

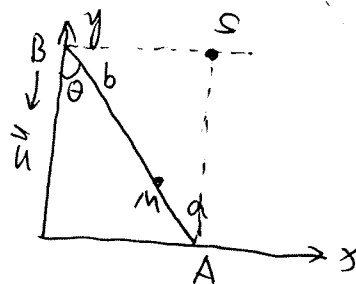
质心的位置

$$\vec{r}_S = \vec{r}_O + \frac{1}{\omega^2} (\vec{\omega} \times \vec{v}_O) = (a+b) \cos \theta \vec{j} + \frac{1}{\theta^2} [\dot{\theta} \vec{k} \times (-u \vec{j})]$$

$$= (a+b) \sin \theta \vec{i} + (a+b) \cos \theta \vec{j}$$

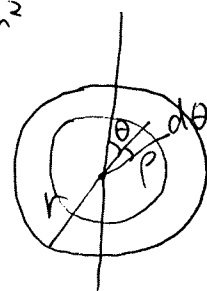
几何方法:

Q3: 解: $J = \int_V \rho^2 dm = \int_V \rho^2 \alpha dv$ dv 到 z 轴的垂直距离



$$J_{zz} = J_z = 2 \int_0^r \int_0^{2\pi} \int_0^r r^2 r dr d\theta \alpha dz = \alpha \pi r^5 = \frac{1}{2} m r^2$$

相对 x 轴的转动惯量 J_x , 即求 J_{xx}
为求之, 可先计算厚度为 dz 的圆盘薄片对其直径的转动

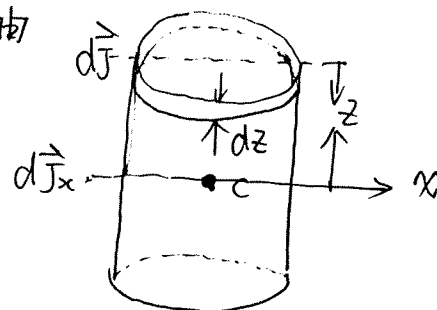


惯量 dJ : $dJ = \int_0^{2\pi} \int_0^r \alpha (\rho \sin \theta)^2 \rho d\rho d\theta dz = \frac{1}{4} \pi r^4 \alpha dz$

由平行轴定理, 可得位于高度为 z 处的圆盘对 x 轴的转动惯量 $dJ_x = dJ + dm \cdot z^2$

$$= \frac{1}{4} \pi r^4 \alpha dz + \alpha \pi r^2 dz \cdot z^2$$

$$= \alpha \pi r^2 \left(\frac{1}{4} r^2 + z^2 \right) dz$$



则 $J_{xx} = J_x = 2 \int_0^r \alpha \pi r^2 \left(\frac{1}{4} r^2 + z^2 \right) dz = \frac{7}{6} \pi \alpha r^5 = \frac{7}{12} m r^2 \rightarrow \alpha \pi r^2 \cdot z r$

由对称性可知 $J_{yy} = J_y = J_z = J_{zz}$

由于坐标轴是惯量主轴, 诸惯量积均为零, 相对 OB' 轴的转动惯量为

$$J_{CB'} = (\alpha \beta \gamma) \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = J_{xx} \alpha^2 + J_{yy} \beta^2 + J_{zz} \gamma^2$$

由题知 $\alpha = \cos(\pi/2) = 0$, $\beta = \cos 30^\circ = \frac{\sqrt{3}}{2}$, $\gamma = \cos 60^\circ = \frac{1}{2}$.

则 $J_{CB'} = \frac{1}{4} (3J_y + J_z) = \frac{9}{16} m r^2$

再用平行轴定理, 求得相对 AB 轴的转动惯量

$$J_{AB} = J_{CB'} + m d^2$$

相对质心 C 相对 A 点距离 $\vec{r}_c = -r \vec{j} + r \vec{k}$, AB 方向单位矢 $\vec{e}_l = \alpha \vec{i} + \beta \vec{j} + \gamma \vec{k} = \frac{\sqrt{3}}{2} \vec{j} + \frac{1}{2} \vec{k}$

