量子光学 PHYS6651P 2021-2022 学年第二学期

作业一

截止时间: 2022 年 3 月 11 日 (周五)

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成绩：__________

第 1 题 得分: __________. 求 $[a^\dagger, a^n]$.

解: $[a^\dagger, a^n] = a[a^\dagger, a^{n-1}] + [a^\dagger, a]a^{n-1} = a[a^\dagger, a^{n-1}] - a^{n-1}$.
利用数学归纳法:

- 当 $n=1$ 时, $[a^\dagger, a] = -1$;
- 当 $n=2$ 时, $[a^\dagger, a^2] = a[a^\dagger, a] + [a^\dagger, a]a = -2a$;
- 假设 $[a^\dagger, a^k] = -ka^{k-1}$, 则当 $n=k+1$ 时, $[a^\dagger, a^{k+1}] = a[a^\dagger, a^k] + [a^\dagger, a]a^k = -a\cdot ka^{k-1} - a^k = -(k+1)a^k$.

故 $[a^\dagger, a^n] = -na^{n-1}$. □

第 2 题 得分: __________. 证明 $[a^\dagger, f(a, a^\dagger)] = -\frac{\partial f}{\partial a}$.

证: 设 $f(a, a^\dagger) = \sum_{m,n} f_{mn} a^m (a^\dagger)^n$.
$[a^\dagger, f] = [a^\dagger, \sum_{m,n} f_{mn} a^m (a^\dagger)^n] = \sum_{m,n} f_{mn}[a^\dagger, a^m (a^\dagger)^n] = \sum_{m,n} f_{mn}[a^\dagger, a^m (a^\dagger)^n] = \sum_{mn} f_{mn}\{a^m[a^\dagger, (a^\dagger)^n] + [a^\dagger, a^m](a^\dagger)^n\} = \sum_{m,n} f_{mn}[a^\dagger, a^n](a^\dagger)^n = -\sum_{m,n} f_{mn} m a^{m-1}(a^\dagger)^n$,
$-\frac{\partial f}{\partial a} = -\sum_{m,n} f_{mn} m a^{m-1}(a^\dagger)^n$,
故 $[a^\dagger, f(a, a^\dagger)] = -\frac{\partial f}{\partial a}$. □