

Tractable Approximate Counting for CQs

报告人：陈鹏宇

哈尔滨工业大学

2023 年 4 月 22 日

Content

Conjunctive Query (CQ)

CQ $Q(\bar{x}) \leftarrow R_1(\bar{y}_1), R_2(\bar{y}_2), \dots, R_n(\bar{y}_n)$, and database $D = (R_1^D, \dots, R_n^D)$

Homomorphism $h: V \cup C \rightarrow C$ s.t.

1. $h(c) = c$ for all $c \in C$

2. $h(\bar{y}_i) = (h(y_{i,1}), \dots, h(y_{i,k_i})) \in R_i^D$ for all $i \in [n]$

Answers to a CQ

$$Q(D) = \{h(\bar{x}) \in C^m \mid h \text{ a homomorphism}\}$$

Conjunctive Query Complexity

Query Evaluation Problem: is $Q(D) = \emptyset$?

- NP-Hard in general, poly-time if $tw(Q)$ or $htw(Q)$ bounded.
- No poly-time for if $tw(\mathcal{G})$ unbounded.

Counting Problem: Output $|Q(D)|$

- #P-Hard in general
- #P-Hard even if $tw(Q) = 1$ [Pichler, Skritek '13]

Approximate Counting: Output R s.t. $R = (1 \pm \epsilon)|Q(D)|$

- No poly-time in general unless P=NP.
- What about bounded (hyper)-treewidth?

Content

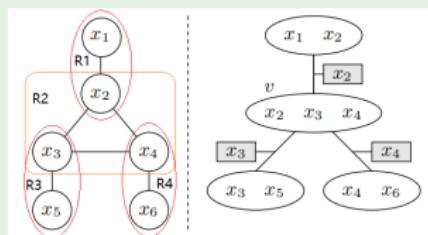
Tree Decomposition

Definition 7.2: Consider a graph $G = \langle V, E \rangle$. A tree decomposition of G is a tree structure T where each node of T is a subset of V , such that:

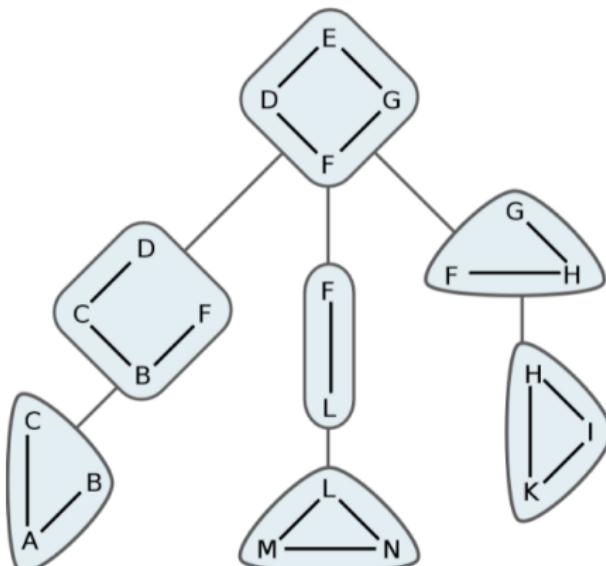
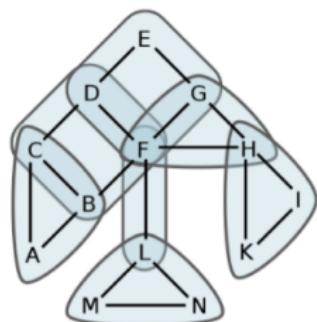
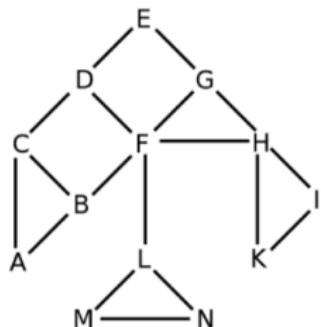
- The union of all nodes of T is V .
- For each edge $(v_1 \rightarrow v_2) \in E$, there is a node N in T such that $v_1, v_2 \in N$.
- For every vertex $v \in V$, the set of nodes of T that contain v form a subtree of T ; equivalently: if two nodes contain v , then all nodes on the path between them also contain v (**connectedness condition**).

