

Unit 3 Bit Hacks

When using bit hacks, keep in mind that usually compiler can do simple bit hacks better. Apply O3 level of optimization might do the job. However, sometimes compiler will not be able to perform bit hacks and we will have to do it by hand.

Set and Clean kth Bit

```
// set kth bit in x
y = x | (1 << k);
// clean kth bit in x
y = x & ~(1 << k);
```

Toggle (flip) kth Bit

```
// flip kth bit in x
y = x ^ (1 << k);
```

Extract a Bit Field

```
// Extract a bit field from x
(x & mask) >> shift;
```

Set a Bit Field

```
// Set a bit field in x to y
x = (x & ~mask) | (y << shift);
```

No-Temp Swap

```
// Swap x and y
x = x ^ y;
y = x ^ y;
x = x ^ y;
```

Minimum of Two Integers

Branching involve branch prediction and if the prediction failed, all the prefetched instruction will have to be discarded. Therefore, branching in program is expensive.

```
// Find the minimum r of two integers x and y
r = y ^ ((x ^ y) & -(x < y));
```

If $x < y$ is true, $-(x < y)$ is -1 which is all 1's in two's complement representation. Therefore, we have $y \wedge (x \wedge y)$ which is x . On the other hand, if $x < y$ is false, $-(x < y)$ will be 0 and $(x \wedge y) \& 0$ will be 0. Therefore, we

have $y \wedge 0$ which is y .

Φ `__restrict` key word can be used in C to tell compiler that the pointer is the only pointer that will point to a certain data. Therefore, compiler will have more freedom to do optimizations.

Modular Addition

Compute $(x + y) \bmod n$, assuming that $0 \leq x < n$ and $0 \leq y < n$.

Φ Division is expensive, unless by a power of 2

```
z = x + y;
r = z - (n & -(z >= 0));
```

Round up to a Power of 2

If n is the power of 2, the bit will not be set

Problem
Compute $2^{\lceil \lg n \rceil}$.

```
uint64_t n;
:
--n;
n |= n >> 1;
n |= n >> 2;
n |= n >> 4;
n |= n >> 8;
n |= n >> 16;
n |= n >> 32;
++n;
```

Bit $\lceil \lg n \rceil - 1$ must be set

Example

0010000001010000
0010000001001111
0011000001101111
0011110001111111
0011111111111111
0100000000000000

Set bit $\lceil \lg n \rceil$

Populate all bits to the right with 1

Log Base 2 of a Power of 2

Problem
Compute $\lg x$, where x is a power of 2.

```
const uint64_t deBruijn = 0x022fdd63cc95386d;
const int convert[64] = {
    0, 1, 2, 53, 3, 7, 54, 27,
    4, 38, 41, 8, 34, 55, 48, 28,
    62, 5, 39, 46, 44, 42, 22, 9,
    24, 35, 59, 56, 49, 18, 29, 11,
    63, 52, 6, 26, 37, 40, 33, 47,
    61, 45, 43, 21, 23, 58, 17, 10,
    51, 25, 36, 32, 60, 20, 57, 16,
    50, 31, 19, 15, 30, 14, 13, 12
};
r = convert[(x * deBruijn) >> 58];
```

Why it works
A *deBruijn sequence* s of length 2^k is a cyclic 0-1 sequence such that each of the 2^k 0-1 strings of length k occurs exactly once as a substring of s .

Example: $k=3$

	00011101
0	000
1	001
2	011
3	111
4	110
5	101
6	010
7	100

```
const int convert[8]
= {0,1,6,2,7,5,4,3};
```