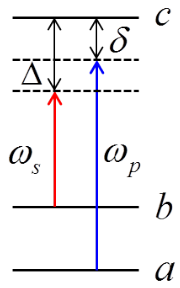
PHYS2202 Nonlinear Optics

Problem Set 4

Due 17:00 on Wednesday, April 15, 2020

1. **(25 points) Electromagnetically induced transparency**
Consider the three-level system represented by the energy diagram below. The excited state, b, is much higher in energy relative to the ground state than the thermal energy ($\hbar\omega_{ba} >> k_B T$) so that we can assume that at equilibrium $\hat{\rho}^{(0)} = |a\rangle\langle a|$. The system is characterized by homogeneous damping rates Γ_{ba}, Γ_{ca}, and Γ_{cb}. Suppose that we use a weak probe pulse at frequency $\omega_p = \omega_{ca} - \delta$ and a strong saturating pump pulse at frequency $\omega_s = \omega_{cb} - \Delta$. (In other words, the probe and pump beams are detuned from resonance by δ and Δ respectively.) Suppose, too, that the detunings are small compared to ω_{ba}: $\delta, \Delta << \omega_{ba}$.



   (a) Using the density-matrix formalism, find the susceptibility associated with the first order *in the probe* response. This requires that we consider the response to infinite order in the pump intensity!

   Hint: although we are discussing the frequency domain and so cannot say that one field is present before another, we are interested in the maximally resonant response. If using a diagrammatic approach, think about what that must mean for the ordering of the actual interactions (not simply when the fields are present but in what order they must interact). If using a non-diagrammatic approach, pay attention to which terms are fully resonant and which are not.

   (b) Assume that $\Gamma_{ba} = 0.01\Gamma_{ca} = 0.01\Gamma_{cb}$. Plot the linear absorption coefficient as a function of probe frequency, ω_s for $\Delta = 0$ in the cases

      i. $\Omega_s = 0$ (this is just the normal linear absorption coefficient)

      ii. $\Omega_s \equiv \left|\frac{\vec{\mu}_{cb}\cdot\vec{E}_s}{\hbar}\right| = 0.5\Gamma_{ca}$

      iii. $\Omega_s \equiv \left|\frac{\vec{\mu}_{cb}\cdot\vec{E}_s}{\hbar}\right| = 5\Gamma_{ca}$