

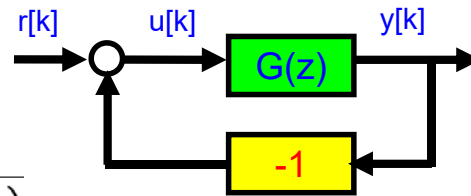
## Homework 4: Stability, Controllability, Observability

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DCS00-HW-11

### Problem 4-1:

- Consider the system and let:

$$G(z) = \frac{K}{z(z - 0.2)(z - 0.4)}$$



- where  $K > 0$ .
- Determine the values of  $K$  for which the closed-loop system is stable.

### Problem 4-2:

- Is the following system (a) stable, (b) observable, (c) reachable?

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.5 & -0.5 \\ 0 & 0.25 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 6 \\ 4 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 2 & -4 \end{bmatrix} \mathbf{x}(k)$$

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## Homework 4: Stability, Controllability, Observability

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DCS00-HW-12

### Problem 4-3:

- Consider the system:

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(k) \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- Determine a control sequence such that the system is taken from the initial state,  $\mathbf{x}(0)$ , to the origin.
- Which is the minimum number steps that solve the problem in (a).
- Explain why it is not possible to find a sequence of control signals such that the state  $[1, 1, 1]^T$  is reached from the origin.

### Problem 4-4:

- Given the system:  $(q^2 + 0.4q)y[k] = u[k]$ 
  - For which values of  $K$  in the proportional controller:  $u[k] = K[r(k) - y(k)]$  is the closed-loop system stable?
  - Determine the stationary error  $r(k) - y(k)$  when  $r(k)$  is a **step** function and when  $K = 0.5$  in the controller in (a).

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