Chapter 9

激光振荡

习题 9.1: 推导方程式 (9.3-22). □

证: 由式 (8.3-4),

$$\frac{\text{感应发射速率/模式}}{\text{自发辐射速率/模式}} = \frac{h\nu V_m W_i}{K} = n_m, \tag{9.1}$$

其中感应发射的跃迁速率

$$W_i = \frac{\lambda^2 I_\nu}{8\pi h\nu n^2 t_{\text{自发}}} g(\nu), \tag{9.2}$$

辐射强度

$$I_\nu = \frac{n_m h\nu}{V_m}\frac{c}{n}, \tag{9.3}$$

频谱增宽

$$\Delta\nu = \frac{1}{g(\nu_0)}. \tag{9.4}$$

故

$$K = \frac{h\nu V_m W_i}{n_m} = \frac{h\nu V_m}{n_m}\frac{\lambda^2 g(\nu_0)}{8\pi h\nu n^2 t_{\text{自发}}} I_\nu = \frac{h\nu\lambda^2 c}{8\pi n^3 t_{\text{自发}}} g(\nu_0) = \frac{h\nu c^3}{8\pi n^3\nu^2\Delta\nu t_{\text{自发}}}. \tag{9.5}$$

□

习题 9.2: 推导由于高斯光束模式的横向束缚, 对激光振荡器共振频率的影响. 在共焦腔的模式和 $z_0 \gg l$ 的模式之间的共振频率有什么变化 (l 为谐振腔长度)? □

证: 谐振腔的共振频率

$$\nu_m = \frac{mc}{2nl} + \frac{c}{2\pi nl}\left(\tan^{-1}\frac{z_2}{z_0} - \tan^{-1}\frac{z_1}{z_0} - \frac{\theta_{m1}+\theta_{m2}}{2}\right). \tag{9.6}$$

对共焦腔模式, $R_1 = R_2 = l$, $\tan^{-1}(z_2/z_0) = -\tan^{-1}(z_1/z_0) = \pi/4$, 共振频率

$$\nu_m = \frac{mc}{2nl} + \frac{c}{2\pi nl}\left(\frac{\pi}{2} + \frac{\theta_{m1}+\theta_{m2}}{2}\right). \tag{9.7}$$

对 $z_0 \gg l$ 的模式, $\tan^{-1}(z_2/z_0) \approx \tan^{-1}(z_1/z_0) \approx 0$, 共振频率

$$\nu_m = \frac{mc}{2nl} + \frac{c}{2\pi nl}\left(\frac{\theta_{m1}+\theta_{m2}}{2}\right). \tag{9.8}$$

两者的共振频率相差 $\frac{c}{4nl}$. □

习题 9.3: 若法布里-珀罗谐振腔内放入极化率为 $\chi(\omega)$ 的原子介质, 试证明模之间频率间隔等于

$$\omega_m - \omega_{m-1} = \frac{\pi c}{nl\left[1 + \frac{\omega}{2n^2}\frac{\partial \chi'(\omega)}{\partial \omega}\right]_{\omega=\omega_m}}.$$

□

证: 该 F-B 腔的谐振条件为

$$k'_m l = \frac{n\omega_m}{c}\left[1 + \frac{\chi'(\omega_m)}{2n^2}\right] l = m\pi, \tag{9.9}$$

$$\Longrightarrow \omega_m\left[1 + \frac{\chi'(\omega_m)}{2n^2}\right] = \frac{m\pi c}{nl}. \tag{9.10}$$

用 $m-1$ 代换 m 得

$$\omega_{m-1}\left[1 + \frac{\chi'(\omega_m)}{2n^2}\right] = \frac{(m-1)\pi c}{nl}. \tag{9.11}$$

以上两式相减得

$$\omega_m - \omega_{m-1} + \frac{\omega_m\chi'(\omega_m) - \omega_{m-1}\chi'(\omega'_{m-1})}{2n^2} = (\omega_m - \omega_{m-1})\left[1 + \frac{\chi'(\omega)}{2n^2} + \frac{\omega}{2n^2}\frac{\partial \chi(\omega)}{\partial \omega}\right]_{\omega=\omega_m} = \frac{\pi c}{nl}, \tag{9.12}$$

从而模式之间频率间隔为

$$\omega_m - \omega_{m-1} = \frac{\pi c}{nl\left[1 + \frac{\chi'(\omega)}{2n^2} + \frac{\omega}{2n^2}\frac{\partial \chi'(\omega)}{\partial \omega}\right]_{\omega=\omega_m}}. \tag{9.13}$$

□

习题 9.4: 若一个光脉冲在原子介质中传播, 脉冲的中心频率等于原子的共振频率 ω_0, 考虑原子介质的色散对脉冲的群速的影响, 分两种情况讨论: (a) 放大介质, (b) 吸收介质. 若不考虑烧孔效应并假定脉冲频谱比 $\Delta\nu$ 窄, 试将群速表示为洛伦兹谱线的峰值增益的函数. □

