主动连通保持

Q2

权重函数

$$w_{ij} = \exp\left[-c_1(\psi_{ij}-\varphi_i)^2 - c_2\varphi_i^2 - c_3(l_{ij}-\frac{\sqrt{a^2+b^2}}{2})^2\right]$$

$$w_{ij} = \exp\left[-c_1(\psi_{ij}-\varphi_i)^2 - c_2\varphi_i^2 - c_3(|\theta_j-\theta_i|-\frac{\pi}{2})^2\right]$$

$$w_{ij} = \exp\left[-c_1\sin^2(\psi_{ij}-\varphi_i)\right] + \exp\left[-c_2\sin^2\varphi_i\right] + \exp\left[-c_3\cos^2(\theta_j-\theta_i)\right]$$

$$w_{ij} = c_1\exp\left[-(\psi_{ij}-\varphi_i)^2/\sigma_1^2\right] + c_2\exp\left(-\varphi_i^2/\sigma_2^2\right) + c_3\exp\left[-c_3(|\theta_j-\theta_i|-\frac{\pi}{2})^2/\sigma_3^2\right]$$

拉普拉斯矩阵

$$\begin{aligned}
\widetilde{w}_{ij} &= w_{ij} + w_{ji} \\
&= \exp\left[-c_1(\psi_{ij}-\varphi_i)^2 - c_2\varphi_i^2 - c_3(l_{ij}-\frac{\sqrt{a^2+b^2}}{2})^2\right] \\
&+ \exp\left[-c_1(\psi_{ji}-\varphi_j)^2 - c_2\varphi_j^2 - c_3(l_{ij}-\frac{\sqrt{a^2+b^2}}{2})^2\right]
\end{aligned}$$

$$L = \begin{bmatrix}
\widetilde{w}_{12}+\widetilde{w}_{13}+\widetilde{w}_{14} & -\widetilde{w}_{12} & -\widetilde{w}_{13} & -\widetilde{w}_{14} \\
-\widetilde{w}_{12} & \widetilde{w}_{12}+\widetilde{w}_{23}+\widetilde{w}_{24} & -\widetilde{w}_{23} & -\widetilde{w}_{24} \\
-\widetilde{w}_{13} & -\widetilde{w}_{23} & \widetilde{w}_{13}+\widetilde{w}_{23}+\widetilde{w}_{34} & -\widetilde{w}_{34} \\
-\widetilde{w}_{14} & -\widetilde{w}_{24} & -\widetilde{w}_{34} & \widetilde{w}_{14}+\widetilde{w}_{24}+\widetilde{w}_{34}
\end{bmatrix}$$

控制律

$$u_i^\theta = \frac{\partial\lambda_2}{\partial\theta_i}$$
$$u_i^\varphi = \frac{\partial\lambda_2}{\partial\varphi_i}$$

Q3

$$\begin{aligned}
u_i &= \dot{p}_i = \frac{\partial\lambda_2}{\partial p_i} \\
\dot{u}_i &= \ddot{p}_i = \frac{\partial}{\partial t}(\frac{\partial\lambda_2}{\partial p_i}) = \frac{\partial}{\partial p_i}(\frac{\partial\lambda_2}{\partial p_i})\cdot\frac{\partial p_i}{\partial t} = \frac{\partial}{\partial p_i}(\frac{\partial\lambda_2}{\partial p_i})u_i = \frac{\partial u_i}{\partial p_i}u_i
\end{aligned}$$

$$
\begin{aligned}
u_i^\theta &= \frac{\partial \lambda_2}{\partial \theta_i} \\
\dot{u}_i^\theta &= \frac{\partial u_i^\theta}{\partial \theta_i} u_i^\theta = \frac{\partial^2 \lambda_2}{\partial \theta_i^2} \cdot \frac{\partial \lambda_2}{\partial \theta_i} \\
u_i^\varphi &= \frac{\partial \lambda_2}{\partial \varphi_i} \\
\dot{u}_i^\varphi &= \frac{\partial u_i^\varphi}{\partial \varphi_i} u_i^\varphi = \frac{\partial^2 \lambda_2}{\partial \varphi_i^2} \cdot \frac{\partial \lambda_2}{\partial \varphi_i}
\end{aligned}
$$

