

解: 取等边三角形为研究对象并受力分析如图。

$$\text{由 } \sum M_{BB'}(\vec{F}) = 0, \quad M + \frac{\sqrt{3}}{2} a \frac{\sqrt{3}}{2} F_6 = 0 \text{ 得 } F_6 = -\frac{4M}{3a}$$

$$\text{由 } \sum M_{CC'}(\vec{F}) = 0, \quad M + \frac{\sqrt{3}}{2} a \frac{\sqrt{3}}{2} F_4 = 0, \text{ 得 } F_4 = -\frac{4M}{3a}$$

$$\text{由 } \sum M_{AA'}(\vec{F}) = 0, \quad M + \frac{\sqrt{3}}{2} a \frac{\sqrt{3}}{2} F_5 = 0, \text{ 得 } F_5 = -\frac{4M}{3a}$$

$$\text{由 } \sum M_{BC}(\vec{F}) = 0, \quad -\frac{\sqrt{3}}{2} a F_1 - \frac{\sqrt{3}}{2} a \frac{1}{2} F_4 = 0, \text{ 得 } F_1 = \frac{2M}{3a}$$

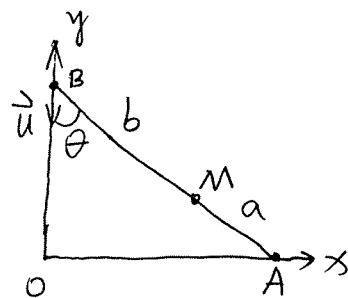
$$\text{由 } \sum M_{AC}(\vec{F}) = 0, \quad \frac{\sqrt{3}}{2} a F_2 + \frac{\sqrt{3}}{2} a \frac{1}{2} F_5 = 0, \text{ 得 } F_2 = \frac{2M}{3a}$$

$$\text{由 } \sum M_{AB}(\vec{F}) = 0, \quad -\frac{\sqrt{3}}{2} a F_3 - \frac{\sqrt{3}}{2} a \frac{1}{2} F_6 = 0, \text{ 得 } F_3 = \frac{2M}{3a}$$

Q2 解: 杆有圆轮是作平面平行运动。

取 B 点为基点, 则有 $\vec{r}_M = b \sin \theta \vec{i} - b \cos \theta \vec{j}$

$$\vec{v}_B = -u \vec{j}, \quad \vec{a}_B = 0, \quad \omega = \dot{\theta} \vec{k}$$



(i) M 点的速度和加速度

$$\vec{v}_M = \vec{v}_B + \vec{\omega} \times \vec{r}_M$$

$$= -u \vec{j} + \dot{\theta} \vec{k} \times (b \sin \theta \vec{i} - b \cos \theta \vec{j}) = \dot{\theta} b \cos \theta \vec{i} + (\dot{\theta} b \sin \theta - u) \vec{j} \quad (1)$$

$$\vec{v}_B = v_A \vec{i} + \dot{\theta} \vec{k} \times [-(a+b) \sin \theta \vec{i} + (a+b) \cos \theta \vec{j}]$$

$$= [v_A - (a+b) \dot{\theta} \cos \theta] \vec{i} - (a+b) \dot{\theta} \sin \theta \vec{j} = -u \vec{j}$$

$$\therefore v_A = (a+b) \dot{\theta} \cos \theta, \quad \dot{\theta} = \frac{u}{(a+b) \sin \theta}, \text{ 代入 (1), 得}$$

$$\vec{v}_M = \dot{\theta} b \cos \theta \vec{i} + (\dot{\theta} b \sin \theta - u) \vec{j} = \frac{u}{a+b} (b \cot \theta \vec{i} - a \vec{j})$$

$$\vec{a}_M = \vec{a}_B + \dot{\vec{\omega}} \times \vec{r}_M + \vec{\omega} \times (\vec{\omega} \times \vec{r}_M)$$

$$= \ddot{\theta} \vec{k} \times \vec{r}_M + \vec{\omega} (\vec{r}_M \cdot \vec{\omega}) - \omega^2 \vec{r}_M = \ddot{\theta} \vec{k} \times (b \sin \theta \vec{i} - b \cos \theta \vec{j}) - \dot{\theta}^2 (b \sin \theta \vec{i} - b \cos \theta \vec{j})$$

$$\text{又 } \dot{\theta} = \frac{u}{(a+b) \sin \theta}, \text{ 则 } \ddot{\theta} = -\frac{u \cos \theta}{(a+b) \sin^2 \theta} \dot{\theta} = -\frac{u^2 \cos \theta}{(a+b)^2 \sin^3 \theta}$$

$$\therefore \vec{a}_M = -\frac{b u^2}{(a+b)^2 \sin^3 \theta} \vec{i} = -\frac{b^4 u^2}{(a+b)^2 x^3} \vec{i}$$