7-4试证,对遵从玻尔兹曼分布的定域系统,熵可表为$S = -Nk\sum_s P_s\ln P_s$,式中P_s–粒子处在量子态s的概率,$P_s = e^{-\alpha-\beta\varepsilon_s}/N = e^{-\beta\varepsilon_s}/Z_1$,$\sum_s$–对粒子所有量子态求和.对满足经典极限条件的非定域系统,熵的表达式有何不同? P_s满足归一化条件$\sum_s P_s = 1$,$\ln P_s = -\ln Z_1 - \beta\varepsilon_s$,粒子平均能量:$E = \sum_s P_s\varepsilon_s$,定域系统的熵:$S = Nk(\ln Z_1 - \beta\frac{\partial}{\partial\beta}\ln Z_1) = Nk(\ln Z_1 + \beta\bar\varepsilon) = Nk\sum_s P_s(\ln Z_1 + \beta\varepsilon_s) = Nk\sum_s P_s\ln P_s$;对满足经典极限条件的定域系统,$S = Nk(\ln Z_1 - \beta\partial\ln Z_1/\partial\beta) - k\ln N! = -Nk\sum_s P_s\ln P_s - k\ln N! = -Nk\sum_s P_s\ln P_s - Nk(\ln N - 1)$

7-6晶体含N个原子,当原子离开正常位置,晶体中就出现了空位和间隙原子,晶体的这种缺陷称为弗仑克尔缺陷,(a)假设正常位置和见习位置数都是N,试证由于在晶体中形成n个空位和见习原子而具有的熵:$S = 2k\ln N!/n!(N-n)!$,(b)设原子在间隙位置和正常位置的能量差为u,试由自由能$F = nu - TS$为极小,证明T下空位和间隙原子数:$n \approx Ne^{-u/2kT}$(设$n \ll N$) (a)可能的微观状态数:$\Omega = [N!/n!(N-n)!]\cdot[N!/n!(N-n)!]$,熵增:$S = k\ln\Omega$,(b)当时,内能增加$U = nu$,自由能改变$F = nu - TS = nu - 2kT[N\ln N - n\ln n - (N-n)\ln(N-n)]$,平衡态$F$极小要求$\frac{\partial F}{\partial n} = u - 2kT\ln\frac{N-n}{n} = 0$,因$n \ll N$,有$n \approx Ne^{-u/2kT}$

7-7若原子脱离正常位置而占据表面上的正常位置,称肖特基缺陷,用自由能极小的条件证明T下$n \approx Ne^{-W/kT}$,其中W–原子在表面位置与正常位置的能量差 可能的微观态:$\Omega = N!/n!(N-n)!$,熵增:$S = k\ln\Omega = k[N\ln N - n\ln n - (N-n)\ln(N-n)]$,内能增加$U = nW$,自由能改变$F = nW - TS = nW - kT[N\ln N - n\ln n - (N-n)\ln(N-n)]$,平衡态$F$极小要求$\partial F/\partial n = W - kT\ln(N-n/n) = 0$,因$n \ll N$有$n \approx Ne^{-W/kT}$

