

第 1 题 得分：_____. 利用 KDP 晶体进行参量放大，若其中有两个光波是 e 光，第三个光波是 o 光，试推导相位匹配角公式，其中信号、空闲和泵浦光中哪一个为寻常光？令 $\omega_3 = 10\,000\text{ cm}^{-1}$, $\omega_1 = \omega_2 = 5000\text{ cm}^{-1}$ 能否实现这种形式的相位匹配？

解：KDP 为负单轴的晶体，故为 II 类相位匹配 ($e + o \rightarrow e$).

要实现相位匹配，需满足

$$\frac{1}{2}[n_e^\omega(\theta) + n_o^\omega] = n_e^{2\omega}(\theta_m), \quad (1)$$

其中传播方向与光轴夹角为 θ 的非常光的折射率满足

$$\frac{1}{[n_e^\omega \text{ 或 } 2\omega(\theta)]^2} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{[n_e^\omega \text{ 或 } 2\omega(\theta)]^2}. \quad (2)$$

由于 $n_o^\omega = 1.4596$, $n_e^\omega = 1.4606$, $n_e^{2\omega} = 1.4999$, $\frac{1}{2}[n_e^\omega(\pi/2) + n_o^\omega] < n_e^{2\omega}(\pi/2)$, 故无角度满足相位匹配条件，无法实现相位匹配. \square

第 2 题 得分：_____. 试证明外加直流电场 $E_x = E_{0j}$ 的 KDP 晶体，光波在 zox 面内、与 x 轴成 45° 方向传播时的电光延迟为下式. 其中 l 为沿光传播方向上的晶体长度， d 为沿外加电场的晶体厚度， U_y 为外加电压.

$$\Delta\varphi \approx \frac{2\pi l}{\lambda} \left[\frac{\sqrt{2}n_on_e}{\sqrt{n_o^2 + n_e^2}} - n_o + \sqrt{2} \left[\frac{1}{n_o^2} + \frac{1}{n_e^2} \right]^{-3/2} \gamma_{41} U_y \frac{l}{d} \right].$$

证：KDP 为负单轴晶体，故无外加电场下，KDP 折射率椭球方程为

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1, \quad (3)$$

光波在 zox 面内、与 x 轴成 45° 方向传播时，垂直于 zox 面的偏振分量感受到的折射率为 $n_\perp = n_o$ ，平行于 zox 面的折射率分量感受到的折射率满足

$$\frac{n_\parallel^2 \sin^2(-45^\circ)}{n_o^2} + \frac{n_\parallel^2 \cos^2(-45^\circ)}{n_e^2} = 1, \quad (4)$$

$$\Rightarrow n_\parallel = \frac{\sqrt{2}n_on_e}{\sqrt{n_o^2 + n_e^2}}. \quad (5)$$

KDP 的电光系数为

$$[\gamma_{ij}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_{41} & 0 & 0 \\ 0 & \gamma_{41} & 0 \\ 0 & 0 & \gamma_{63} \end{bmatrix}, \quad (6)$$

故外加电场 $E_x = E_{0j} = \frac{U_y}{d}$ 下，KDP 折射率椭球系数变化量为

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \begin{bmatrix} E_{0j} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \gamma_{41} E_{0j} \\ 0 \end{bmatrix}, \quad (7)$$

折射率椭球方程为

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2\gamma_{41}E_{0j}xz = 1, \quad (8)$$

光波在 zox 面内、与 x 轴成 45° 方向传播时, 垂直于 zox 面的偏振分量感受到的折射率满足

$$\frac{[n'_\perp]^2}{n_o^2} = 1, \quad (9)$$

$$\Rightarrow n'_\perp = n_o, \quad (10)$$

平行于 zox 面的偏振分量感受到的折射率满足

$$\frac{[n'_\parallel]^2 \sin^2(-45^\circ)}{n_o^2} + \frac{[n'_\parallel]^2 \cos^2(-45^\circ)}{n_e^2} + 2\gamma_{41}E_{0j}n'_\parallel \sin(-45^\circ)n'_\parallel \cos(-45^\circ) = 1, \quad (11)$$

$$\Rightarrow n'_\parallel = \frac{1}{\sqrt{\frac{n_o^2+n_e^2}{2n_o^2n_e^2} - r_{41}E_{0j}}} = \frac{\sqrt{2}n_on_e}{\sqrt{n_e^2+n_o^2}} \frac{1}{\sqrt{1 - \frac{2n_o^2n_e^2}{n_e^2+n_o^2}r_{41}E_{0j}}} \approx \sqrt{2}\frac{n_on_e}{\sqrt{n_o^2+n_e^2}} + \sqrt{2}\left(\frac{n_o^2n_e^2}{n_o^2+n_e^2}\right)^{3/2}\gamma_{41}E_{0j}, \quad (12)$$