例 1 (Treewidth=2)

$$\begin{aligned}\varphi(x_1, x_5, x_6) &\leftarrow R_1(x_1, x_2), \\ &R_2(x_2, x_3, x_4), \\ &R_3(x_3, x_5), \\ &R_4(x_4, x_6)\end{aligned}$$



Tree Decomposition



Tree Width

The treewidth of a graph defines how “tree-like” it is:

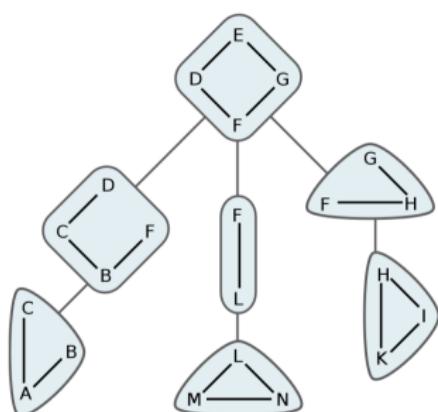
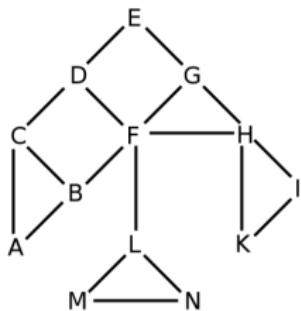
Definition 7.3: The **width** of a tree decomposition is the size of its largest bag minus one.

The **treewidth** of a graph G , denoted $\text{tw}(G)$, is the smallest width of any of its tree decompositions.

Simple observations:

- If G is a tree, then we can decompose it into bags that contain only one edge
 ~ trees have treewidth 1
- Every graph has at least one tree decomposition where all vertices are in one bag
 ~ maximal treewidth = number of vertices – 1

Tree Width



~ tree decomposition of width 3

~ tree decomposition of width 2 = treewidth of the example graph

Tree Width via Games

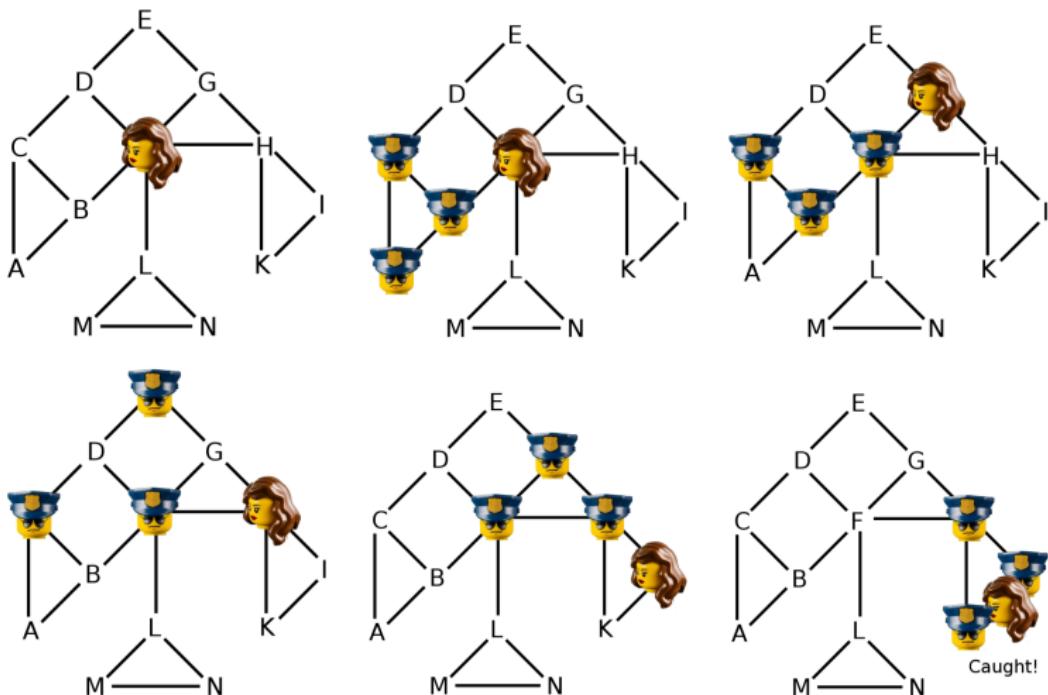
Seymour and Thomas [1993] gave an alternative characterisation of treewidth:

