112-2 生物統計學一

單一樣本與兩組樣本檢定

2024/04/16

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假說檢定(Hypothesis Testing)

- 假說檢定：根據樣本提供的證據判斷母體參數假設真偽的統計推論

- $X_1, X_2, \dots, X_n \sim f_X(x;\theta)$ 我們可以針對母體參數θ做一些假設，再由樣本來幫助我們決定支持哪一個假設

- 進行假說檢定時，我們沒辦法避免檢定的決策錯誤，可以分為型一錯誤(type I error)與型二錯誤(type II error)

單一樣本Z檢定(One-sample z-test)

- 目的：檢定某連續型變數其母體平均值是否為某特定值
- 前提假設：
  1. 隨機樣本
  2. 母體服從常態分配
  3. 母體標準差已知
- 虛無假設(null hypothesis)與對立假設(alternative hypothesis)：

  H_0: 母體平均數等於某數值($\mu = \mu_0$)
  H_a: 母體平均數不等於某數值($\mu \neq \mu_0$)

  包含母體平均數的所有可能, 只有一個是對的

- 檢定統計量 Z 服從標準常態分佈：$Z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$ (想要計算 $\bar{x}$ 與 μ_0 差距有多大)

單一樣本Z檢定(One-sample z-test)

- 檢定統計量： $Z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$

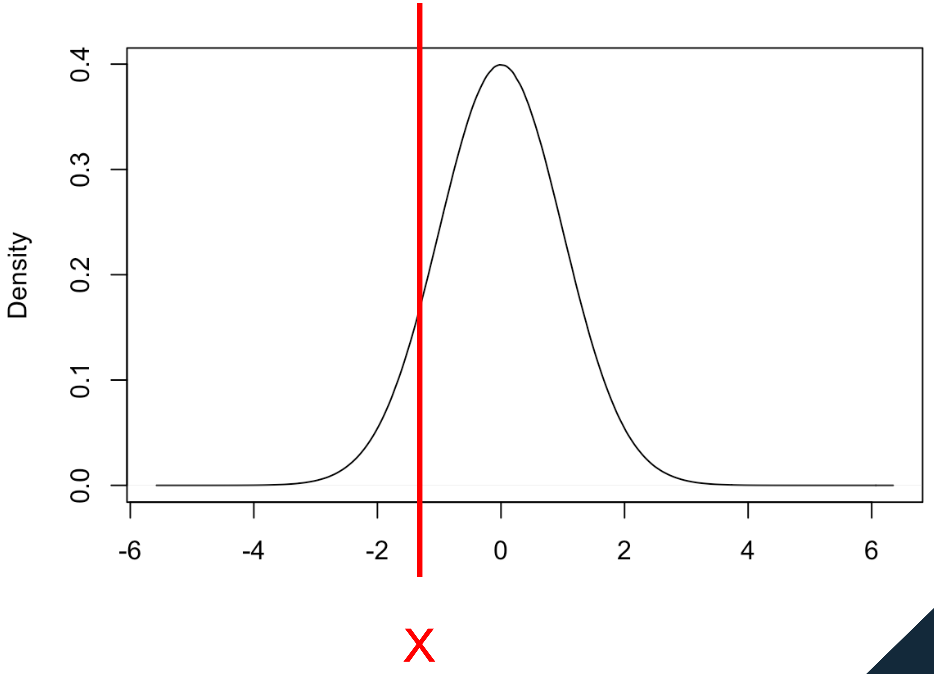
```
> z_obs <- (mean(x) - mu0) / (sigma / sqrt(n))
```

- 計算P-value(在H_0下，能得到比手上這筆樣本更極端的機率)：

```
> pnorm(x, 0, 1)
```

- 給定顯著水準$\alpha = 0.05$下：

1. 若p-value < 0.05，拒絕虛無假設，表示有足夠的證據可以推論$\mu \neq \mu_0$。(μ與μ_0有達統計顯著上的差異)
2. 若p-value > 0.05，無法拒絕虛無假設，表示沒有足夠的證據可以推論$\mu \neq \mu_0$



單一樣本T檢定(One-sample t-test)

- 目的：檢定某連續型變數其母體平均值是否為某特定值
- 前提假設：
  1. 隨機樣本
  2. 母體服從常態分配
  3. 母體標準差未知
- 虛無假設(null hypothesis)與對立假設(alternative hypothesis)：

  H_0: 母體平均數等於某數值($\mu = \mu_0$)

  H_a: 母體平均數不等於某數值($\mu \neq \mu_0$)

- 檢定統計量 T 服從 Student's t 分佈: $T = \dfrac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}}$

單一樣本T檢定(One-sample t-test)

- 檢定統計量：$T = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}}$

```
> t_obs <- (mean(x) - mu) / (S / sqrt(n))
```

- 計算P-value：

```
> pt(x, df)
```

- 也可以使用函數：

```
> t.test(studata1$Age, alternative = "two.sided", mu = 18)

        One Sample t-test

data:  studata1$Age
t = 2.2341, df = 43, p-value = 0.03073
alternative hypothesis: true mean is not equal to 18
95 percent confidence interval:
 18.13714 20.68105
sample estimates:
mean of x
 19.40909
```

兩獨立樣本T檢定(Two independent sample t-test)

- Step 1. 設立虛無與對立假說
- Step 2. 判斷變異數同質性

```
> var.test(y ~ x)
```

- Step 3. 依據變異數同質/異質，算出對應的檢定統計量 t、決定自由度
  - 若變異數同質 (相等) (equal variance)

