PHYS2202 Nonlinear Optics
Problem Set 8: Make-up assignment
Due at 17:00 Thursday, July 2, 2020

This problem set is optional and intended for students who have not turned in all assignments or have done poorly on the previous problem sets. Your percentage score on this problem set will replace your lowest score among the previous problem sets.

(20 points)
When a medium exhibiting the optical Kerr-effect (an intensity-dependent change in the refractive index $n = n_0 + n_2 I$) is placed inside a cavity, one introduces an optical feedback to the nonlinear interaction between light and matter. This feedback leads to phenomena where the system is latched into different stable states with the transmitted intensity taking two possible stable values for a given input, where the two states of the system can be pictured as "low" (switched off) and "high" (switched on).

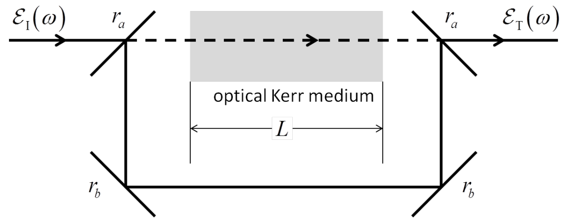

Figure 1

1. First consider the linear medium. For the ring cavity shown in Fig. 1, formulate the equation that relates the transmitted intensity $I_{\mathrm{T}} = \frac{1}{2}\epsilon_0 c \left|E_\omega^T\right|^2$ to the incident intensity $I_{\mathrm{I}} = \frac{1}{2}\epsilon_0 c \left|E_\omega^{\mathrm{inc}}\right|^2$ as a function of the round trip phase acquired by the waves running around the cavity. Assume that the two semitransparent mirrors have a complex amplitude reflectance of r_a each, while the other two mirrors have a reflectance of unity. For simplicity, neglect the reflectance of the optical beam at the surfaces of the linear medium, hence neglecting any back-reflected wave in the ring cavity. You may assume the light to be linearly polarized (TE or TM with respect to the plane of incidence on the mirrors), which will allow you to use a scalar theory of wave propagation.

2. Repeat the preceding for the case where the medium has a Kerr nonlinearity.

3. Part 2 should yield a transcendental equation. Obtain the solutions for the stable states of the cavity (states of constant output I_{T} for a given I_{I}). One approach to solving transcendental equations is the graphical approach. Plot (1) the transmission coefficient $T \equiv I_{\mathrm{T}}/I_{\mathrm{I}}$ of the cavity versus round-trip phase (this is irrespective of the source of the phase evolution) and (2) the dependence of the transmission coefficient on the round-trip phase associated specifically with the constant and nonlinear contributions to the round trip phase.

4. Based on the preceding, explain in words how the obtained relation between transmitted and incident intensities leads to multiple stable states of the output intensity.

5. Starting with zero input intensity and a cavity tuned away from high transmission, plot the output intensity as a function of input intensity as the input intensity gradually rises from 0, through a region with at least two stable solutions of I_{T} for a given I_{I}, into a region of a single stable solution, and then back to zero input intensity. What happens to the light energy contained in the cavity as the system switches from the upper (high I_{T}) to the lower state of transmission?

Such a bistable optical device can be used in optical switching in a manner similar to the electro-optic switch of Problem **??**. Such a device may look as shown in Fig. 2. In principle, the all-optical device may be much faster than the electro-optic device.

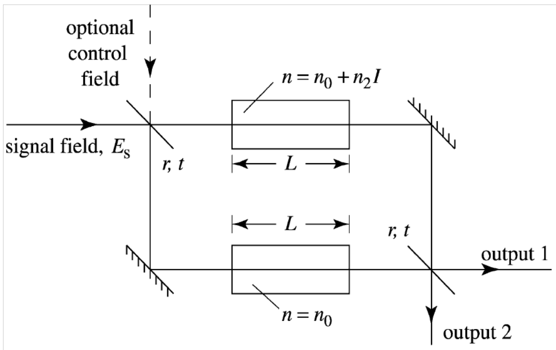

Figure 2