故电光延迟为

$$\Delta\varphi = \frac{2\pi l(n'_\parallel - n'_\perp)}{\lambda} \approx \frac{2\pi l}{\lambda} \left[\frac{\sqrt{2}n_on_e}{n_o^2+n_e^2} - n_o + \sqrt{2}\left[\frac{1}{n_o^2} + \frac{1}{n_e^2}\right]^{-3/2}\gamma_{41}\frac{U_y}{d} \right]. \quad (13)$$

□

第 3 题 得分: _____. 考虑 LiNbO_3 浸提中的 II 型 ($o+e \rightarrow e$) 相位匹配下的共线传播倍频过程 $\omega + \omega \rightarrow 2\omega$:

- 1) 设光波矢沿 (θ, φ) , 求出此时有效非线性系数 d_{eff} 的表达式. 注: 已知 LiNbO_3 晶体 (负单轴晶体) 的非线性系数矩阵为

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}.$$

- 2) 用折射率曲面的方法画出实现相位匹配的示意图.
- 3) 若要得到最佳倍频输出, 问光波矢的方向 (θ, φ) 应取何值?

解: 1)

$$\mathbf{a}^o = \begin{bmatrix} \sin \varphi \\ -\cos \varphi \\ 0 \end{bmatrix}, \quad (14)$$

$$\mathbf{a}^e = \begin{bmatrix} -\cos \theta \cos \varphi \\ -\cos \theta \sin \varphi \\ \sin \theta \end{bmatrix}, \quad (15)$$

$$\mathbf{a}^o \mathbf{a}^e = \begin{bmatrix} -\cos \theta \sin \varphi \cos \varphi \\ -\cos \theta \sin \varphi \cos \varphi \\ 0 \\ -2 \sin \theta \cos \varphi \\ 2 \sin \theta \sin \varphi \\ \cos \theta \cos 2\varphi \end{bmatrix}. \quad (16)$$

II 型相位匹配下, 有效非线性系数为

$$\begin{aligned}
 d_{eff} &= \mathbf{a}^e \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \mathbf{a}^o \mathbf{a}^e \\
 &= \begin{bmatrix} -\cos \theta \cos \varphi & -\cos \theta \sin \varphi & \sin \theta \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\cos \theta \sin \varphi \cos \varphi \\ -\cos \theta \sin \varphi \cos \varphi \\ 0 \\ -2 \sin \theta \cos \varphi \\ 2 \sin \theta \sin \varphi \\ \cos \theta \cos 2\varphi \end{bmatrix} \\
 &= \begin{bmatrix} -\cos \theta \cos \varphi & -\cos \theta \sin \varphi & \sin \theta \end{bmatrix} \begin{bmatrix} 2d_{15} \sin \theta \sin \varphi - d_{22} \cos \theta \cos 2\varphi \\ -2d_{15} \sin \theta \cos \varphi \\ -2d_{13} \cos \theta \sin \varphi \cos \varphi \end{bmatrix} \\
 &= d_{22} \cos^2 \theta \cos \varphi \cos 2\varphi - 2d_{13} \cos \theta \sin \theta \sin \varphi \cos \varphi.
 \end{aligned} \tag{17}$$

2) 如图 1.

