作业一
(随堂测验)

姓名：陈　稼　霖
学号：SA21038052
成绩：__________

第 1 题 得分: _________. 利用 KDP 晶体进行参量放大, 若其中有两个光波是 e 光, 第三个光波是 o 光, 试推导相位匹配角公式, 其中信号、空闲和泵浦光中哪一个为寻常光? 令 $\omega_3 = 10\,000\,\mathrm{cm}^{-1}$, $\omega_1 = \omega_2 = 5000\,\mathrm{cm}^{-1}$ 能否实现这种形式的相位匹配?

解: KDP 为负单轴的晶体, 故为 II 类相位匹配 $(e + o \to e)$.

要实现相位匹配, 需满足

$$\frac{1}{2}[n_e^{\omega}(\theta) + n_o^{\omega}] = n_e^{2\omega}(\theta_m), \tag{1}$$

其中传播方向与光轴夹角为 θ 的非常光的折射率满足

$$\frac{1}{[n_e^{\omega\text{ 或 }2\omega}(\theta)]^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{[n_e^{\omega\text{ 或 }2\omega}(\theta)]^2}. \tag{2}$$

由于 $n_o^{\omega} = 1.4596$, $n_e^{\omega} = 1.4606$, $n_e^{2\omega} = 1.4999$, $\frac{1}{2}[n_e^{\omega}(\pi/2) + n_o^{\omega}] < n_e^{2\omega}(\pi/2)$, 故无角度满足相位匹配条件, 无法实现相位匹配. □

第 2 题 得分: _________. 试证明外加直流电场 $E_x = E_{0j}$ 的 KDP 晶体, 光波在 zox 面内、与 x 轴成 45° 方向传播时的电光延迟为下式. 其中 l 为沿光传播方向上的晶体长度, d 为沿外加电场的晶体厚度, U_y 为外加电压.

$$\Delta\varphi \approx \frac{2\pi l}{\lambda}\left[\frac{\sqrt{2}n_o n_e}{\sqrt{n_o^2 + n_e^2}} - n_o + \sqrt{2}\left[\frac{1}{n_o^2} + \frac{1}{n_e^2}\right]^{-3/2}\gamma_{41}U_y\frac{l}{d}\right].$$

证: KDP 为负单轴晶体, 故无外加电场下, KDP 折射率椭球方程为

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1, \tag{3}$$

光波在 zox 面内、与 x 轴成 45° 方向传播时, 垂直于 zox 面的偏振分量感受到的折射率为 $n_\perp = n_o$, 平行于 zox 面的折射率分量感受到的折射率满足

$$\frac{n_\parallel^2\sin^2(-45^\circ)}{n_o^2} + \frac{n_\parallel^2\cos^2(-45^\circ)}{n_e^2} = 1, \tag{4}$$

$$\Longrightarrow n_\parallel = \frac{\sqrt{2}n_o n_e}{\sqrt{n_o^2 + n_e^2}}. \tag{5}$$

KDP 的电光系数为

$$[\gamma_{ij}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_{41} & 0 & 0 \\ 0 & \gamma_{41} & 0 \\ 0 & 0 & \gamma_{63} \end{bmatrix}, \tag{6}$$

故外加电场 $E_x = E_{0j} = \frac{U_y}{d}$ 下, KDP 折射率椭球系数变化量为

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n_2}\right)_2 \\ \Delta\left(\frac{1}{n_2}\right)_3 \\ \Delta\left(\frac{1}{n_2}\right)_4 \\ \Delta\left(\frac{1}{n_2}\right)_5 \\ \Delta\left(\frac{1}{n_2}\right)_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}\begin{bmatrix} E_{0j} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \gamma_{41}E_{0j} \\ 0 \end{bmatrix}, \tag{7}$$

折射率椭球方程为

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}+2\gamma_{41}E_{0j}xz=1, \tag{8}$$

