# Quantum Mechanics
# Homework Assignment 07
## Fall, 2019

1. Starting from the time-dependent Schrödinger equation in the Dirac notation, $i\hbar\dfrac{d\,|\psi(t)\rangle}{dt} = \left[\dfrac{\hat{\vec{p}}^{\,2}}{2m} + \hat{V}(\hat{\vec{r}})\right]|\psi(t)\rangle$, derive the time-dependent Schrödinger equation in the $\{|\vec{p}\rangle\}$ representation,
$$i\hbar\frac{\partial}{\partial t}\overline{\psi}(\vec{p},t) = \left[\frac{\vec{p}^{\,2}}{2m} + \hat{V}(i\hbar\vec{\nabla}_{\vec{p}})\right]\overline{\psi}(\vec{p},t)$$

2. Introducing the Fourier transform of the potential energy $V(\vec{r})$ in the $\{|\vec{r}\rangle\}$ representation, $\overline{V}(\vec{p}) = \dfrac{1}{(2\pi\hbar)^{3/2}}\displaystyle\int d^3r\; e^{-i\vec{p}\cdot\vec{r}/\hbar}V(\vec{r})$, show that the time-dependent Schrödinger equation in the $\{|\vec{p}\rangle\}$ representation can be also written as
$$i\hbar\frac{\partial}{\partial t}\overline{\psi}(\vec{p},t) = \frac{\vec{p}^{\,2}}{2m}\overline{\psi}(\vec{p},t) + \frac{1}{(2\pi\hbar)^{3/2}}\int d^3p'\;\overline{V}(\vec{p}-\vec{p}\,')\overline{\psi}(\vec{p}\,',t).$$

3. In the $\{|p_x\rangle\}$ representation, find the energy eigenvalue and eigenfunction of a particle of mass $m$ in the one-dimensional $\delta$-function potential well
$$V(x) = -\lambda\delta(x),\ \lambda > 0.$$

4. In the $\{|\vec{p}\rangle\}$ representation, the wave function of a particle at a given time is given by $\overline{\psi}(\vec{p}) = Ne^{-\alpha|\vec{p}|/\hbar}$ with $\alpha > 0$. Find the value of the normalization constant $N$ and the wave function $\psi(\vec{r})$ in the $\{|\vec{r}\rangle\}$ representation.

5. For a particle in one-dimensional space, find the expression of the operator $\hat{x}^{-1} = \dfrac{1}{\hat{x}}$ in the $\{|p_x\rangle\}$ representation and the expression of the operator $\hat{p}_x^{-1} = \dfrac{1}{\hat{p}_x}$ in the $\{|x\rangle\}$ representation.

   Note that $\hat{x}^{-1}$ is the inverse of $\hat{x}$ and that $\hat{p}_x^{-1}$ is the inverse of $\hat{p}_x$.