Q4

λ_2 与 v_2 的估计

$$
u_i = \frac{\partial \lambda_2}{\partial p_i} = v_2^T \frac{\partial L}{\partial p_i} v_2
$$

$$
\begin{aligned}
\alpha_1^i &= \tilde{v}_2^i \\
\alpha_2^i &= (\tilde{v}_2^i)^2 \\
z_1^i &= \mathrm{Ave}(\tilde{v}_2^i) \\
z_2^i &= \mathrm{Ave}((\tilde{v}_2^i)^2)
\end{aligned}
$$

$$
\begin{aligned}
\dot{z}^i &= \gamma(\alpha^i - z^i) - K_p \sum_{j \in \mathcal{N}_i} (z^i - z^j) + K_i \sum_{j \in \mathcal{N}_i} (\omega^i - \omega^j) \\
\dot{\omega}^i &= -K_i \sum_{j \in \mathcal{N}_i} (z^i - z^j) \\
\dot{\tilde{v}}_2^i &= -k_1 z_1^i - k_2 \sum_{j \in \mathcal{N}_i} a_{ij} (\tilde{v}_2^i - \tilde{v}_2^j) - k_3 (z_2^i - 1) \tilde{v}_2^i
\end{aligned}
$$

$$
\begin{aligned}
\dot{z}^i &= \gamma(\alpha^i - z^i) - K_p \sum_{j \in \mathcal{N}_i} (z^i - z^j) + K_i \sum_{j \in \mathcal{N}_i} (\omega^i - \omega^j) \\
\dot{\omega}^i &= -K_i \sum_{j \in \mathcal{N}_i} (z^i - z^j) \\
\dot{\tilde{v}}_2^i &= -k_1 z_1^i - k_2 \sum_{j \in \mathcal{N}_i} a_{ij} (\tilde{v}_2^i - \tilde{v}_2^j)
\end{aligned}
$$

$$
\tilde{v}_2^i = \frac{\tilde{v}_2^i}{|\tilde{v}_2^i|}
$$

$$
\begin{aligned}
u_i = \tilde{v}_2^T \frac{\partial L}{\partial p_i} \tilde{v}_2 &= \sum_{j \in \mathcal{N}_i} \frac{\partial \widetilde{w}_{ij}}{\partial p_i} (\tilde{v}_2^i - \tilde{v}_2^j)^2 \\
\widetilde{w}_{ij} = \exp &\left[-c_1 (\psi_{ij} - \varphi_i)^2 - c_2 \varphi_i^2 - c_3 (l_{ij} - \frac{\sqrt{a^2 + b^2}}{2})^2 \right] \\
+ \exp &\left[-c_1 (\psi_{ji} - \varphi_j)^2 - c_2 \varphi_j^2 - c_3 (l_{ij} - \frac{\sqrt{a^2 + b^2}}{2})^2 \right]
\end{aligned}
$$

模型优化

$$
\begin{aligned}
\widetilde{w}_{ij} &= w_{ij} + w_{ji} \\
&= \exp\left[-c_1 \sin^2(\psi_{ij} - \varphi_i) \right] + \exp\left[-c_2 \sin^2 \varphi_i \right] + \exp\left[-c_3 \cos^2(\theta_j - \theta_i) \right] \\
&+ \exp\left[-c_1 \sin^2(\psi_{ji} - \varphi_j) \right] + \exp\left[-c_2 \sin^2 \varphi_j \right] + \exp\left[-c_3 \cos^2(\theta_j - \theta_i) \right]
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{w}_{ij} &= w_{ij} + w_{ji} \\
&= \exp\left[-c_1 \cos^2(\frac{\theta_i + \theta_j}{2} - \varphi_i)\right] + \exp\left[-c_2 \sin^2 \varphi_i\right] + \exp\left[-c_3 \cos^2(\theta_j - \theta_i)\right] \\
&+ \exp\left[-c_1 \cos^2(\frac{\theta_i + \theta_j}{2} - \varphi_j)\right] + \exp\left[-c_2 \sin^2 \varphi_j\right] + \exp\left[-c_3 \cos^2(\theta_j - \theta_i)\right]
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{w}_{ij} &= w_{ij} + w_{ji} \\
&= \exp\left[-c_1 \sin^2(\arctan\frac{\sin\theta_j - \sin\theta_i}{\cos\theta_j - \cos\theta_i} - \varphi_i)\right] + \exp\left[-c_2 \sin^2 \varphi_i\right] + \exp\left[-c_3 \cos^2(\theta_j - \theta_i)\right] \\
&+ \exp\left[-c_1 \sin^2(\arctan\frac{\sin\theta_i - \sin\theta_j}{\cos\theta_i - \cos\theta_j} - \varphi_j)\right] + \exp\left[-c_2 \sin^2 \varphi_j\right] + \exp\left[-c_3 \cos^2(\theta_j - \theta_i)\right]
\end{aligned}
$$

$$
\begin{aligned}
u_i^\theta &= \sum_{j \in \mathcal{N}_i} \frac{\partial \widetilde{w}_{ij}}{\partial \theta_i} (\tilde{v}_2^i - \tilde{v}_2^j)^2 \\
u_i^\varphi &= \sum_{j \in \mathcal{N}_i} \frac{\partial \widetilde{w}_{ij}}{\partial \varphi_i} (\tilde{v}_2 - \tilde{v}_j)^2
\end{aligned}
$$