cases}$$

## 5

将下列二阶偏微分方程化为标准形式.

$$(1) \frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 5 \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0;$$

解：此方程的特征方程为

$$\left( \frac{dy}{dx} \right)^2 - 4 \frac{dy}{dx} + 5 = 0$$

解得

$$y - 4x = C_1, \quad y - x = C_2$$

做变换

$$\xi = y - 4x, \quad \eta = y - x$$

原方程化为标准形式

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = -\frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial u}{\partial \eta}$$

$$(2) \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial y^2} + \frac{1}{2} \frac{\partial u}{\partial y} = 0;$$

解：此方程的特征方程为

$$\left( \frac{dy}{dx} \right)^2 + y = 0$$

4 ☐ ☐ ☐ ☐ 10 / 10

✓ - 0 pts Correct

- 6 pts Lack or error of differential equation
- 4 pts Lack or error of boundary conditions
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在径向，根据牛二律有

$$\rho_m \cdot \rho \Delta \phi \cdot \Delta \rho \frac{\partial^2 u}{\partial t^2} = -T \rho \Delta \phi \sin \alpha|_{\rho} + T(\rho + \Delta \rho) \Delta \phi \sin \alpha|_{\rho + \Delta \rho} + T \Delta \rho \sin \beta|_{\phi} - T \Delta \rho \sin \beta|_{\phi + \Delta \phi}$$

在小振动下有如下近似

$$\sin \alpha \approx \frac{\partial u}{\partial \rho}, \quad \sin \beta \approx \frac{1}{\rho} \frac{\partial u}{\partial \phi}$$

故原方程可化为

$$\begin{aligned} \rho_m \rho \Delta \phi \Delta \rho \frac{\partial^2 u}{\partial t^2} &= T \Delta \phi \left[ \left( \rho \frac{\partial u}{\partial \rho} \right) \Big|_{\rho + \Delta \rho} - \left( \rho \frac{\partial u}{\partial \rho} \right) \Big|_{\rho} \right] + T \Delta \rho \frac{1}{\rho} \left[ \frac{\partial u}{\partial \phi} \Big|_{\phi + \Delta \phi} - \frac{\partial u}{\partial \phi} \Big|_{\phi} \right] \\ &= T \Delta \phi \frac{\partial}{\partial \rho} \left( \rho \frac{\partial u}{\partial \rho} \right) \Delta \rho + T \Delta \rho \frac{1}{\rho} \frac{\partial}{\partial \phi} \left( \frac{\partial u}{\partial \phi} \right) \Delta \phi \\ &\Rightarrow \frac{\partial^2 u}{\partial t^2} = \frac{T}{\rho_m} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial u}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \phi^2} \right] \end{aligned}$$

由于四周固定，故

$$u|_{\rho=R} = 0$$

转换到直角坐标系，振动方程及边界条件为

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = \frac{T}{\rho_m} \nabla^2 u \\ u|_{\sqrt{x^2+y^2}=R} = 0 \end{cases}$$

## 5

将下列二阶偏微分方程化为标准形式.

$$(1) \frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 5 \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0;$$

解：此方程的特征方程为

$$\left( \frac{dy}{dx} \right)^2 - 4 \frac{dy}{dx} + 5 = 0$$

解得

$$y - 4x = C_1, \quad y - x = C_2$$

做变换

$$\xi = y - 4x, \quad \eta = y - x$$

原方程化为标准形式

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = -\frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial u}{\partial \eta}$$

$$(2) \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial y^2} + \frac{1}{2} \frac{\partial u}{\partial y} = 0;$$

解：此方程的特征方程为

$$\left( \frac{dy}{dx} \right)^2 + y = 0$$

判别式为

$$\Delta = -y$$

当 $\Delta > 0$ 即 $y < 0$ 时, 解得

$$x \pm 2(-y)^{1/2} = \gamma$$

做变换

$$\xi = x + 2(-y)^{1/2}, \quad \eta = x - 2(-y)^{1/2}$$

原方程化为标准形式

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = 0$$

当 $\Delta = 0$ 即 $y = 0$ 时, 原方程为

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{2} \frac{\partial u}{\partial y} = 0$$

当 $\Delta < 0$ 即 $y > 0$ 时, 特征方程解得

$$x \pm 2iy^{1/2} = \gamma$$

做变换

$$\xi = x, \quad \eta = 2y^{1/2}$$

原方程化为标准形式

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} = 0$$

$$(3) \quad \frac{\partial^2 u}{\partial x^2} - 2 \cos x \frac{\partial^2 u}{\partial x \partial y} - (3 + \sin^2 x) \frac{\partial^2 u}{\partial y^2} - y - \frac{\partial u}{\partial y} = 0$$

解: 此方程的特征方程为

$$\left(\frac{dy}{dx}\right) + 2 \cos x \frac{dy}{dx} - (3 + \sin^2 x) = 0$$

判别式为

$$\Delta = \cos^2 x + 3 + \sin^2 x = 4 > 0$$

解得

$$y \pm \sin x = \gamma$$

做变换

$$\xi = y + \sin x, \quad \eta = y - \sin x$$

原方程化为标准形式

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = -\frac{1}{8} \left( \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \eta} + u \right)$$

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- 0 pts Correct

- 12 pts Please select the page of question / Not answer

✓ - 4 pts (1) is wrong

- 4 pts (2) is wrong

✓ - 4 pts (3) is wrong

- 2 pts Half wrong in (2)

- 2 pts Half wrong in (3)