量子光学
PHYS6251P
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期末考试

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成绩：_____

第 1 题 (20 分) 得分: _____. 产生湮灭算符 $a^\dagger$, a 满足对易关系 $[a, a^\dagger] = 1$, 且 $[a, (a^\dagger)^n] = n(a^\dagger)^{n-1}$, 试证:

(i) $[a^\dagger, a^m] = -ma^{m-1}$;

(ii) $a(a^\dagger)^n a^m a^\dagger = (a^\dagger)^{n+1} a^{m+1} + (m+n+1)(a^\dagger)^n a^m + mn(a^\dagger)^{n-1} a^{m-1}$.

证: (i) 利用数学归纳法证明:

– 当 $m = 1$ 时,

$$[a^\dagger, a^1] = -1 = -1 \cdot a^{1-1}. \tag{1}$$

– 假设当 $m = k$ 时,

$$[a^\dagger, a^k] = -ka^{k-1}, \tag{2}$$

则当 $m = k+1$ 时,

$$[a^\dagger, a^{k+1}] = [a^\dagger, a^k]a + a^k[a^\dagger, a] = -ka^{k-1}a + a^k \cdot (-1) = -(k+1)a^k = -(k+1)a^{(k+1)-1}. \tag{3}$$

综上, $[a^\dagger, a^m] = -ma^{m-1}$.

(ii)

$$\begin{aligned} a(a^\dagger)^n a^m a^\dagger =& [(a^\dagger)^n a + n(a^\dagger)^{n-1}]a^m a^\dagger = (a^\dagger)^n a^{m+1} a^\dagger + n(a^\dagger)^{n-1} a^m a^\dagger \\ =& (a^\dagger)^n [a^\dagger a^{m+1} + (m+1)a^m] + n(a^\dagger)^{n-1}[a^\dagger a^m + ma^{m-1}] \\ =& (a^\dagger)^{n+1} a^{m+1} + (m+n+1)(a^\dagger)^n a^m + mn(a^\dagger)^{n-1} a^{m-1}. \end{aligned} \tag{4}$$

□