光波在 zox 面内、与 x 轴成 $45°$ 方向传播时, 垂直于 zox 面的偏振分量感受到的折射率满足

$$\frac{[n'_\perp]^2}{n_o^2}=1, \tag{9}$$

$$\Longrightarrow n'_\perp=n_o, \tag{10}$$

平行于 zox 面的偏振分量感受到的折射率满足

$$\frac{[n'_\parallel]^2\sin^2(-45°)}{n_o^2}+\frac{[n'_\parallel]^2\cos^2(-45°)}{n_e^2}+2\gamma_{41}E_{0j}n'_\parallel\sin(-45°)n'_\parallel\cos(-45°)=1, \tag{11}$$

$$\Longrightarrow n'_\parallel=\frac{1}{\sqrt{\frac{n_o^2+n_e^2}{2n_o^2n_e^2}-r_{41}E_{0j}}}=\frac{\sqrt{2}n_on_e}{\sqrt{n_e^2+n_o^2}}\frac{1}{\sqrt{1-\frac{2n_o^2n_e^2}{n_e^2+n_o^2}r_{41}E_{0j}}}\approx\sqrt{2}\frac{n_on_e}{\sqrt{n_o^2+n_e^2}}+\sqrt{2}\left(\frac{n_o^2n_e^2}{n_o^2+n_e^2}\right)^{3/2}\gamma_{41}E_{0j}, \tag{12}$$

故电光延迟为

$$\Delta\varphi=\frac{2\pi l(n'_\parallel-n'_\perp)}{\lambda}\approx\frac{2\pi l}{\lambda}\left[\frac{\sqrt{2}n_on_e}{n_o^2+n_e^2}-n_o+\sqrt{2}\left[\frac{1}{n_o^2}+\frac{1}{n_e^2}\right]^{-3/2}\gamma_{41}\frac{U_y}{d}\right]. \tag{13}$$

□

第 3 题 得分: __________. 考虑 $\mathrm{LiNbO_3}$ 浸提中的 II 型 $(o+e\to e)$ 相位匹配下的共线传播倍频过程 $\omega+\omega\to 2\omega$:

1) 设光波矢沿 (θ,φ), 求出此时有效非线性系数 d_{eff} 的表达式. 注: 已知 $\mathrm{LiNbO_3}$ 晶体 (负单轴晶体) 的非线性系数矩阵为

$$\begin{Bmatrix}0&0&0&0&d_{15}&-d_{22}\\-d_{22}&d_{22}&0&d_{15}&0&0\\d_{31}&d_{31}&d_{33}&0&0&0\end{Bmatrix}.$$

2) 用折射率曲面的方法画出实现相位匹配的示意图.

3) 若要得到最佳倍频输出, 问光波矢的方向 (θ,φ) 应取何值?

解: 1)

$$\boldsymbol{a}^o=\begin{bmatrix}\sin\varphi\\-\cos\varphi\\0\end{bmatrix}, \tag{14}$$

$$\boldsymbol{a}^e=\begin{bmatrix}-\cos\theta\cos\varphi\\-\cos\theta\sin\varphi\\\sin\theta\end{bmatrix}, \tag{15}$$

$$\boldsymbol{a}^o\boldsymbol{a}^e=\begin{bmatrix}-\cos\theta\sin\varphi\cos\varphi\\\cos\theta\sin\varphi\cos\varphi\\0\\-\sin\theta\cos\varphi\\\sin\theta\sin\varphi\\\cos\theta\cos2\varphi\end{bmatrix}. \tag{16}$$

II 型相位匹配下, 有效非线性系数为

$$
\begin{aligned}
d_{eff} =&\boldsymbol{a}^e\cdot\begin{bmatrix}0&0&0&0&d_{15}&-d_{22}\\-d_{22}&d_{22}&0&d_{15}&0&0\\d_{31}&d_{31}&d_{33}&0&0&0\end{bmatrix}\boldsymbol{a}^o\boldsymbol{a}^e\\
=&\begin{bmatrix}-\cos\theta\cos\varphi&-\cos\theta\sin\varphi&\sin\theta\end{bmatrix}\begin{bmatrix}0&0&0&0&d_{15}&-d_{22}\\-d_{22}&d_{22}&0&d_{15}&0&0\\d_{31}&d_{31}&d_{33}&0&0&0\end{bmatrix}\begin{bmatrix}-\cos\theta\sin\varphi\cos\varphi\\\cos\theta\sin\varphi\cos\varphi\\0\\-\sin\theta\cos\varphi\\\sin\theta\sin\varphi\\\cos\theta\cos2\varphi\end{bmatrix}\\
=&\begin{bmatrix}-\cos\theta\cos\varphi&-\cos\theta\sin\varphi&\sin\theta\end{bmatrix}\begin{bmatrix}d_{15}\sin\theta\sin\varphi-d_{22}\cos\theta\cos2\varphi\\2d_{22}\sin\theta\sin\varphi\cos\varphi-d_{15}\sin\theta\cos\varphi\\0\end{bmatrix}\\
=&d_{22}\cos^2\theta\cos^2\cos\phi(\cos2\varphi-2\sin^2\phi)=d_{22}\cos^2\theta\cos^2\cos\phi(1-4\sin^2\phi).
\end{aligned}
\tag{17}
$$

2) 如图 1.