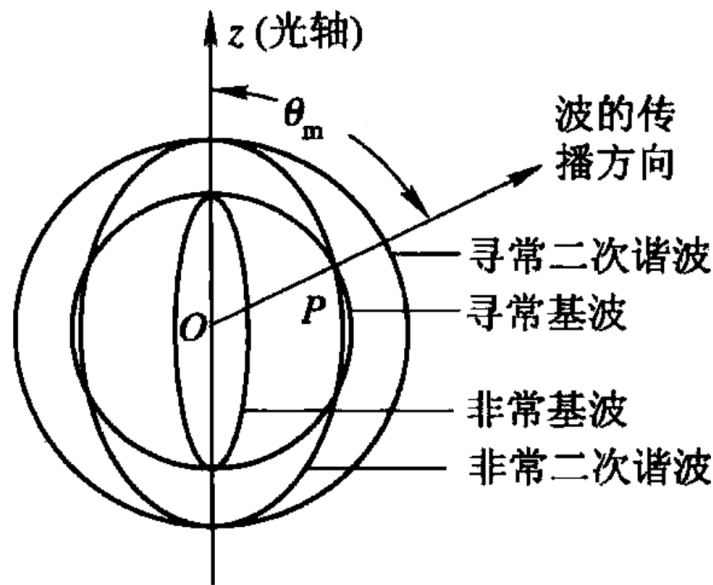


图 1: II 型相位匹配示意图.

3) 为得到最佳倍频输出, 取 θ_m 满足 $\frac{1}{2}[n_e^\omega(\theta) + n_o^\omega] = n_e^{2\omega}(\theta)$, 在此基础上取 φ 使 d_{eff} 极大.

□

第 4 题 得分: _____. 试证明在非共线相位匹配的条件下, 为获得远红外差频光 ($\omega_1, \omega_2 \gg \omega_3$), 晶体必须具有反常色散特性.

证: 为获得远红外差频光, $\omega_1 - \omega_2 = \omega_3$, $n_1 \approx n_2 \equiv n$, $n_3 \approx n + \Delta n$. 由余弦定理, \mathbf{k}_1 和 \mathbf{k}_2 之间的夹角 θ 满足

$$\cos \theta = \frac{k_1^2 + k_2^2 - k_3^2}{2k_1 k_2} = \frac{(n_1 \omega_1)^2 + (n_2 \omega_2)^2 - (n_3 \omega_3)^2}{2n_1 \omega_1 n_2 \omega_2} \approx \frac{n^2(\omega_1^2 + \omega_2^2) - (n + \Delta n)^2(\omega_1 - \omega_2)^2}{2n^2 \omega_1 \omega_2} \approx 1 - \frac{\Delta n}{n} \frac{(\omega_1 - \omega_2)^2}{\omega_1 \omega_2}. \tag{18}$$

由 $\cos \theta < 1$ 得 $\Delta n > 0$, 即反常色散.

□

第 5 题 得分: _____. 试证明, 如果二次谐波产生过程的基频光 ω 是寻常光, 倍频光 2ω 是非常光, θ_m 是其相位匹配角, 则有

$$\Delta k(\theta)L|_{\theta=\theta_m} = \frac{2\omega L}{c} \sin(2\theta_m) \frac{(n_e^{2\omega})^{-2} - (n_o^{2\omega})^{-2}}{2(2n_o^{2\omega})^{-3}} (\theta - \theta_m).$$

证: 该过程为负单轴晶体的 I 类相位匹配, 匹配条件为

$$n_o^\omega = n_e^{2\omega}(\theta_m). \quad (19)$$

相位失配为

$$\Delta k = 2 \frac{n_o^\omega \omega}{c} - \frac{n_e^{2\omega}(\theta) 2\omega}{c} = - \frac{2\omega}{c} \frac{\partial n_e^{2\omega}(\theta)}{\partial \theta} (\theta - \theta_m), \quad (20)$$

其中

$$\frac{[n_e^{2\omega}(\theta)]^2 \cos^2 \theta}{[n_o^{2\omega}]^2} + \frac{[n_e^{2\omega}(\theta)]^2 \sin^2 \theta}{[n_e^{2\omega}]^2} = 1, \quad (21)$$

$$\Rightarrow n_e^{2\omega}(\theta) = \frac{n_o^{2\omega} n_e^{2\omega}}{\sqrt{[n_e^{2\omega}]^2 \cos^2 \theta + [n_o^{2\omega}]^2 \sin^2 \theta}}, \quad (22)$$