The Cops-and-Robber Game

- The game is played on a graph G
- There are k cops and one robber that may be positioned at vertices
- In the first turn, the robber places herself at an arbitrary vertex of the graph; the cops are all in a “helicopter” (i.e., not yet placed on any vertex)
- In each turn:
 - one of the cops can decide to “fly” to an arbitrary vertex in the graph
 - if the moving cop is already in the game, he is lifted from his vertex
 - before “landing” (i.e. positioning the cop at his new vertex), the target vertex is announced to the robber (the robber sees the helicopter approaching)
 - the robber can run along the edges of the graph, as far as she likes, as long as she does not use any vertex currently occupied by a cop
 - the moving cop arrives at his destination vertex
- The cops’ goal is to catch the robber; the robber’s goal is never to be caught



Tree Width via Games



Theorem 7.6 (Seymour and Thomas): A graph G is of treewidth $\leq k - 1$ if and only if k cops have a winning strategy in the cops & robber game on G .

Summary of Tree Width

Graphs of bounded treewidth as a generalisation of (undirected) trees:

- Trees have treewidth 1
- Graphs of higher treewidth resemble trees with “thicker branches”
- It is (in theory) not hard to check if a graph has treewidth $\leq k$ for some k
- It is (in theory) not hard to answer BCQs whose primal graph has a bounded treewidth

Practically feasible only for lower treewidths

However, bounded treewidth does not generalise the notion of hypergraph acyclicity
(acyclic families of hypergraphs may have unbounded treewidth)

Is there a better notion of tree-likeness for hypergraphs?

Content

Query Width

Idea of Chekuri and Rajamaran [1997]:

- Create tree structure similar to tree decomposition
- But consider bags of query atoms instead of bags of variables
- Two connectedness conditions:
 - (1) Bags that refer to a certain variable must be connected
 - (2) Bags that refer to a certain query atom must be connected

Query width: least number of atoms needed in bags of a query decomposition

Theorem 8.1: Given a query decomposition for a BCQ, the query answering problem can be decided in time polynomial in the query width.

Theorem 8.2 (Gottlob et al. 1999): Deciding if a query has query width at most k is NP-complete.

In particular, it is also hard to find a query decomposition

~> Query answering complexity drops from NP to P ...
... but we need to solve another NP-hard problem first!

Hyper Tree Decomposition¹

Definition 8.3: Consider a hypergraph $G = \langle V, E \rangle$. A **hypertree decomposition** of G is a tree structure T where each node n of T is associated with a bag of variables $B_n \subseteq V$ and with a set of edges $G_n \subseteq E$, such that:

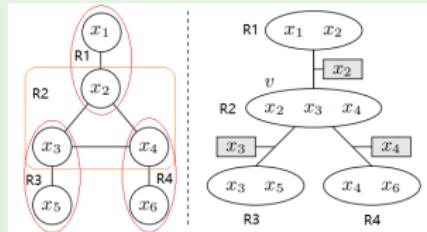
- T with B_n yields a tree decomposition of the primal graph of G .
- For each node n of T :
 - (1) the vertices used in the edges G_n are a superset of B_n ,
 - (2) if a vertex v occurs in an edge of G_n and this vertex also occurs in B_m for some node m below n in T , then $v \in B_n$.

The **width** to T is the largest number of edges in a set G_n .

The **hypertree width** of G , $hw(G)$, is the least width of its hypertree decompositions.

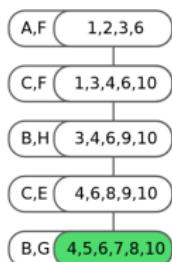
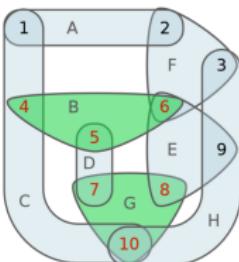
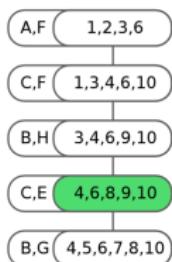
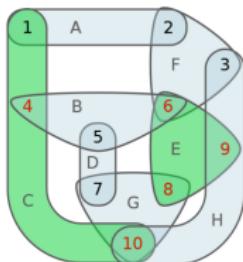
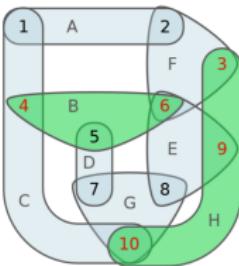
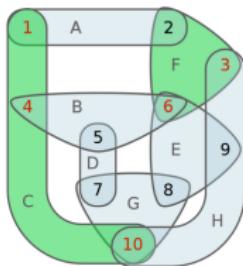
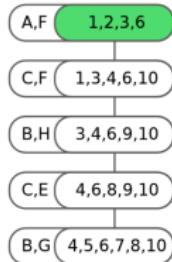
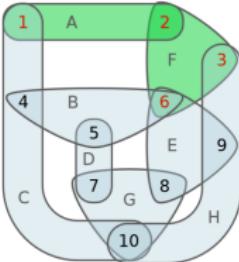
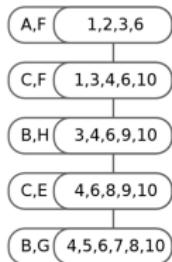
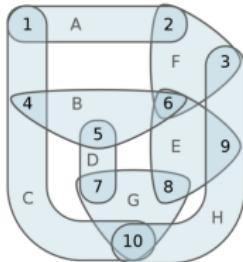
例 2 (Treewidth=2, Hyperwidth=1)