    ```
    > t.test(y ~ x, var.equal = T)
    ```

  - 若變異數異質 (不相等) (unequal variance)

    ```
    > t.test(y ~ x, var.equal = F)
    ```

- Step 4. 判斷顯著性並下結論

配對樣本T檢定(Paired t-test)

```
> t.test(studata1$Credit_Last, studata1$Credit_Current, paired = T)

        Paired t-test

data:  studata1$Credit_Last and studata1$Credit_Current
t = 1.1757, df = 49, p-value = 0.2454
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.035608  3.955608
sample estimates:
mean of the differences
                   1.46
```

練習

請先匯入Studata.csv，並命名為studata，並進行以下檢定：

(假設本學期生統一修課同學是一組由台大學生抽出的隨機樣本)

- 台大學生的平均身高 (Ht) 是否為 165 cm?
- 台大女學生的平均身高 (Ht) 是否為 160 cm?
- 請問台大男、女同學的平均身高 (Ht) 是否相同?

(補充)單一樣本

Summary: One-Sample, Two-Sided Hypothesis Tests for the Mean

Standard deviation	Known	Unknown
Null hypothesis	$H_0 : \mu = \mu_0$ or $H_0 : \mu - \mu_0 = 0$	$H_0 : \mu = \mu_0$ or $H_0 : \mu - \mu_0 = 0$
Alternative hypothesis	$H_A : \mu \neq \mu_0$ or $H_A : \mu - \mu_0 \neq 0$	$H_A : \mu \neq \mu_0$ or $H_A : \mu - \mu_0 \neq 0$
Test	One-sample, two-sided z-test	One sample, two-sided t-test
Test statistic	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$
Distribution of test statistic	Standard normal	t distribution with $n - 1$ degrees of freedom

Summary: One-Sample, One-Sided Hypothesis Test for the Mean

Variance	Known	Unknown
Null hypothesis	$H_0 : \mu \geq \mu_0$ or $H_0 : \mu - \mu_0 \geq 0$	$H_0 : \mu \geq \mu_0$ or $H_0 : \mu - \mu_0 \geq 0$
Alternative hypothesis	$H_A : \mu < \mu_0$ or $H_A : \mu - \mu_0 < 0$	$H_A : \mu < \mu_0$ or $H_A : \mu - \mu_0 < 0$
Test	One-sample, one-sided z-test	One-sample, one-sided t-test
Test statistic	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$
Distribution of test statistic	Standard normal	t distribution with $n - 1$ degrees of freedom

Source: Pagano, M., Gauvreau, K., & Mattie, H. (2022). *Principles of biostatistics*. Chapman and Hall/CRC.

(補充)配對樣本

Summary: Two-Sample Hypothesis Test for Means, Paired Samples

Null hypothesis	$H_0 : \mu_1 = \mu_2$ or $H_0 : \delta = \mu_1 - \mu_2 = 0$
Alternative hypothesis	$H_A : \mu_1 \neq \mu_2$ or $H_A : \delta = \mu_1 - \mu_2 \neq 0$
Test	Paired t-test
Test statistic	$t = \dfrac{\bar{d} - \delta}{s_d/\sqrt{n}}$
Distribution of test statistic	t distribution with $n - 1$ degrees of freedom

Source: Pagano, M., Gauvreau, K., & Mattie, H. (2022). *Principles of biostatistics*. Chapman and Hall/CRC.

(補充)兩獨立樣本

Summary: Two-Sample Hypothesis Tests for Means, Independent Samples, Equal Variances

Variance	Known	Unknown
Null hypothesis	$H_0 : \mu_1 = \mu_2$ or $H_0 : \mu_1 - \mu_2 = 0$	$H_0 : \mu_1 = \mu_2$ or $H_0 : \mu_1 - \mu_2 = 0$
Alternative hypothesis	$H_A : \mu_1 \neq \mu_2$ or $H_A : \mu_1 - \mu_2 \neq 0$	$H_A : \mu_1 \neq \mu_2$ or $H_A : \mu_1 - \mu_2 \neq 0$
Test	Two-sample, two-sided z-test with equal variances	Two-sample, two-sided t-test with equal variances
Test statistic	$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma^2[(1/n_1) + (1/n_2)]}}$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2[(1/n_1) + (1/n_2)]}}$
Distribution of test statistic	Standard normal	t distribution with $n_1 + n_2 - 2$ degrees of freedom

Equation for pooled estimate of variance: $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Summary: Two-Sample Hypothesis Test for Means, Independent Samples, Unequal Variances

Variance	Known	Unknown
Null hypothesis	$H_0 : \mu_1 = \mu_2$ or $H_0 : \mu_1 - \mu_2 = 0$	$H_0 : \mu_1 = \mu_2$ or $H_0 : \mu_1 - \mu_2 = 0$
Alternative hypothesis	$H_A : \mu_1 \neq \mu_2$ or $H_A : \mu_1 - \mu_2 \neq 0$	$H_A : \mu_1 \neq \mu_2$ or $H_A : \mu_1 - \mu_2 \neq 0$
Test	Two-sample, two-sided z-test with unequal variances	Two-sample, two-sided t-test with unequal variances
Test statistic	$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$
Distribution of test statistic	Standard normal	t distribution with ν degrees of freedom

Degrees of freedom: $\nu = \dfrac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\left[(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)\right]}$

Source: Pagano, M., Gauvreau, K., & Mattie, H. (2022). *Principles of biostatistics*. Chapman and Hall/CRC.