PHYS1303
数学物理方法II
Spring 2019
Homework 2

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**Problem 1.** 解无界弦上的振动问题

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + \cos(\omega t)\cos(x), (-\infty < x < +\infty, t > 0) \\ u|_{t=0} = e^{-2x^2}, (-\infty < x < +\infty) \\ \frac{\partial u}{\partial t}|_{t=0} = \sin(x), (-\infty < x < +\infty) \end{cases}$$

*Solution:*

$$\phi(x) = e^{-2x^2}, \ \ \psi = \sin x, \ \ f(x,t) = \cos(\omega t)\cos x$$

利用达朗贝尔公式

$$\begin{aligned} u(x,t) =& \frac{1}{2}[\phi(x+at) + \phi(x-at)] + \frac{1}{2a}\int_{x-at}^{x+at} \psi(\xi)d\xi + \frac{1}{2a}\int_0^t \int_{x-a(t-\tau)}^{x+a(t-\tau)} f(\xi,\tau)d\xi d\tau \\ =& \frac{1}{2}[e^{-2(x+at)^2} + e^{-2(x-at)^2}] + \frac{1}{2a}\int_{x-at}^{x+at} \sin\xi d\xi + \frac{1}{2a}\int_0^t \int_{x-a(t-\tau)}^{x+a(t-\tau)} \cos(\omega\tau)\cos\xi d\xi d\tau \\ =& \frac{1}{2}[e^{-2(x+at)^2} + e^{-2(x-at)^2}] + \frac{1}{2a}[-\cos(x+at) + \cos(x-at)] \\ &+ \frac{1}{2a}\int_0^t \cos(\omega\tau)\{\sin[x+a(t-\tau)] - \sin[x-a(t-\tau)]\}d\tau \\ =& \frac{1}{2}[e^{-2(x+at)^2} + e^{-2(x-at)^2}] + \frac{1}{a}\sin x\sin(at) + \frac{1}{a}\int_0^t \cos(\omega\tau)\cos x\sin[a(t-\tau)]d\tau \\ =& \frac{1}{2}[e^{-2(x+at)^2} + e^{-2(x-at)^2}] + \frac{1}{a}\sin x\sin(at) \\ &+ \frac{\cos x}{2a}\int_0^t \{\sin[\omega\tau + a(t-\tau)] - \sin[\omega\tau - a(t-\tau)]\}d\tau \\ =& \frac{1}{2}[e^{-2(x+at)^2} + e^{-2(x-at)^2}] + \frac{1}{a}\sin x\sin(at) + \frac{1}{\omega^2 - a^2}\cos x[\cos(\omega t) - \cos(at)] \end{aligned}$$

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