Quantum Mechanics
Homework Assignment 04
Fall, 2019

1. **[C-T Exercise 1-7]** Consider a particle of mass m placed in the one-dimensional potential

$$V(x) = \begin{cases} \infty, & x < 0, \\ -V_0, & 0 \leq x < a, \\ 0, & x \geq a. \end{cases}$$

Let $\varphi(x)$ be a wave function associated with a stationary state of the particle.

   (a) Show that $\varphi(x)$ can be extended to give an odd wave function which corresponds to a stationary state for a square well of width $2a$ and depth V_0.

   (b) Discuss, with respect to a and V_0, the number of bound states of the particle. Is there always at least one such state as for the symmetric square well?

2. Consider a particle of mass m placed in the one-dimensional potential

$$V(x) = \begin{cases} \lambda\delta(x), & |x| < a, \\ \infty, & |x| \geq a. \end{cases}$$

Here $\lambda > 0$. Find the energies and wave functions of the stationary states for the particle. The wave functions are not required to be normalized.

3. Consider a particle of mass m placed in the one-dimensional potential

$$V(x) = \begin{cases} V_1, & x \leq 0, \\ 0, & 0 < x < a, \\ V_2, & x \geq a. \end{cases}$$

Here $V_1 > V_2$. Find the equation that determines the energies of the bound states of the particle.

4. Consider a particle of mass m placed in a one-dimensional infinite-depth potential well with the potential energy given by

$$V(x) = \begin{cases} 0, & 0 < x < a, \\ \infty, & x \leq 0, x \geq a. \end{cases}$$

The particle is in a state described by the wave function $\psi(x) = Ax(x-a)\theta(x)\theta(a-x)$ with $A = \sqrt{30}\,a^{-5/2}$.

   (a) If the energy of the particle is measured, what are the possible results? What are the probabilities of obtaining these results?

   (b) What is the mean of all possible experimental results when the energy of the particle is measured? What is the standard deviation?

5. **[C-T Exercise 1-5]** Consider a particle of mass m whose potential energy is $V(x) = -\alpha\delta(x) - \alpha\delta(x-\ell)$, where α is greater than zero and ℓ is a constant length.

   (a) Calculate the bound states of the particle, setting $E = -\dfrac{\hbar^2\rho^2}{2m}$. Show that the possible energies are given by the relation $e^{-\rho\ell} = \pm\left(1 - \dfrac{2\rho}{\mu}\right)$ with $\mu = \dfrac{2m\alpha}{\hbar^2}$. Give a graphic solution of this equation.

      i. *Ground state.* Show that this state is even (invariant with respect to reflection about the point $x = \ell/2$), and that its energy E_S is less than the energy $-E_L = -\dfrac{m\alpha^2}{2\hbar^2}$. Interpret this result physically. Represent graphically the corresponding wave function.

      ii. *Excited state.* Show that, when ℓ is greater than a value which you are to specify, there exists an odd excited state of energy E_A greater than $-E_L$. Find the corresponding wave function.

iii. Explain how the preceding calculations enable us to construct a model which represents an ionized diatomic molecule (H_2^+, for example) whose nuclei are separated by a distance ℓ. How do the energies of the two levels vary with respect to ℓ? What happens at the limit where $\ell \to 0$ and at the limit where $\ell \to \infty$? If the repulsion of the two nuclei is taken into account, what is the total energy of the system? Show that the curve which gives the variation with respect to ℓ of the energies thus obtained enables us to predict in certain cases the existence of bound states of H_2^+, and to determine the value of ℓ at equilibrium. In this way we obtain a very elementary model of the chemical bond.

(b) Calculate the reflection and transmission coefficients of the system of two delta function barriers. Study their variations with respect to ℓ. Do the resonances thus obtained occur when ℓ is an integral multiple of the de Broglie wavelength of the particle? Why?