$$\begin{aligned} \Rightarrow \frac{\partial n_e^{2\omega}}{\partial \theta} &= - \frac{1}{2} n_o^{2\omega} n_e^{2\omega} \{ [n_e^{2\omega} \cos \theta]^2 + [n_o^{2\omega} \sin \theta]^2 \}^{-3/2} \{ [n_o^{2\omega}]^2 - [n_e^{2\omega}]^2 \} \sin 2\theta \\ &= - \frac{1}{2} [n_o^{2\omega}]^{-2} [n_e^{2\omega}]^{-2} \left\{ \frac{\cos^2 \theta}{[n_o^{2\omega}]^2} + \frac{\sin^2 \theta}{[n_e^{2\omega}]^2} \right\}^{-3/2} \{ [n_o^{2\omega}]^2 - [n_e^{2\omega}]^2 \} \sin 2\theta \\ &= - \frac{1}{2} [n_o^{2\omega}]^{-2} [n_e^{2\omega}]^{-2} [n_e^{2\omega}(\theta)]^{-3/2} \{ [n_o^{2\omega}]^2 - [n_e^{2\omega}]^2 \} \sin 2\theta, \end{aligned}$$

由于 $n_o^\omega = n_e^{2\omega}(\theta_m)$, 故

$$\begin{aligned} \frac{\partial n_e^{2\omega}(\theta_m)}{\partial \theta} &= - \frac{1}{2} [n_o^{2\omega}]^{-2} [n_e^{2\omega}]^{-2} [n_e^{2\omega}(\theta_m)]^{-3/2} \{ [n_o^{2\omega}]^2 - [n_e^{2\omega}]^2 \} \sin 2\theta_m \\ &= - \frac{1}{2} [n_o^{2\omega}]^{-2} [n_e^{2\omega}]^{-2} [n_o^\omega]^{-3} \{ [n_o^{2\omega}]^2 - [n_e^{2\omega}]^2 \} \sin 2\theta_m \\ &= - \frac{1}{2} \frac{[n_e^{2\omega}]^2 - [n_o^{2\omega}]^2}{[n_o^\omega]^3} \sin 2\theta_m. \end{aligned}$$

将上式代回相位适配表达式中得

$$\Delta k = \frac{2\omega}{c} \frac{[n_e^{2\omega}]^{-2} - [n_o^{2\omega}]^{-2}}{2[n_o^\omega]^3} \sin(2\theta_m) (\theta - \theta_m), \quad (23)$$

故

$$\Delta k(\theta)L|_{\theta=\theta_m} = \frac{2\omega L}{c} \sin(2\theta_m) \frac{(n_e^{2\omega})^{-2} - (n_o^{2\omega})^{-2}}{2(2n_o^{2\omega})^{-3}} (\theta - \theta_m). \quad (24)$$

□

第 6 题 得分: _____. 参量振荡器可以看作是一种新型的激光器, 利用它可以实现频率的调谐输出, 请以负单轴 II 类晶体为例, 分析角度调谐输出.

解: 负单轴晶体的 II 类相位匹配中, $o + e \rightarrow e$, 相位匹配条件为

$$k_3 = k_1 + k_2, \quad (25)$$

$$\Rightarrow n_e^{\omega_3}(\theta_m) \omega_3 = n_o^{\omega_1} \omega_1 + n_e^{\omega_2}(\theta) \omega_2, \quad (26)$$

若将晶体旋转至 $\theta_m + \Delta\theta$, 则输出频率变为 $\omega_1 + \Delta\omega$ 和 $\omega_2 + \Delta\omega$, 上式变为

$$\begin{aligned} & \omega_3 \left[n_e^{\omega_3}(\theta_m) + \frac{\partial n_e^{\omega_3}(\theta_m)}{\partial \theta} \Delta\theta \right] \\ &= \left(n_o^{\omega_1} + \frac{\partial n_o^{\omega_1}}{\partial \omega} \Delta\omega \right) (\omega_1 + \Delta\omega) + \left[n_e^{\omega_2}(\theta_m) + \frac{\partial n_e^{\omega_2}(\theta)}{\partial \theta} \Delta\theta - \frac{\partial n_e^{\omega_2}(\theta_m)}{\partial \omega} \Delta\omega \right] (\omega_2 + \Delta\omega_2). \end{aligned} \quad (27)$$

