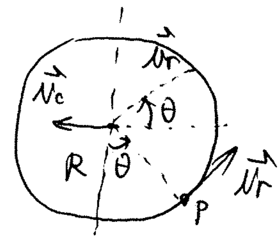
Q3 解: 系统 $s=1$, 选 $\theta$ 为广义坐标。圆盘绕质心的转动速度为

$\vec{\omega} = \dot{\theta}$.




圆盘质心平动速度大小为 $v_c = R\dot{\theta}$

质点p的速度为 $\vec{v} = \vec{v}_c + \vec{v}_r$

其中相对速度 $\vec{v}_r = \vec{\omega} \times \vec{r}_{cp} = R\dot{\theta}\cos\theta\,\vec{i} + \sin\theta\,\vec{j}$

$\therefore$ p的速度 $\vec{v} = \vec{i}R\dot{\theta}(\cos\theta - 1) + \vec{j}R\dot{\theta}\sin\theta$

$\therefore$ $v^2 = 2R^2\dot{\theta}^2(1-\cos\theta)$

系统动能: $T = T_1 + T_2 = \left(\frac{1}{2}M v_c^2 + \frac{1}{2}I_c\omega^2\right) + \frac{1}{2}mv^2$

$$= \frac{1}{2}\left[\frac{3}{2}M + 2m(1-\cos\theta)\right]R^2\dot{\theta}^2$$

体系势能为(取圆盘质心处势能零点 $V_c = 0$), $V = -mgR\cos\theta$

则拉氏函数 $L = \frac{1}{2}\left[\frac{3}{2}M + 2m(1-\cos\theta)\right]R^2\dot{\theta}^2 + mgR\cos\theta$

广义动量 $p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \left[\frac{3}{2}M + 2m(1-\cos\theta)\right]R^2\dot{\theta}$

$$\therefore \dot{\theta} = \frac{p_\theta}{\left[\frac{3}{2}M + 2m(1-\cos\theta)\right]R^2}$$

则 $H = \sum_{\alpha=1}^{1}\frac{\partial L}{\partial \dot{q}_\alpha}\dot{q}_\alpha - L = \frac{\partial L}{\partial \dot{\theta}}\dot{\theta} - L$

$$= \frac{1}{2}\left[\frac{3}{2}M + 2m(1-\cos\theta)\right]R^2\dot{\theta}^2 - mgR\cos\theta$$

$$= \frac{p_\theta^2}{2\left[\frac{3}{2}M + 2m(1-\cos\theta)\right]R^2} - mgR\cos\theta$$

代入正则方程:

$$\begin{cases} \dot{\theta} = \frac{\partial H}{\partial p_\theta} = \frac{p_\theta}{\left[\frac{3}{2}M + 2m(1-\cos\theta)\right]R^2} \\ \dot{p}_\theta = -\frac{\partial H}{\partial \theta} = \frac{p_\theta^2 m\sin\theta}{\left[\frac{3}{2}M + 2m(1-\cos\theta)\right]^2 R^2} - mgR\sin\theta \end{cases}$$