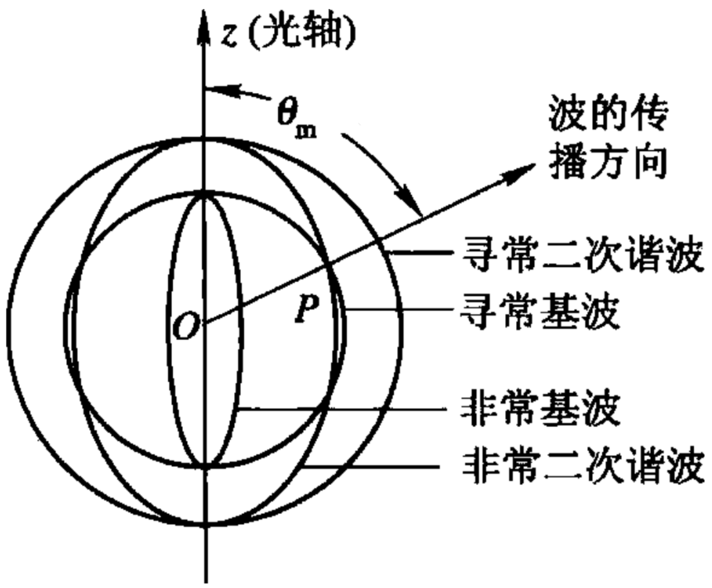

图 1: II 型相位匹配示意图.

3) 为得到最佳倍频输出, 取 θ_m 满足 $\frac{1}{2}[n_e^\omega(\theta)+n_o^\omega]=n_e^{2\omega}(\theta)$, 在此基础上取 φ 使 $|d_{eff}|$ 极大.

$$\frac{\partial d_{eff}}{\partial\varphi}\propto-\sin\varphi(1-4\sin^2\varphi)-8\cos^2\varphi\sin\varphi=-3\sin\varphi(1-4\sin^2\varphi)=0, \tag{18}$$

$$\Longrightarrow\varphi=0,\pm\frac{\pi}{3},\pm\frac{2\pi}{3}. \tag{19}$$

□

第 4 题 得分: _________. 试证明在非共线相位匹配的条件下, 为获得远红外差频光 (ω_1、$\omega_2\gg\omega_3$), 晶体必须具有反常色散特性.

证: 为获得远红外差频光, $\omega_1-\omega_2=\omega_3$, $n_1\approx n_2\equiv n$, $n_3\approx n+\Delta n$. 由余弦定理, $\boldsymbol{k}_1$ 和 $\boldsymbol{k}_2$ 之间的夹角 θ 满足

$$\cos\theta=\frac{k_1^2+k^2-k_3^2}{2k_1k_2}=\frac{(n_1\omega_1)^2+(n_2\omega_2)^2-(n_3\omega_3)^2}{2n_1\omega_1n_2\omega_2}\approx\frac{n^2(\omega_1^2+\omega_2^2)-(n+\Delta n)^2(\omega_1-\omega_2)^2}{2n^2\omega_1\omega_2}\approx 1-\frac{\Delta n}{n}\frac{(\omega_1-\omega_2)^2}{\omega_1\omega_2}. \tag{20}$$

由 $\cos\theta<1$ 得 $\Delta n>0$, 即反常色散. □

第 5 题 得分: _________. 试证明, 如果二次谐波产生过程的基频光 ω 是寻常光, 倍频光 2ω 是非常光, θ_m 是其相位匹配角, 则有

$$\Delta k(\theta)L|_{\theta=\theta_m}=\frac{2\omega L}{c}\sin(2\theta_m)\frac{(n_e^{2\omega})^{-2}-(n_o^{2\omega})^{-2}}{2(2n_o^{2\omega})^{-3}}(\theta-\theta_m).$$

证: 该过程为负单轴晶体的 I 类相位匹配, 匹配条件为

$$n_o^{\omega}=n_e^{2\omega}(\theta_m). \tag{21}$$

相位失配为

$$\Delta k=2\frac{n_o^{\omega}\omega}{c}-\frac{n_e^{2\omega}(\theta)2\omega}{c}=-\frac{2\omega}{c}\frac{\partial n_e^{2\omega}(\theta)}{\partial\theta}(\theta-\theta_m), \tag{22}$$