7-8稀薄气体由某种原子组成,原子的两个能级能量之差:$\varepsilon_2 - \varepsilon_1 = \hbar\omega$,跃迁辐射光子,由于气体原子的速度分布和多普勒效应,光谱仪观察到的不是单一频率的ω_0的谱线,而有多普勒展宽,求展宽表达式 设观测方向为z轴方向,原子质量m,初态处于能级ε_2,速度$\boldsymbol{v}_2$,沿z轴发射能量$\hbar\omega$,动量$\hbar k$的光子后跃迁至能级ε_1,速度变为$\boldsymbol{v}_1$,动量守恒和能量守恒,$m\boldsymbol{v}_1 + \hbar\boldsymbol{k} = m\boldsymbol{v}_2$,$\varepsilon_1 + mv_1^2/2 + \hbar\omega = \varepsilon_2 + mv_2^2/2$,式(1)平方并处以$2m$得$mv_1^2/2 + \hbar^2k^2/2m + \hbar\boldsymbol{v}_1\cdot\boldsymbol{k} = \frac{1}{2}mv_2^2$,代入式(2)得$\hbar\omega_0 = \hbar\omega - \hbar\boldsymbol{v}_1\cdot\boldsymbol{k} - \hbar^2k^2/2m$,或$\omega_0 = \omega - v_{1z}\omega/c - \hbar\omega^2/2mc^2$,考虑$m \sim 10^{-26}\mathrm{kg}$,$v_{1z} \sim 3\times10^2\mathrm{m\cdot s^{-1}}$,$\omega \sim 10^{15}\mathrm{s^{-1}}$有$1 \gg v_{1z} \gg \hbar\omega/2mc^2$,右侧第三项可忽略,$\omega = \omega/(1 - v_{1z}/c) \approx \omega_0(1 + v_{1z}/c)$,$T$下气体原子速度$z$分量在$v_z \sim v_z + dv_z$范围内的概率$\propto e^{-mv_z^2/2kT}dv_z$,$\omega \sim \omega + d\omega$范围内概率$\propto e^{-(m/2kT)(c^2(\omega-\omega_0)^2/\omega_0^2)}c/\omega_0 d\omega$,归一化得$F(\omega) = (2\pi\delta^2)^{-1/2}e^{-(\omega-\omega_0)^2/2\delta^2}$,$\delta = \omega(kT/mc^2)^{1/2}$

7-11表面活性物质的分子在液面熵作二维自由运动,可视为二维气体,试写出二维气体中分子的速度分布和速率分布,并求平均速率$\bar v$,最概然速率v_m和方均根速率v_s 速度分布:$(m/2\pi kT)e^{-m(v_x^2+v_y^2)/2kT}dv_xdv_y$,速率分布:$(m/kT)e^{-mv^2/2kT}vdv$,$\bar v = (m/kT)\int_0^\infty e^{-mv^2/2kT}vdv = \sqrt{\pi kT/2m}$,$\bar{v^2} = (m/kT)\int_0^{+\infty} e^{-mv^2/2kT}v^3dv = 2kT/m$,$v_s = \sqrt{2kT/m}$,$d(e^{-mv^2/2kT}v)/dv = 0 \Rightarrow v_m = \sqrt{kT/m}$

7-12根据麦克斯韦速度分布律导出两分子的相对速度$\boldsymbol{v}_r = \boldsymbol{v}_2 - \boldsymbol{v}_1$和相对速率$v_r$的概率分布,并求相对速率的平均值$\bar v_r$ 分子1和分子2各自处在速度间隔$d\boldsymbol{v}_1$和$d\boldsymbol{v}_2$的概率:$dW = dW_1\cdot dW_2 = (m/2\pi kT)^{3/2}e^{-mv_1^2/2kT}d\boldsymbol{v}_1\cdot(m/2\pi kT)^{3/2}e^{-mv_2^2/2kT}d\boldsymbol{v}_2$,质心速度:$\boldsymbol{v}_c = (m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2)/(m_1+m_2)$,相对速度:$\boldsymbol{v}_r = \boldsymbol{v}_2 - \boldsymbol{v}_1$,当$m_1 = m_2 = m$,简化为$\boldsymbol{v}_c = (\boldsymbol{v}_1 + \boldsymbol{v}_2)/2$,$\boldsymbol{v}_r = \boldsymbol{v}_2 - \boldsymbol{v}_1$,动能:$E_k = m_1v_1^2/2 + m_2v_2^2/2 = m'v_c^2/2 + \mu v_r^2/2$,其中$m' = m_1 + m_2$,$\mu = m_1m_2/(m_1+m_2)$,当$m_1 = m_2 = m$,$m' = 2m$,$\mu = m/2$,$dW = (m'/2\pi kT)^{3/2}e^{-m'v_c^2/2kT}d\boldsymbol{v}_c\cdot(\mu/2\pi kT)^{3/2}e^{-\mu v_r^2/2kT}d\boldsymbol{v}_r = dW_cdW_r$,相对速度的概率分布$dW_r = (\mu/2\pi kT)^{3/2}e^{-\mu v_r^2/2kT}d\boldsymbol{v}_r$,相对速率的分布:$4\pi(\mu/2\pi kT)^{3/2}e^{-\mu v_r^2/2kT}v_r^2dv_r$,相对速率的平均值:$\bar v_r = 4\pi(\mu/2\pi kT)^{3/2}\int_0^\infty e^{-\mu v_r^2/2kT}v_r^3dv_r = \sqrt{8kT/\pi\mu}$