以上两式相减并略去二阶小量得

$$\omega_3 \frac{\partial n_e^{\omega_3}(\theta_m)}{\partial \theta} \Delta\theta = n_o^{\omega_1} \Delta\omega + \omega_1 \frac{\partial n_o^{\omega_1}}{\partial \omega} \Delta\omega + n_e^{\omega_2}(\theta_m) \Delta\omega + \omega_2 \frac{\partial n_e^{\omega_2}(\theta_m)}{\partial \theta} \Delta\theta - \omega_2 \frac{\partial n_e^{\omega_2}(\theta_m)}{\partial \omega} \Delta\omega \quad (28)$$

$$\Rightarrow \frac{\Delta\omega}{\Delta\theta} = \frac{\omega_3 \frac{\partial n_e^{\omega_3}(\theta_m)}{\partial \theta} - \omega_2 \frac{\partial n_e^{\omega_2}(\theta_m)}{\partial \theta}}{n_o^{\omega_1} + n_e^{\omega_2}(\theta) + \omega_1 \frac{\partial n_o^{\omega_1}}{\partial \omega} - \omega_2 \frac{\partial n_e^{\omega_2}(\theta_m)}{\partial \omega}}, \quad (29)$$

其中

$$\frac{n_e^{\omega}(\theta_m) \cos^2 \theta_m}{n_o^2(\omega)} + \frac{n_e^{\omega}(\theta_m) \sin^2 \theta_m}{n_e^2(\omega)} = 1, \quad (30)$$

$$\Rightarrow n_e^{\omega}(\theta_m) = \frac{n_o(\omega) n_e(\omega)}{\sqrt{n_e^2(\omega) \cos^2 \theta_m + n_o^2(\omega) \sin^2 \theta_m}}. \quad (31)$$

□

第 7 题 得分: _____. 请以负单轴晶体为例, 按以下条件分别推导 I 型和 II 型二次谐波的匹配带宽:

- 1) 简并共线;
- 2) 简并非共线,

并比较 I 型和 II 型的带宽特点.

证: 对 I 型相位匹配下的二次谐波, $o + o \rightarrow e$. 简并共线情况下, 相位匹配条件为

$$\Delta k = 2k_o(\omega_0) - k_e(2\omega_0) = 2 \frac{n_o^{\omega_0} \omega_0}{c} - \frac{n_e^{2\omega_0}(\theta) 2\omega_0}{c} = \frac{2\omega_0}{c} [n_o^{\omega_0} - n_e^{2\omega_0}(\theta_m)] = 0. \quad (32)$$

当基频变化 $\Delta\omega$ 时, 相位失配为

$$\Delta k = \frac{2\omega_0}{c} \left[\frac{\partial n_o^{\omega_0}}{\partial \omega} \Delta\omega - \frac{\partial n_e^{2\omega_0}(\theta_m)}{\partial \omega} 2\Delta\omega \right]. \quad (33)$$

令

$$\frac{\Delta k L}{2} = \frac{\pi}{2}, \quad (34)$$

得带宽为

$$\Delta\omega = \frac{2\pi c}{\omega_0 L} \left[\frac{\partial n_o^{\omega_0}}{\partial \omega_0} - 2 \frac{\partial n_e^{2\omega_0}(\theta_m)}{\partial \omega} \right]^{-1}. \quad (35)$$

非简并共线情况下, 设 $\mathbf{k}_{o,1}(\omega_0)$ 与 $\mathbf{k}_{o,2}(\omega_0)$ 与 $\mathbf{k}_e(2\omega_0)$ 夹角分别为 θ_1 和 θ_2 , 沿着 $\mathbf{k}_e(2\omega_0)$ 方向相位匹配条件为

$$\begin{aligned} \Delta k &= k_{o,1}(\omega_0) \cos \theta_1 + k_{o,2}(\omega_0) \cos \theta_2 - k_e(2\omega_0) = \frac{n_o^{\omega_0} \omega_0}{c} \cos \theta_1 + \frac{n_o^{\omega_0} \omega_0}{c} \cos \theta_2 - \frac{n_e^{2\omega_0} 2\omega_0}{c} \\ &= \frac{\omega_0}{c} [n_o^{\omega_0} (\cos \theta_1 + \cos \theta_2) - 2n_e^{2\omega_0}] = 0. \end{aligned} \quad (36)$$