其中

$$\frac{[n_e^{2\omega}(\theta)]^2\cos^2\theta}{[n_o^{2\omega}]^2}+\frac{[n_e^{2\omega}(\theta)]^2\sin^2\theta}{[n_e^{2\omega}]^2}=1, \tag{23}$$

$$\Longrightarrow n_e^{2\omega}(\theta)=\frac{n_o^{2\omega}n_e^{2\omega}}{\sqrt{[n_e^{2\omega}]^2\cos^2\theta+[n_o^{2\omega}]^2\sin^2\theta}}, \tag{24}$$

$$\begin{aligned}\Longrightarrow\frac{\partial n_e^{2\omega}}{\partial\theta}&=-\frac{1}{2}n_o^{2\omega}n_e^{2\omega}\{[n_e^{2\omega}\cos\theta]^2+[n_o^{2\omega}\sin\theta]^2\}^{-3/2}\{[n_o^{2\omega}]^2-[n_e^{2\omega}]^2\}\sin 2\theta\\&=-\frac{1}{2}[n_o^{2\omega}]^{-2}[n_e^{2\omega}]^{-2}\left\{\frac{\cos^2\theta}{[n_o^{2\omega}]^2}+\frac{\sin^2\theta}{[n_e^{2\omega}]^2}\right\}^{-3/2}\{[n_o^{2\omega}]^2-[n_e^{2\omega}]^2\}\sin 2\theta\\&=-\frac{1}{2}[n_o^{2\omega}]^{-2}[n_e^{2\omega}]^{-2}[n_e^{2\omega}(\theta)]^{-3/2}\{[n_o^{2\omega}]^2-[n_e^{2\omega}]^2\}\sin 2\theta,\end{aligned}$$

由于 $n_o^{\omega}=n_e^{2\omega}(\theta_m)$, 故

$$\begin{aligned}\frac{\partial n_e^{2\omega}(\theta_m)}{\partial\theta}&=-\frac{1}{2}[n_o^{2\omega}]^{-2}[n_e^{2\omega}]^{-2}[n_e^{2\omega}(\theta_m)]^{-3/2}\{[n_o^{2\omega}]^2-[n_e^{2\omega}]^2\}\sin 2\theta_m\\&=-\frac{1}{2}[n_o^{2\omega}]^{-2}[n_e^{2\omega}]^{-2}[n_o^{\omega}]^{-3}\{[n_o^{2\omega}]^2-[n_e^{2\omega}]^2\}\sin 2\theta_m\\&=-\frac{1}{2}\frac{[n_e^{2\omega}]^2-[n_o^{2\omega}]^2}{[n_o^{\omega}]^3}\sin 2\theta_m.\end{aligned}$$

将上式代回相位适配表达式中得

$$\Delta k=\frac{2\omega}{c}\frac{[n_e^{2\omega}]^{-2}-[n_o^{2\omega}]^{-2}}{2[n_o^{\omega}]^3}\sin(2\theta_m)(\theta-\theta_m), \tag{25}$$

故

$$\Delta k(\theta)L|_{\theta=\theta_m}=\frac{2\omega L}{c}\sin(2\theta_m)\frac{(n_e^{2\omega})^{-2}-(n_o^{2\omega})^{-2}}{2(2n_o^{2\omega})^{-3}}(\theta-\theta_m). \tag{26}$$

□

第 6 题 得分: ________. 参量振荡器可以看作是一种新型的激光器, 利用它可以实现频率的调谐输出, 请以负单轴 II 类晶体为例, 分析角度调谐输出.

解: 负单轴晶体的 II 类相位匹配中, $o+e\to e$, 相位匹配条件为

$$k_3 = k_1 + k_2, \tag{27}$$

$$\Longrightarrow n_e^{\omega_3}(\theta_m)\omega_3 = n_o^{\omega_1}\omega_1 + n_e^{\omega_2}(\theta)\omega_2, \tag{28}$$

若将晶体旋转至 $\theta_m+\Delta\theta$, 则输出频率变为 $\omega_1+\Delta\omega$ 和 $\omega_2+\Delta\omega$, 上式变为

$$\begin{aligned}&\omega_3\left[n_e^{\omega_3}(\theta_m)+\frac{\partial n_e^{\omega_3}(\theta_m)}{\partial\theta}\Delta\theta\right]\\&=\left(n_o^{\omega_1}+\frac{\partial n_o^{\omega_1}}{\partial\omega}\Delta\omega\right)(\omega_1+\Delta\omega)+\left[n_e^{\omega_2}(\theta_m)+\frac{\partial n_e^{\omega_2}(\theta)}{\partial\theta}\Delta\theta-\frac{\partial n_e^{\omega_2}(\theta_m)}{\partial\omega}\Delta\omega\right](\omega_2+\Delta\omega_2).\end{aligned} \tag{29}$$