7-14分子从器壁的小孔中射出,求在射出的分子束中,分子的平均速率/方均根速率/平均能量 相当于单位时间内碰到单位面积器壁上$v \sim v + dv$范围内的分子数:$d\Gamma(v) = \pi n(m/2\pi kT)^{3/2}e^{-mv^2/2kT}v^3dv$,平均速率:$\bar v = (\int_0^{+\infty} vd\Gamma(v))/(\int_0^{+\infty} d\Gamma) = \sqrt{9\pi kT/8m}$,$\bar{v^2} = (\int_0^{+\infty} v^5e^{-mv^2/2kT}dv)/(\int_0^{+\infty} v^3e^{-mv^2/2kT}dv) = 4kT/m$,$v_s = \sqrt{\bar{v^2}} = \sqrt{4kT/m}$,平均动能:$m\bar{v^2}/2 = 2kT$

7-17已知粒子遵经典玻尔兹曼分布,其能量表达式:$\varepsilon = (p_x^2 + p_y^2 + p_z^2)/2m + ax^2 + bx$,求粒子的平均能量 配方$\varepsilon = (p_x^2 + p_y^2 + p_z^2)/2m + a(x + b/2a)^2 - b^2/4a$,能量均分定理:$\bar\varepsilon = 2kT - b^2/4a$

7-23对双原子分子,常温下$kT \gg$转动的能级间距,求转动熵 转动配分函数:$Z_1^r = (1/h^2)\int e^{-\beta(p_\theta^2 + p_\varphi^2/\sin^2\theta)/2I}dp_\theta dp_\varphi d\theta d\varphi = 2I/\beta\hbar^2$,$S^r = Nk(\ln Z_1^r - \beta\partial(\ln Z_1^r)/\partial\beta) = Nk[\ln(2I/\beta\hbar^2) + 1]$

7-28晶体中原子密度n,角动量量子数1,外场B下,原子磁矩可有三种不同取向,忽略磁矩间相互作用,求T下,磁化强度M,及其在高温弱场和低温强场下近似 配分函数:$Z_1 = e^{\beta\mu B} + 1 + e^{-\beta\mu B} = 1 + 2\cosh(\beta\mu B)$,磁化强度$M = (n/\beta)\partial(\ln Z_1)/\partial B = n\mu(2\sinh\beta\mu B)/(1 + 2\cosh\beta\mu B)$,高温弱场下,$\beta\mu B \ll 1$,$\sinh\beta\mu B \approx \beta\mu B$,$\cosh\beta\mu B \approx 1$,$\boldsymbol{M} = (2/3)(n\mu^2/kT)B$,反之,$\sinh\beta\mu B \approx \cosh\beta\mu B \approx e^{\beta\mu B}/2$,$M \approx n\mu$

8-2试证,理想费米系统的熵可表为$S_{\mathrm{F.D.}} = -k\sum_s[f_s\ln f_s + (1 - f_s)\ln(1 - f_s)]$,其中$f_s$–量子态$s$上的平均粒子数,$\sum_s$–对各量子态求和,并证当$f_s \ll 1$,$S_{\mathrm{B.E.}} \approx S_{\mathrm{F.D.}} \approx S_{\mathrm{M.B.}} = -k\sum_s(f_s\ln f_s - f_s)$ 理想费米系统的熵:$S_{\mathrm{F.D.}} = k\sum_l[\omega_l\ln\omega_l - a_l\ln a_l - (\omega_l - a_l)\ln(\omega_l - a_l)] = -k\sum_l[(\omega_l - a_l)\ln((\omega_l - a_l)/\omega_l) + a_l\ln(a_l/\omega_l)] = -k\sum_l\omega_l[(1 - a_l/\omega_l)\ln(1 - a_l/\omega_l) - (a_l/\omega_l)\ln(a_l/\omega_l)]$,式中$\sum_l$–对各能级求和,因$f_s = a_l/\omega_l$,$\sum_l \sim\sim_s$,$S_{\mathrm{F.D.}} = -k\sum_s[f_s\ln f_s + (1 - f_s)\ln(1 - f_s)]$,当$f_s \ll 1$,$\pm(1 \mp f_s)\ln(1 \pm f_s) \approx \pm(1 \mp f_s)(\mp f_s) \approx -f_s$,故得