$$\begin{aligned}\varphi(x_1, x_5, x_6) &\leftarrow R_1(x_1, x_2), \\ &R_2(x_2, x_3, x_4), \\ &R_3(x_3, x_5), \\ &R_4(x_4, x_6)\end{aligned}$$

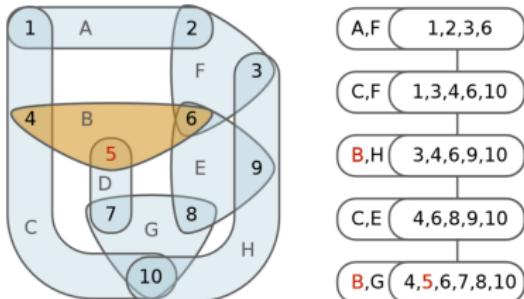


¹(2) is the “special condition” : without it we get the generalized hypertree width

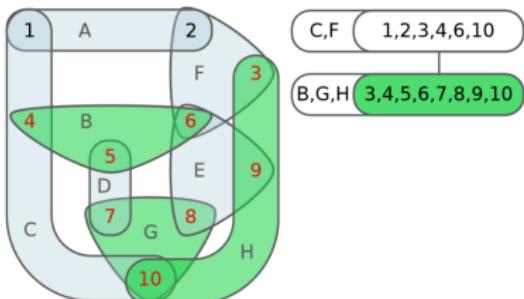
(Generalized) Hyper Tree Decomposition



(Generalized) Hyper Tree Decomposition



Special condition violated \rightsquigarrow no hypertree decomposition
 \rightsquigarrow But generalised hypertree decomposition of width 2



Special condition satisfied \rightsquigarrow hypertree decomposition of width 3

(Generalized) Hyper Tree Results

- Relationships of hypergraph tree-likeness measures:
generalised hypertree width \leq hypertree width \leq query width
(both inequalities might be $<$ in some cases)
- Acyclic graphs have hypertree width 1
- Deciding “query width $< k$?” is NP-complete
- Deciding “generalised hypertree width < 4 ?” is NP-complete
- Deciding “hypertree width $< k$?” is polynomial (LOGCFL)
- Hypertree decompositions can be computed in polynomial time if k is fixed

Theorem 8.9: For a BCQ of (generalised) hypertree width k , query answering can be decided in polynomial time, and is complete for LOGCFL.

Summary of (Hyper) Tree Width

Treewidth

Graph $G(Q) = (V, E)$

- $V :=$ variables in Q
- $(u, v) \in E$ if $u, v \in \bar{y}_i$ for some i
- $tw(G) :=$ measures how close G is to a tree
- $tw(G) = 1 \leftrightarrow G$ acyclic

$$tw(Q) = tw(G(Q))$$

Hyper-Treewidth

Hyper-Graph $H(Q) = (V, \mathcal{E})$

- $V :=$ variables in Q
- $\mathcal{E} := \{ \{v \mid v \in \bar{y}_i\} \mid i \in [n] \}$
- $htw(H) :=$ how close H is to acyclic
- [Gottlob, Leone, Scarcello '02]

$$htw(Q) \leq tw(Q)$$

Content

Main Results

Theorem: Let Q be a CQ with $htw(Q) = O(1)$, and D a database. Then $Q(D)$ admits an FPRAS.

- Extends to unions of CQ's with bounded htw
- $htw(Q) \leq tw(Q)$, so we obtain an FPRAS for bounded treewidth.