解: 当脉冲的中心频率等于原子的共振频率 ω_0 时, 脉冲群速度

$$\begin{aligned} v_g = \left.\frac{\partial \omega}{\partial k'}\right|_{\omega=\omega_0} &= \frac{1}{\pi}\left(\frac{\partial k'}{\partial \nu}\right)^{-1}_{\nu=\nu_0} = \frac{1}{2\pi}\left\{\frac{\partial}{\partial \nu}\frac{2\pi n\nu}{c}\left[1 + \frac{\chi'(\nu)}{2n^2}\right]\right\}^{-1}_{\nu=\nu_0} \\ &= \left\{\frac{\partial}{\partial \nu}\frac{n\nu}{c}\left[1 + \frac{\nu - \nu_0}{\Delta\nu}\frac{\chi''(\nu)}{2n^2}\right]\right\}^{-1}_{\nu=\nu_0} \\ &= \frac{c}{n\left[1 + \frac{\nu}{\Delta\nu_0}\frac{\chi''(\nu_0)}{2n^2}\right]}. \end{aligned} \tag{9.14}$$

洛伦兹谱线的峰值增益系数为

$$\gamma(\nu_0) = -k\frac{\chi''(\nu)}{n^2} = -\frac{2\pi n\nu_0}{c}\frac{\chi''(\nu_0)}{n^2}, \tag{9.15}$$

故脉冲群速度

$$v_g = \frac{c}{n\left[1 - \frac{\gamma(\nu_0)c}{2\pi n\Delta\nu}\right]}. \tag{9.16}$$

(a) 对放大介质, $\gamma(\nu_0) > 0$, 故 $v_g > \frac{c}{n}$.

(b) 对吸收介质, $\gamma(\nu_0) < 0$, 故 $v_g < \frac{c}{n}$.

□

习题 9.5: 证明式 (9.2-14) 等效于式 (9.1-10). □

证: 式 (9.2-14):

$$(\omega_l^2-\omega^2)+i\frac{\sigma\omega}{\varepsilon}=\frac{\omega^2\varepsilon_0 f}{\varepsilon}(\chi'-i\chi''), \tag{9.17}$$

其中 $\omega=2\pi\nu$ 为激光频率, $\omega_l=2\pi\nu_m$ 为空腔谐振频率 ($\nu_m=\frac{mc}{2nl}+\frac{c}{2\pi nl}\left(\tan^{-1}\frac{z_2}{z_0}-\tan^{-1}\frac{z_1}{z_0}-\frac{\theta_{m1}+\theta_{m2}}{2}\right)$), $\frac{\sigma}{\varepsilon}=\frac{n}{c}\frac{1}{t_c}=\alpha-\frac{1}{l}\ln(r_1r_2)=\gamma_t$ (t_c 为腔寿命, α 为介质非共振损耗常数, r_1,r_2 分别为两面反射镜的反射率, 这里将反射镜处的损耗也包含在 $\frac{\sigma}{\varepsilon}$ 中), $\frac{\varepsilon_0}{\varepsilon}=\frac{1}{n^2}$. 在完全填充谐振腔且均匀翻转的情况下, $f=1$, 当 $\omega\approx\omega_l$ 时, $\omega_l^2-\omega^2\approx 2\omega(\omega_l-\omega)$, 此时

$$\nu_m-\nu+i\gamma_t=\frac{\nu}{2n^2}(\chi'-i\chi''), \tag{9.18}$$

$$\Longrightarrow \nu\left[1+\frac{\chi'}{2n^2}\right]-i\nu\frac{\chi''}{2n^2}-i\frac{\gamma_t}{2}=\nu_m, \tag{9.19}$$

此即等效于 (9.1-10)

$$e^{-i2[k'l-\tan^{-1}(z_2/z_0)+\tan^{-1}(z_1/z_0)]}r_1r_2e^{-i(\theta_{m1}+\theta_{m2})}=e^{-i2m\pi}, \tag{9.20}$$

其中 $k'=k\left[1+\frac{\chi'}{2n^2}\right]-ik\frac{\chi''}{2n^2}-i\frac{\alpha}{2}$, $k=\frac{2\pi\nu}{c}$. □

习题 9.6: 推导式 (9.3-5). □

证: 由式 (9.3-4), 平衡态下,

$$\frac{\mathrm{d}N_2}{\mathrm{d}t}=R_2-\frac{N_2}{t_2}-\left(N_2-\frac{g_2}{g_1}N_1\right)W_i(\nu)=0, \tag{9.21}$$

$$\frac{\mathrm{d}N_1}{\mathrm{d}t}=R_1-\frac{N_1}{t_1}+\frac{N_2}{t_{21}}+\left(N_2-\frac{g_2}{g_1}N_1\right)W_i(\nu)=0. \tag{9.22}$$

上面两式相加得

$$R_2-\frac{N_2}{t_2}+R_1-\frac{N_1}{t_1}+\frac{N_2}{t_{21}}=0, \tag{9.23}$$

$$\Longrightarrow N_1=\left(\frac{1}{t_{21}}-\frac{1}{t_2}\right)t_1N_2+(R_1+R_2)t_1. \tag{9.24}$$

将上式代入式 (9.21) 中得

$$N_2=\frac{R_2t_2+(R_1+R_2)t_1t_2\frac{g_2}{g_1}W_i}{1+\left[t_2+(1-\delta)t_1\frac{g_2}{g_1}\right]W_i(\nu)}. \tag{9.25}$$

利用上式和式 (9.24) 得

$$\Delta N\equiv N_2-\frac{g_2}{g_1}N_1=\frac{R_2t_2-(R_1+\delta R_2)t_1\frac{g_2}{g_1}}{1+\left[t_1+(1-\delta)t_1\frac{g_2}{g_1}\right]W_i(\nu)}. \tag{9.26}$$

□