PHYS2202 Nonlinear Optics

Problem Set 5

Due 17:00 on Wednesday, April 29, 2020

1. **(20 points) Photon Echo?**
The Bloch vector model for the optical response of a two-level system gives us a simple, visual picture for understanding the photon echo. A $\pi/2$ pulse at time $t = 0$ followed by a π pulse at time T will produce a rephasing of an inhomogeneously broadened ensemble of oscillators with the appearance of an "echo" at time $2T$ even if $2T >> T_2^*$, where T_2^* is the inhomogeneous dephasing time of the coherence between states.

Suppose that we use a pair of pulses characterized by wave vectors $\vec{k}_1$ and $\vec{k}_2$ and separated by time T (pulse 1 arrives at the sample first) that are weak, corresponding to "tipping angles" $\theta = \int R_n(t)dt << \pi/2$, where $R_n(t)$ is the generalized Rabi flopping frequency of pulse n.

For simplicity, we will make the following assumptions:

(a) The pulses are incident on a sample that is thin compared to the length scale over which dispersion matters (i.e., we can neglect propagation effects in the sample).

(b) There are N atoms per unit volume.

(c) The pulses are characterized by a central frequency $\omega = \omega_{10}$, i.e., they are resonant.

(d) The pulses are very short in time compared to the inhomogeneous dephasing time $\tau_{\text{pulse}} << T_2^*$.

The problem:
Show whether an echo appears in the direction $\vec{k}_1$ in the limit of $\int R_1(t)dt, \int R_2(t)dt << \pi/2$. Do this in two ways:

(a) Qualitatively sketch on the Bloch sphere the evolution of different sub-ensembles (corresponding to different resonant frequencies) when the tipping angle is small.

(b) Based on the Bloch vector formalism, write explicitly what the value of the polarization of the system is at time $2T$.