

This repository provides the codes for the numerical simulations in the paper ‘On zonal instability and the effect of magnetic field’ by Chen Wang, Joanne Mason and Andrew Gilbert. Following is a brief guidance for using the codes.

1. We provide the Matlab codes that solve the full MHD equations in two dimensional space with random forcing, i.e. equations (3.1)-(3.3) in the paper. The code to compute the theoretical growth rate is not presented here, as its equation is algebraic and straight forward to solve numerically.
2. The main code to perform the computation is `simulation.m`. It will generate the data of the numerical solution and plot some sample figures during the computation. The essential descriptions have been given inside the code as comments. `simulation.m` has used the function files `fft_df_2d.m`, `Laplacian_fft.m` and `Laplacian_fft_inv.m`, which compute a variable’s gradient, Laplacian and inverse Laplacian, respectively. `simulation.m` has also used the Matlab function `"bm"` to generate a time sequence of Brownian motion (lines 55-65); one may need to install the Financial Toolbox to access this function.
3. Once a realisation is performed using `simulation.m`, one may use `"plot_zeta_U.m"` and `"plot_zeta_j.m"` to plot snapshots of vorticity, current density and mean-flow profile, in the style of figures 2, 10, 12, and 14.
4. The code `simulation.m` performs one realisation of the stochastic process. To compare the results of numerical simulation and theoretical analysis, one needs to perform multiple realisations, compute the ensemble average, and then compare to the expectation given by the theory.
5. The white noises that drive the flow shown in figures 10, 12 and 14 in the paper have been provided in the data files `"noise_fig_10.mat"`, `"noise_fig_12.mat"` and `"noise_fig_14.mat"`. To use these data, one needs to load them to the data space and then comment out the commands in lines 55-65 in `simulation.m` as they would and overwrite the data.
6. The code encounters numerical difficulties if the magnetic diffusivity is too weak. At the lowest magnetic diffusivity in the paper, $\eta = 10^{-5}$, with 512×512 grid points/Fourier modes and other parameter specified in the paper, roughly in one out of ten realisations the simulation runs away. In such situations, when the small-scale filaments in the magnetic field forms, grid-size numerical errors quickly blow up, making the simulation fail. This problem becomes more severe when η becomes even smaller. Our preliminary investigation suggests this difficulty might be alleviated via higher resolution or higher-order temporal scheme, which has to be left for future study. Nevertheless, we have confirmed that as long as the simulation does not run away, the numerical solution is robust via testing convergence with respect to grid size and time steps.