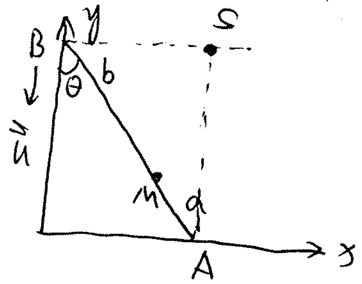
瞬心S的位置

$$\vec r_S = \vec r_B + \frac{1}{\omega^2}(\vec\omega\times\vec v_B) = (a+b)\cos\theta\,\vec j + \frac{1}{\dot\theta^2}\left[\dot\theta\vec k\times(-u\vec j)\right]$$

$$= (a+b)\sin\theta\,\vec i + (a+b)\cos\theta\,\vec j$$

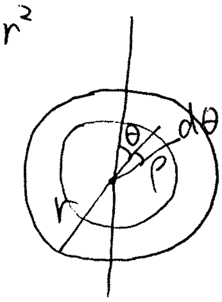
几何方法：



Q3：解：

$$J=\int_V \rho^2 dm = \int_V \rho^2 \sigma dV$$ dV到z轴的垂直距离

$$J_{zz}=J_z = 2\int_0^r\int_0^{2\pi}\int_0^r r^2 \underbrace{r\,dr\,d\theta}_{ds}\, \sigma dz = \sigma\pi r^5 = \frac12 mr^2$$



柱体对x轴的转动惯量J_x，即矩阵元J_{xx}

为求之，可先计算厚度为dz的圆盘薄片对其直径的转动

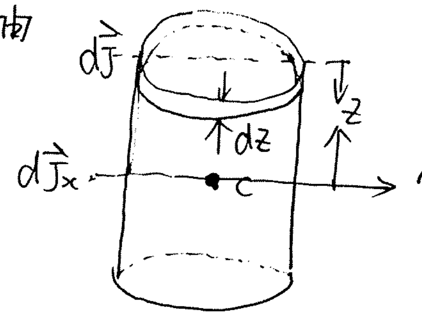
惯量dJ： $$dJ = \int_0^{2\pi}\int_0^r \sigma(\rho\sin\theta)^2 \underbrace{\rho\,d\rho\,d\theta}_{ds}\, dz = \frac14\pi r^4\sigma dz$$

由平行轴定理，可得位于高度为z处的圆盘对x轴

的转动惯量 $$dJ_x = dJ + dm\cdot z^2$$

$$= \frac14\pi r^4\sigma dz + \sigma\pi r^2 dz\cdot z^2$$

$$= \sigma\pi r^2\left(\frac14 r^2 + z^2\right)dz$$



则 $$J_{xx}=J_x = 2\int_0^r \sigma\pi r^2\left(\frac14 r^2+z^2\right)dz = \frac76\pi\sigma r^5 = \frac{7}{12}mr^2 \longrightarrow \sigma\pi r^2\cdot 2r$$

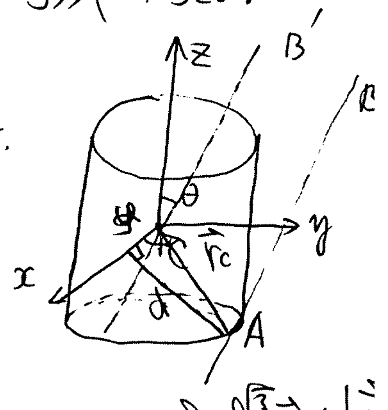
由对称性可知 $J_{yy}=J_y=J_x=J_{xx}$

由于坐标轴是惯量主轴，诸惯量积均为零，柱体对CB'轴的转动惯量为

$$J_{CB'} = (\alpha\ \beta\ \gamma)\begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz}\end{bmatrix}\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = J_{xx}\alpha^2 + J_{yy}\beta^2 + J_{zz}\gamma^2$$

由题知 $\alpha=\cos(\pi/2)=0$，$\beta=\cos30^\circ=\frac{\sqrt3}{2}$，$\gamma=\cos60^\circ=\frac12$.

则 $J_{CB'}=\frac14(3J_y+J_z)=\frac{9}{16}mr^2$



再利用平行轴定理，得柱体对AB轴的转动惯量

$$J_{AB}=J_{CB'}+md^2$$

柱体质心C相对A点位矢 $\vec r_C = -r\vec j + r\vec k$，AB方向单位矢 $\vec e_l = \alpha\vec i+\beta\vec j+\gamma\vec k = \frac{\sqrt3}{2}\vec j+\frac12\vec k$