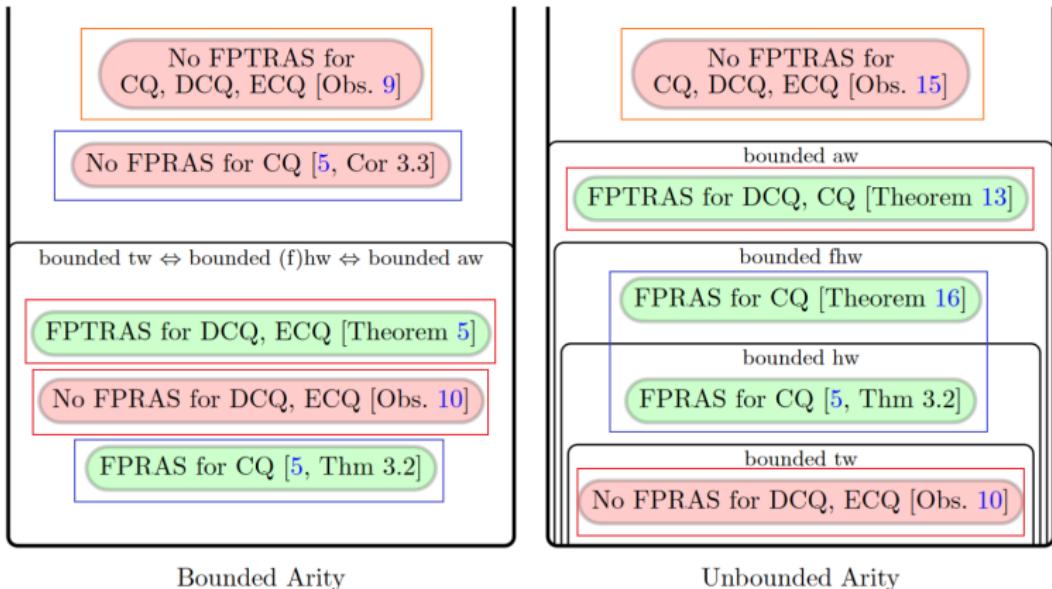
Proof Approach:

1. Construct automata A , with $|\mathcal{L}_n(A)| = |Q(D)|$, where $\mathcal{L}_n(A) :=$ set of inputs of size n accepted by A .
 2. Design FPRAS for $\mathcal{L}_n(A)$
-
- FPRAS²: 时间为 $\text{poly}(|\mathcal{D}| + |\varphi|, \varepsilon^{-1}, \log \delta^{-1})$
 - FPTRAS³: 时间为 $f(|\varphi|) \cdot \text{poly}(|\mathcal{D}|, \varepsilon^{-1}, \log \delta^{-1})$

²fully polynomial randomised approximation scheme

³fixed-parameter tractable randomised approximation scheme

Main Results



- 橙色：宽松条件下的复杂性下界
- 红色：#DCQ、#ECQ 的_(参数化) 算法及复杂性下界
- 蓝色：#CQ 的_(FPRAS) 算法及复杂性下界

Content

Tree Automata (TA)

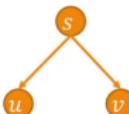
“Branching” Generalization of NFA’s.

➤ Tree automata T accepts a language $\mathcal{L}(T)$ of *ordered trees*.

➤ NFA: one state to another

➤ TA: one state to **one or more** child states.

$(s, u, v) \in \Delta$ means:



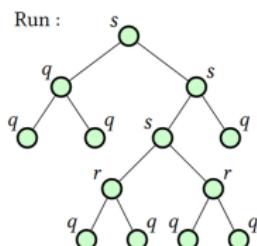
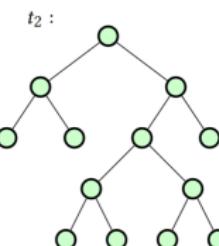
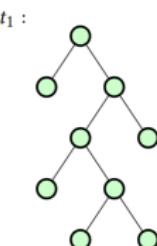
$s \in \Delta$ means
s a leaf



This talk, Binary Tree Automata: $T = (S, \Delta, s_{init})$

1. $S :=$ set of states
2. $\Delta \subseteq (S \times S \times S) \times S$ is a *transition relation*
3. $s_{init} \in S$ is the *initial state*.

