SI 140
Probability and Statistics
Semester Spring 2019
Assignment 1

Name:
StudentID:

Problem 1. If two sets have identical complements, then they are themselves identical. Show this in two ways:(i) by verbal definition, (ii) by using formula$(A^c)^c$

Solution: Write your solution here. □

Problem 2. Show that

$$(A \cup B) \cap C \neq A \cup (B \cap C)$$

but also give some special cases where there is equality.

Solution: Write your solution here. □

Problem 3. Show that A $\subset$ B if and only if AB = A; or A$\cup$ B = B. (So the relation of inclusion can be defined through identity and the operations.)

Solution: Write your solution here. □

Problem 4. Show that there is a distributive law also for difference:

$$(A \backslash B) \cap C = (A \cap C) \backslash (B \cap C).$$

Is the dual

$$(A \cap B) \backslash C = (A \backslash C) \cap (B \backslash C)$$

also true?

Solution: Write your solution here. □

Problem 5. Show that A $\subset$ B if and only if $I_A \leq I_B$; and A $\cap$ B = $\emptyset$ if and only if $I_A I_B$ =0.

Solution: Write your solution here. □

Problem 6. Given n events $A_1, A_2, ..., A_n$ and indicators $I_j, j = 1, ..., n$ ($I_j = 1$ if A_j occur, else $I_j = 0$). Let $X = \sum_{j=1}^{n} I_j$ be the number of events that occur. You need to find the number of pairs of distinct events that occur: (i) Write your answer in terms of X. (ii) Write your answer in terms of indicators.

Solution: Write your solution here. □

Problem 7. Express $I_{A \cup B \cup C}$ as a polynomial of I_A, I_B, I_C.

Solution: Write your solution here. □

Problem 8. Show that

$$I_{ABC} = I_A + I_B + I_C - I_{A \cup B} - -I_{A \cup C} - I_{B \cup C} + I_{A \cup B \cup C}$$

You can verify this directly, but it is nicer to derive it from problem 7 by duality.

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Solution: Write your solution here. □

Problem 9. Prove that the set of all rational numbers is countable.

Solution: Write your solution here. □

Problem 10. Let A be the set of all sequences whose elements are the digits 0 and 1. For example, the following sequence is a element of A.

$$1, 0, 1, 0, 0, 0, 1, 1, ...$$

Prove that set A is uncountable. (Hint: You can prove it by using Cantor's diagonal process.)

Solution: Write your solution here. □