当基频变化 $\Delta\omega$ 时, 相位失配为

$$\Delta k = \frac{\omega_0}{c} \left[\frac{\partial n_o^{\omega_0}}{\partial \omega} \Delta\omega (\cos \theta_1 + \cos \theta_2) - 2 \frac{\partial n_e^{2\omega_0}}{\partial \omega} 2\Delta\omega \right]. \quad (37)$$

令

$$\frac{\Delta k L}{2} = \frac{\pi}{2}, \quad (38)$$

得带宽为

$$\Delta\omega = \frac{\pi c}{\omega_0 L} \left[\frac{\partial n_o^{\omega_0}}{\partial \omega} (\cos \theta_1 + \cos \theta_2) - 4 \frac{\partial n_e^{2\omega_0}}{\partial \omega} \right]^{-1}. \quad (39)$$

对 II 型相位匹配下的二次谐波, $o + e \rightarrow e$. 简并共线情况下, 相位匹配条件为

$$\Delta k = k_o(\omega_0) + k_e(\omega_0) - k_e(2\omega_0) = \frac{n_o^{\omega_0} \omega_0}{c} + \frac{n_e^{\omega_0}(\theta_m) \omega_0}{c} - \frac{n_e^{\omega_0}(\theta_m) 2\omega_0}{c} = \frac{\omega_0}{c} [n_o^{\omega_0} + n_e^{\omega_0}(\theta_m) - 2n_e^{2\omega_0}(\theta_m)] = 0. \quad (40)$$

当基频变化 $\Delta\omega$ 时, 相位失配为

$$\Delta k = \frac{\omega_0}{c} \left[\frac{\partial n_o^{\omega_0}}{\partial \omega} \Delta\omega + \frac{\partial n_e^{\omega_0}(\theta_m)}{\partial \omega} \Delta\omega - 2 \frac{\partial n_e^{2\omega_0}(\theta_m)}{\partial \omega} 2\Delta\omega \right]. \quad (41)$$

令

$$\frac{\Delta k L}{2} = \frac{\pi}{2}, \quad (42)$$

得带宽为

$$\Delta\omega = \frac{\pi c}{\omega_0 L} \left[\frac{\partial n_o^{\omega_0}}{\partial \omega} + \frac{\partial n_e^{\omega_0}(\theta_m)}{\partial \omega} - 4 \frac{\partial n_e^{2\omega_0}(\theta_m)}{\partial \omega} \right]^{-1}. \quad (43)$$

非简并共线情况下, 设 $\mathbf{k}_o(\omega_0)$, $\mathbf{k}_e(\omega_0)$ 与 $\mathbf{k}_e(2\omega_0)$ 夹角分别为 θ_1 和 θ_2 , 沿着 $\mathbf{k}_e(2\omega_0)$ 方向相位匹配条件为

$$\begin{aligned} \Delta k &= k_o(\omega_0) \cos \theta_1 + k_e(\omega_0) \cos \theta_2 - k_e(2\omega_0) = \frac{n_o^{\omega_0} \omega_0}{c} \cos \theta_1 + \frac{n_e^{\omega_0} \omega_0}{c} \cos \theta_2 - \frac{n_e^{2\omega_0} 2\omega_0}{c} \\ &= \frac{\omega_0}{c} [n_o^{\omega_0} \cos \theta_1 + n_e^{\omega_0} \cos \theta_2 - 2n_e^{2\omega_0}] = 0. \end{aligned} \quad (44)$$

当基频变化 $\Delta\omega$ 时, 相位失配为

$$\Delta k = \frac{\omega_0}{c} \left[\frac{\partial n_o^{\omega_0}}{\partial \omega} \Delta\omega \cos \theta_1 + \frac{\partial n_e^{\omega_0}}{\partial \omega} \Delta\omega \cos \theta_2 - 2 \frac{\partial n_e^{2\omega_0}}{\partial \omega} \Delta\omega \right]. \quad (45)$$

令

$$\frac{\Delta k L}{2} = \frac{\pi}{2}, \quad (46)$$

得带宽为

$$\Delta\omega = \frac{\pi c}{\omega_0 L} \left[\frac{\partial n_o^{\omega_0}}{\partial \omega} \cos \theta_1 + \frac{\partial n_e^{\omega_0}}{\partial \omega} \cos \theta_2 - 4 \frac{\partial n_e^{2\omega_0}}{\partial \omega} \right]^{-1}. \quad (47)$$

□