8-4试证,在热力学极限下均匀二维理想玻色气体不会发生玻-爱凝聚 临界温度由$\int_0^{+\infty} D(\varepsilon)d\varepsilon/(e^{\varepsilon/kT_c} - 1) = n$确定,态密度:$D(\varepsilon)d\varepsilon = (2\pi L/h^2)md\varepsilon$,代入得$(2\pi L^2/h^2)m\int_0^{+\infty} d\varepsilon/(e^{\varepsilon/kT_c} - 1) = n$,令$x = \varepsilon/kT_c$,$(2\pi L^2/h^2)mkT_c\int_0^{+\infty} dx/(e^x - 1) = n$,展开$1/(e^x - 1) = 1/(e^x(1 - e^{-x})) = e^{-x}(1 + e^{-x} + \cdots)$,$\int_0^\infty dx/(e^x - 1) = \sum_{n=1}^\infty 1/n$,级数发散意味着有限温度下化学势不可能趋于0,故

8-5约束在磁光陷阱中的理想原子气体,在三维谐振势场$V = m(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)/2$中运动,若为玻色子,试证$T \le T_c$下,有大量原子凝聚在基态,$T_c$满足$N = 1.202(kT_c/\hbar\bar\omega)^3$ 三维谐振子能量:$\varepsilon = \hbar\omega_x(n_x + 1/2) + \hbar\omega_y(n_y + 1/2) + \hbar\omega_z(n_z + 1/2)$,$n_{x/y/z} = 0, 1, \cdots$,在量子态$n_x,n_y,n_z$上的粒子数:$a_{n_x,n_y,n_z} = (e^{(\varepsilon-\mu)/kT} - 1)^{-1}$,化学势$\mu < \varepsilon_0 = (\hbar/2)(\omega_x + \omega_y + \omega_z)$,且满足$N = \sum_s a_{n_x,n_y,n_z}$,$\mu\uparrow$随$T\downarrow$,$T_c$下,$N = \sum_{n_x,n_y,n_z}(e^{\tilde n_x + \tilde n_y + \tilde n_z} - 1)^{-1}$,其中$\tilde n_i = (\hbar\omega_i/kT_c)n_i$,题设极限下化求和为积分,$N = (kT_c/\hbar\bar\omega)\int d\tilde n_xd\tilde n_yd\tilde n_z/(e^{\tilde n_x + \tilde n_y + \tilde n_z} - 1)$,将被积函数如上题展开即得

8-7求T下V内光子气体的平均总光子数 V内$\omega \sim \omega + d\omega$范围内光子的量子态数:$D(\omega)d\omega = (V/\pi^2c^3)\omega^2d\omega$,平均光子数:$\overline{N}(\omega, T)d\omega = D(\omega)d\omega/(e^{\hbar\omega/kT} - 1)$,总光子数:$\overline{N}(T) = (V/\pi^2c^3)\int_0^{+\infty}\omega^2d\omega/(e^{\hbar\omega/kT} - 1)$,设$x = \hbar\omega/kT$,$\overline{N}(T) = (V/\pi^2c^2)(kT/\hbar)^3\int_0^{+\infty} x^2dx/(e^x - 1) = 2.404(k^3/\pi^2c^3\hbar^3)VT^3$,$n = \overline{N}/V$,$n(1000) \approx 2\times10^{16}\mathrm{m^{-3}}$,$n(3) \approx 5.5\times10^8\mathrm{m^{-3}}$

8-8试根据普朗克公式证明平衡辐射内能密度按波长的分布:$u(\lambda, T)d\lambda = (8\pi hc/\lambda^5)d\lambda/(e^{hc/\lambda kT} - 1)$ 内能按圆频率的分布:$u(\lambda, T) = (\pi^2c^3)^{-1}\hbar\omega^3d\omega/(e^{\hbar\omega/kT} - 1)$,由$|d\omega| = (2\pi c/\lambda^2)|d\lambda|$得 **8-14**费米能级用$\hbar$!