以上两式相减并略去二阶小量得

$$\omega_3\frac{\partial n_e^{\omega_3}(\theta_m)}{\partial\theta}\Delta\theta = n_o^{\omega_1}\Delta\omega+\omega_1\frac{\partial n_o^{\omega_1}}{\partial\omega}\Delta\omega+n_e^{\omega_2}(\theta_m)\Delta\omega+\omega_2\frac{\partial n_e^{\omega_2}(\theta_m)}{\partial\theta}\Delta\theta-\omega_2\frac{\partial n_e^{\omega_2}(\theta_m)}{\partial\omega}\Delta\omega \tag{30}$$

$$\Longrightarrow \frac{\Delta\omega}{\Delta\theta}=\frac{\omega_3\frac{\partial n_e^{\omega_3}(\theta_m)}{\partial\theta}-\omega_2\frac{\partial\theta_e^{\omega_2}(\theta_m)}{\partial\theta}}{n_o^{\omega_1}+n_e^{\omega_2}(\theta)+\omega_1\frac{\partial n_o^{\omega_1}}{\partial\omega}-\omega_2\frac{\partial n_e^{\omega_2}(\theta_m)}{\partial\omega}}, \tag{31}$$

其中

$$\frac{n_e^{\omega}(\theta_m)\cos^2\theta_m}{n_o^2(\omega)}+\frac{n_e^{\omega}(\theta_m)\sin^2\theta_m}{n_e^2(\omega)}=1, \tag{32}$$

$$\Longrightarrow n_e^{\omega}(\theta_m)=\frac{n_o(\omega)n_e(\omega)}{\sqrt{n_e^2(\omega)\cos^2\theta_m+n_o^2(\omega)\sin^2\theta_m}}. \tag{33}$$

□

第 7 题 得分: ________. 请以负单轴晶体为例, 按以下条件分别推导 I 型和 II 型二次谐波的匹配带宽:

1) 简并共线;

2) 简并非共线,

并比较 I 型和 II 型的带宽特点.

证: 对 I 型相位匹配下的二次谐波, $o+o\to e$. 简并共线情况下, 相位匹配条件为

$$\Delta k = 2k_o(\omega_0)-k_e(2\omega_0)=2\frac{n_o^{\omega_0}\omega_0}{c}-\frac{n_e^{2\omega_0}(\theta)2\omega_0}{c}=\frac{2\omega_0}{c}[n_o^{\omega_0}-n_e^{2\omega_0}(\theta_m)]=0. \tag{34}$$

当基频变化 $\Delta\omega$ 时, 相位失配为

$$\Delta k=\frac{2\omega_0}{c}\left[\frac{\partial n_o^{\omega_0}}{\partial\omega}\Delta\omega-\frac{\partial n_e^{2\omega_0}(\theta_m)}{\partial\omega}2\Delta\omega\right]. \tag{35}$$

令

$$\frac{\Delta kL}{2}=\frac{\pi}{2}, \tag{36}$$

得带宽为

$$\Delta\omega=\frac{2\pi c}{\omega_0L}\left[\frac{\partial n_o^{\omega_0}}{\partial\omega_0}-2\frac{\partial n_e^{2\omega_0}(\theta_m)}{\partial\omega}\right]^{-1}. \tag{37}$$

非简并共线情况下, 设 $\boldsymbol{k}_{o,1}(\omega_0)$ 与 $\boldsymbol{k}_{o,2}(\omega_0)$ 与 $\boldsymbol{k}_e(2\omega_0)$ 夹角分别为 θ_1 和 θ_2, 沿着 $\boldsymbol{k}_e(2\omega_0)$ 方向相位匹配条件为

$$\begin{aligned}\Delta k =&k_{o,1}(\omega_0)\cos\theta_1+k_{o,2}(\omega_0)\cos\theta_2-k_e(2\omega_0)=\frac{n_o^{\omega_0}\omega_0}{c}\cos\theta_1+\frac{n_o^{\omega_0}\omega_0}{c}\cos\theta_2-\frac{n_e^{2\omega_0}2\omega_0}{c}\\ =&\frac{\omega_0}{c}[n_o^{\omega_0}(\cos\theta_1+\cos\theta_2)-2n_e^{2\omega_0}]=0.\end{aligned}\tag{38}$$

