Q2 解：(1) $\because L=\frac{1}{2}me^{\alpha t}(\dot{x}^2-\omega^2x^2)$，$\therefore \frac{\partial L}{\partial \dot{x}}=me^{+\alpha t}\dot{x}$，$\frac{\partial L}{\partial x}=-m\omega^2e^{\alpha t}x$

代入 $\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_\alpha})-\frac{\partial L}{\partial q_\alpha}=0$，得 $\ddot{x}+\alpha\dot{x}+\omega^2x=0$

(2) $S=1$，选 x 为广义坐标，则系统广义动量为

$$p_x=\frac{\partial L}{\partial \dot{x}}=me^{\alpha t}\dot{x}$$

$\therefore$ 广义速度为 $\dot{x}=\frac{p_x}{m}e^{-\alpha t}$

代入Hamiltonian，并表示成 x，p_x 的函数

$$\begin{aligned}H&=\sum_{k=1}^{s}p_k\dot{q}_k-L\\&=p_x\dot{x}-\frac{m}{2}e^{\alpha t}(\dot{x}^2-\omega^2x^2)\\&=p_x(\frac{p_x}{m}e^{-\alpha t})-\frac{m}{2}e^{\alpha t}[(\frac{p_x}{m}e^{-\alpha t})^2-\omega^2x^2]\\&=\frac{p_x^2}{2m}e^{-\alpha t}+\frac{m}{2}e^{\alpha t}\omega^2x^2\end{aligned}$$

代入 Hamilton正则方程，得

$$\dot{p}_x=-\frac{\partial H}{\partial x}=-\frac{m}{2}e^{\alpha t}\omega^2 2x \quad (1)$$

又 $\dot{x}=\frac{p_x}{m}e^{-\alpha t}$

$$\therefore \ddot{x}=\frac{\dot{p}_x}{m}e^{-\alpha t}-r\frac{p_x}{m}e^{-\alpha t} \quad (2)$$

式(1)代入式(2)，得 $\ddot{x}+\alpha\dot{x}+\omega^2x=0$，结果一致。