8-18绝热压缩系数:$\kappa_S = -(1/V)(\partial V/\partial p)_S$,试证0K下,理想费米气体有$\kappa_T(0) = \kappa_S = (3/2)(1/n\mu(0))$ 0K下理想费米气体压强:$p = (2/5)n\mu(0) = (2/5)(\hbar^2/2m)(3\pi^2)^{2/3}(N/V)^{5/3}$,$(\partial p/\partial V)_T = (3/2)(\hbar^2/2m)(3\pi)^{2/3}(N/V)^{2/3}(-N/V^2)$,故得$\kappa_T = (3/2)(1/\mu(0))$,由能斯特定理,0K下等温线与等熵线重合,$(\partial V/\partial p)_T = (\partial V/\partial p)_S$

8-26由N个自旋极化的粒子组成的理想费米气体处在径向频率ω_r,轴向频率$\lambda\omega_r$的磁光陷阱内,粒子能量:$\varepsilon = (p_x^2 + p_y^2 + p_z^2)/2m + m\omega_r^2(x^2 + y^2 + \lambda^2z^2)/2$,求0K下化学势(以费米温度表示)和粒子的平均能量 粒子能量本征值:$\omega = \hbar\omega_r(n_x + n_y + \lambda n_z)$,$n_i = 0, 1, \cdots$,式中能量零点取$\hbar\omega_r(1 + \lambda/2)$;$\mu$满足$N = \sum_{n_x,n_y,n_z}(e^{\beta[\hbar\omega_r(n_x+n_y+\lambda n_z)-\mu]} + 1)^{-1}$,令$\varepsilon_i = n_i\hbar\omega_r$,$d\varepsilon_i = \hbar\omega_r$,$N = (1/\lambda(\hbar\omega_r)^3)\int d\varepsilon_xd\varepsilon_yd\varepsilon_z/(e^{\beta(\varepsilon_x+\varepsilon_y+\varepsilon_z-\mu)} + 1)$,设$\varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z$,$N = (1/\lambda(\hbar\omega_r)^3)\int d\varepsilon/(e^{\beta(\varepsilon-\mu)} + 1)\int d\varepsilon_x\int d\varepsilon_y$,积分面积:$\varepsilon^2/2$,$N = \int D(\varepsilon)d\varepsilon/(e^{\beta(\varepsilon-\mu)} + 1)$,其中$D(\varepsilon)d\varepsilon = \varepsilon^2d\varepsilon/(2\lambda(\hbar\omega_r)^3)$,解得$\mu(0) = \hbar\omega_r(6\lambda N)^{1/3}$,$E = \int_0^{\mu(0)} D(\varepsilon)\varepsilon d\varepsilon = (3/4)N\mu(0)$,除以$N$得

9-1试证微正则系综理论中熵可表为$S = -k\sum_s\rho_s\ln\rho_s$,其中$\rho_s = 1/\Omega$–系统处在状态$s$的概率,$\Omega$–系统可能微观态数 由归一化条件$\sum_s\rho_s = 1$和$\Omega = 1/\rho_s$得$S = k\ln\Omega = k\sum_s\rho_s\ln\Omega =$

9-2试证正则分布中熵可表为$S = -k\sum_s\rho_s\ln\rho_s$,其中$\rho_s = (1/Z)e^{-\beta E_s}$–系统处在能量$E_s$的状态的概率 由归一化条件$\sum_s\rho_s = 1$和$\ln\rho_s = -(\ln Z + \beta E_s)$有$S = k(\ln Z + \beta U) = k\sum_s\rho_s(\ln Z + \beta E_s) =$