当基频变化 $\Delta\omega$ 时, 相位失配为

$$\Delta k=\frac{\omega_0}{c}\left[\frac{\partial n_o^{\omega_0}}{\partial\omega}\Delta\omega(\cos\theta_1+\cos\theta_2)-2\frac{\partial n_e^{2\omega_0}}{\partial\omega}2\Delta\omega\right].\tag{39}$$

令

$$\frac{\Delta kL}{2}=\frac{\pi}{2},\tag{40}$$

得带宽为

$$\Delta\omega=\frac{\pi c}{\omega_0L}\left[\frac{\partial n_o^{\omega_0}}{\partial\omega}(\cos\theta_1+\cos\theta_2)-4\frac{\partial n_e^{2\omega_0}}{\partial\omega}\right]^{-1}.\tag{41}$$

对 II 型相位匹配下的二次谐波, $o+e\to e$. 简并共线情况下, 相位匹配条件为

$$\Delta k=k_o(\omega_0)+k_e(\omega_0)-k_e(2\omega_0)=\frac{n_o^{\omega_0}\omega_0}{c}+\frac{n_e^{\omega_0}(\theta_m)\omega_0}{c}-\frac{n_e^{\omega_0}(\theta_m)2\omega}{c}=\frac{\omega_0}{c}[n_o^{\omega_0}+n_e^{\omega_0}(\theta_m)-2n_e^{2\omega_0}(\theta_m)]=0.\tag{42}$$

当基频变化 $\Delta\omega$ 时, 相位失配为

$$\Delta k=\frac{\omega_0}{c}\left[\frac{\partial n_o^{\omega_0}}{\partial\omega}\Delta\omega+\frac{\partial n_e^{\omega_0}(\theta_m)}{\partial\omega}\Delta\omega-2\frac{\partial n_e^{2\omega_0}(\theta_m)}{\partial\omega}2\Delta\omega\right].\tag{43}$$

令

$$\frac{\Delta kL}{2}=\frac{\pi}{2},\tag{44}$$

得带宽为

$$\Delta\omega=\frac{\pi c}{\omega_0L}\left[\frac{\partial n_o^{\omega_0}}{\partial\omega}+\frac{\partial n_e^{\omega_0}(\theta_m)}{\partial\omega}-4\frac{\partial n_e^{2\omega_0}(\theta_m)}{\partial\omega}\right]^{-1}.\tag{45}$$

非简并共线情况下, 设 $\boldsymbol{k}_o(\omega_0)$, $\boldsymbol{k}_e(\omega_0)$ 与 $\boldsymbol{k}_e(2\omega_0)$ 夹角分别为 θ_1 和 θ_2, 沿着 $\boldsymbol{k}_e(2\omega_0)$ 方向相位匹配条件为

$$\begin{aligned}\Delta k =&k_o(\omega_0)\cos\theta_1+k_e(\omega_0)\cos\theta_2-k_e(2\omega_0)=\frac{n_o^{\omega_0}\omega_0}{c}\cos\theta_1+\frac{n_e^{\omega_0}\omega_0}{c}\cos\theta_2-\frac{n_e^{2\omega_0}2\omega_0}{c}\\ =&\frac{\omega_0}{c}[n_o^{\omega_0}\cos\theta_1+n_e^{\omega_0}\cos\theta_2-2n_e^{2\omega_0}]=0.\end{aligned}\tag{46}$$

当基频变化 $\Delta\omega$ 时, 相位失配为

$$\Delta k=\frac{\omega_0}{c}\left[\frac{\partial n_o^{\omega_0}}{\partial\omega}\Delta\omega\cos\theta_1+\frac{\partial n_e^{\omega_0}}{\partial\omega}\Delta\omega\cos\theta_2-2\frac{\partial n_e^{2\omega_0}}{\partial\omega}\Delta\omega\right].\tag{47}$$

令

$$\frac{\Delta kL}{2}=\frac{\pi}{2},\tag{48}$$

得带宽为

$$\Delta\omega=\frac{\pi c}{\omega_0L}\left[\frac{\partial n_o^{\omega_0}}{\partial\omega}\cos\theta_1+\frac{\partial n_e^{\omega_0}}{\partial\omega}\cos\theta_2-4\frac{\partial n_e^{2\omega_0}}{\partial\omega}\right]^{-1}.\tag{49}$$

□