$$\begin{array}{ll}\mathcal{T} := & \begin{array}{l}s \rightarrow sq \\ s \rightarrow qs \\ s \rightarrow rr \\ r \rightarrow qq \\ q \rightarrow qq \\ q \rightarrow \cdot \\ s_{init} = s\end{array}\end{array}$$

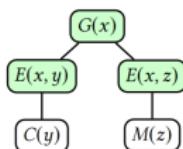


From Conjunctive Query to Tree Automata

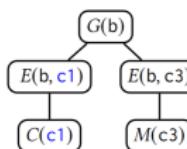
Theorem: Given Q, D with $htw(Q) = O(1)$, there is a $\text{poly}(|Q|, |D|)$ -time algorithm which outputs a TA T and $n = \text{poly}(|Q|, |D|)$ with:

$$|\mathcal{L}_n(T)| = |Q(D)|$$

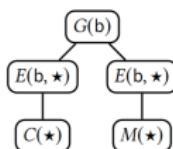
- $Q(x) \leftarrow G(x), E(x, y), E(x, z), C(y), M(z)$



(a) A join tree.



(b) Two witness trees for the answer b.



(c) An anonymous tree for b.

Theorem: Let T be a tree automaton and $n \geq 1$. Then $\mathcal{L}_n(T)$ admits and FPRAS (running in $\text{poly}(n, |T|, \frac{1}{\epsilon})$ -time).

Content

Counting via Sampling

Dynamic Programming via sampling

- $\mathcal{L}_i(s) :=$ trees of size i accepted by T with initial state $s \in S$

$$|\mathcal{L}_i(s)| = \left| \bigcup_{\substack{(s,x,y) \in \Delta \\ j+k=i-1}} \mathcal{L}_j(x) \times \mathcal{L}_k(y) \right|$$

- **Challenge:** union non-disjoint, (size of union) \neq (sum of sizes)

- **Solution:** Estimate $|\mathcal{L}_i(s)|$ via *sample sets* $\mathcal{S}_j(x)$ of $\mathcal{L}_j(x)$ for all $j < i$, $x \in S$

- $\mathcal{S}_j(x) :=$ uniform samples from $\mathcal{L}_j(x)$.

- Given sample sets $\mathcal{S}_j(x)$, straightforward to estimate $|\mathcal{L}_i(s)|$.

- Similar approach in [ACJR'19] for #NFA.

Counting via Sampling

$$|\mathcal{L}_i(s)| = \left| \bigcup_{\substack{(s,x,y) \in \Delta \\ j+k=i-1}} \mathcal{L}_j(x) \times \mathcal{L}_k(y) \right|$$

➤ **Main Challenge:** How to generate samples from $\mathcal{L}_j(x)$ to form $\mathcal{S}_j(x)$?

➤ **Karp-Luby Sampling** in QPRAS of [Gore, Jerrum, Kannan, Sweedyk, and Mahaney '97]

1. Sample a $\mathcal{L}_j(x) \times \mathcal{L}_k(y)$ proportional to $|\mathcal{L}_j(x) \times \mathcal{L}_k(y)|$
2. Recursively Sample $e \sim \mathcal{L}_j(x) \times \mathcal{L}_k(y)$
3. Accept e with probability $= \frac{1}{|\text{other sets containing } e|}$

➤ **Our Solution:** Partition-and-Sample

1. Partition $\bigcup \mathcal{L}_j(x) \times \mathcal{L}_k(y)$ into disjoint T_1, T_2, \dots, T_k
2. Sample T_i proportional to $\approx |T_i|$
3. Recursively sample $e \sim T_i$

$$\sum_{j=1}^k \tilde{N}(T_j) \left(\frac{|\tilde{T}_j \setminus \bigcup_{j' < j} T_{j'}|}{|\tilde{T}_j|} \right)$$

➤ **Technical contribution:** How to partition, and how to estimate $|T_i|$.

- Much harder for TA than NFA

Summary and Open Problem

NFA

$\mathcal{L}_n(N)$:= size n words accepted by N

- Decision problem = PTIME
- Exact counting = #P-Hard
- Approximate counting = FPRAS

Tree Automata

$\mathcal{L}_n(T)$:= size n trees accepted by T

- Decision problem = PTIME
- Exact counting = #P-Hard
- Approximate counting = FPRAS

Context Free Grammar

$\mathcal{L}_n(G)$:= size n words accepted by CFG G

- Decision problem = PTIME
- Exact counting = #P-hard
- Approximate counting = ??