9-5体积V内有A,B两种单原子分子混合理想气体,原子数N_A,N_B,温度T,用正则系综理论求物态方程/内能/熵 能量经典表达式:$Z = (1/N_A!N_B!h^{3N_A}h^{3N_B})\int e^{-\beta(E_A+E_B)}d\Omega_Ad\Omega_B = (V^{N_A}/N_A!)(2\pi m_A/\beta h^2)^{3N_A/2}(V^{N_B}/N_B!)(2\pi m_B/\beta h^2)^{3N_B/2}$,配分函数:$\ln Z = \ln Z_A + \ln Z_B$,物方:$p = (N_A + N_B)kT/V$,内能:$U = (3/2)(N_A + N_B)kT$,熵:$S = N_Ak[(V/N_A)(2\pi m_AkT/h^2)^{3/2}] + N_Bk\ln[(V/N_A)(2\pi m_BkT/h^2)^{3/2}] + (5/2)(N_A + N_B)k$

9-8被吸附在液面的分子形成二维气体,考虑分子间相互作用,试证物态方程:$pA = NkT(1 + (N/N_A)\cdot(B/A))$,其中$B = -(N_A/2)\int(e^{-\phi/kT} - 1)2\pi rdr$,$A$–液面面积,$\phi$–两分子相互作用势 二维气体能量:$E = \sum_{i=1}^{2N}p_i^2/2m + \sum_{i<j}\phi(r_{ij})$,配分函数:$Z = (1/N!h^{2N})\int e^{-\beta E}d\boldsymbol{r}_1\cdots d\boldsymbol{r}_2d\boldsymbol{p}_1\cdots dp_N = (1/N!h^{2N})(2\pi m/\beta h^2)^NQ$,其中$Q = \int e^{-\beta\sum_{i<j}\phi(r_{ij})}d\boldsymbol{r}_1\cdots d\boldsymbol{r}_N$,设$f_{ij} = e^{-\beta\phi(r_{ij})} - 1$,$e^{-\beta\sum_{i<j}\phi(r_{ij})} = \prod_{i<j}(1 + f_{ij}) \approx 1 + \sum_{i<j}f_{ij}$,$Q = A^N + (N^2/2)\int f_{12}d\boldsymbol{r}_1\cdots d\boldsymbol{r}_N = A^N[1 + (N^2/2A)\int_0^{+\infty}(e^{-\beta\phi(r)} - 1)2\pi rdr] = A^N(1 - (N^2/N_AA)B)$,配分函数:$Z = (1/Z!)(2\pi m/\beta h^2)^NA^N(1 - (N^2/N_AA)B)$,$p = (1/\beta)\partial(lnZ)/\partial A$

9-12求长度L的线性原子链在高温和低温下的内能 dk_xdk_y范围内波矢数:$(L^2/4\pi^2)dk_xdk_y$,$k \sim k + dk$内:$(L^2/2\pi)kdk$,$\omega \sim \omega + d\omega$范围内振动模式数:$D(\omega)d\omega = 2(L/2\pi)(1/c) = B_2\omega d\omega$,满足$B_2\int_0^{\omega_D}\omega d\omega = 3N$,德拜频率:$\omega_D^2 = 6N/B_2$,内能:$U = U_0 + B_2\int_0^{\omega_D}\hbar\omega^2d\omega/(e^{\hbar\omega/kT} - 1)$,高温极限下,$U = U_0 + 3NkT$,$C_V =$,高温极限下,$U = U_0 + B_2\int_0^{+\infty} = U_0 + 3Nk\cdot 4.808(T^3/\theta_D^2)$

9-15玻色准粒子有色散关系$\omega = Ak^2$,试证低温下$C_V \propto T^{3/2}$ V内,$k \sim k + dk$范围内准粒子状态数:$V4\pi k^2dk/(2\pi)^3$,$\omega \sim \omega + d\omega$范围内:$B\omega^{1/2}d\omega$,式中$B = (V/4\pi^2)A^{-3/2}$,$\omega \sim \omega + d\omega$准粒子数:$N(\omega)d\omega = B\omega^{1/2}d\omega/(e^{\hbar\omega/kT} - 1)$,贡献内能:$U = B\int_0^{+\infty}\hbar\omega^{3/2}d\omega/(e^{\hbar\omega/kT} - 1) = B(kT/\hbar)^{5/2}\hbar\int_0^{+\infty}x^{3/2}dx/(e^x - 1)$

9-20试证在巨正则系综理论中熵可表为$S = -k\ln_n\ln_s\rho_{N,s}\ln\rho_{N,s}$,其中$\rho_{N,s} = (1/\Xi)e^{-\alpha N-\beta E_s}$–系统有$N$个粒子,处在状态$s$的概率 由归一化条件$\sum_{N,s}\rho_{N,s} = 1$和$\ln\rho_{N,s} = -(\ln\Xi + \alpha N + \beta E_s)$有$S = k(\ln\Xi + \alpha N + \beta U) =$

9-21V内含N个粒子,用正则系综理论证明,小体积v中有n个粒子的概率:$P_n = (1/n!)e^{\bar n}(\bar n)^n$ 视v为系统,$V - r$为粒子源和热源,$P_n = \sum_s\rho_{ns} = (1/\Xi)e^{-\alpha/n}\sum_s e^{-\beta E_s} = (1/\Xi)e^{-\alpha n}Z_n$,$Z_n = \sum_s e^{-\beta E_s} = (1/n!)Z_1^n$–$n$个粒子的正则配分函数,$\ln\Xi = e^{-\alpha}Z_1$,$\bar n = -\partial(\ln\Xi)/\partial\alpha = e^{-\alpha}Z_1 = \ln\Xi$,代入得

9-23单原子分子理想气体与固体吸附面接触达平衡,被吸附分子可在吸附面上二维运动,能量:$p^2/2m - \varepsilon_0$,$\varepsilon_0 > 0$,用巨正则系综理论求被吸附分子面密度 视被吸附分子为系统,理想气体为热源和粒子源,巨配分函数:$\Xi = \sum_{N=0}^\infty\sum_s e^{-\alpha N-\beta E_s} = \sum_{N=0}^\infty e^{-\alpha N}Z_N$,其中$Z_N = (1/N!)Z_1^N$–有$N$个分子吸附时的正则配分函数,$Z_1 = (1/\hbar^2)\int e^{-\beta(p^2/2m-\varepsilon_0)}dxdydp_xdp_y = A(2\pi m/\beta h^2)e^{\beta\varepsilon_0}$–单粒子配分函数,代入得$\Xi = \exp[e^{-\alpha}A(2\pi m/\beta h^2)e^{\beta\varepsilon_0}]$,$\overline{N} = A(2\pi mkT/h^2)e^{(\varepsilon+\mu)/kT}$,代入单原子分子理想气体化学势并处以$A$得

9-25试证玻尔兹曼分布的涨落:$\overline{(a_l - \bar a_l)^2} = \bar a_l$ 视能级ε_l上的粒子为一开系,$\overline{(a_l - \bar a_l)^2} = -\partial\bar a_l/\partial\alpha$得

9-26试证光子气体,$\overline{(a_l - \bar a_l)^2} = -(1/\beta)\partial\bar a_l/\partial\varepsilon_l$ $\Xi = \sum_{a_l}e^{-\beta\varepsilon_la_l}$,$\bar a_l = (1/\Xi)\sum_{a_l}a_le^{-\beta\varepsilon_la_l} = (\sum_{a_l}a_le^{-\beta\varepsilon_la_l})/(\sum_{a_l}e^{-\beta\varepsilon_la_l})$,求导即得,可代入$\bar a_l = 1/(e^{\beta\varepsilon_l} - 1)$

10-8三维布朗颗粒在各项同性介质中运动,郎之万方程:$dp_i/dt = -\gamma p_i + F_i(t)$,$i = 1, 2, 3$,涨落力满足$\overline{F_i(t)} = 0$,$\overline{F_i(t)F_j(t')} = 2m\gamma kT\delta_{ij}\delta(t - t')$,试证$t$后位移平方平均值$\overline{[\boldsymbol{x} - \boldsymbol{x}(0)]^2} = \sum_i\overline{[x_i - x_i(0)]^2} = 6kTt/m\gamma$ 一维布朗:$\overline{[x_i - x_i(0)]^2} = 2kTt/m\gamma$,即得