Adaptive Control Systems: HW 1
(Refer to Book: Robust Adaptive Control)

1.

Consider the third-order plant

$$y = G(s)u,$$

where

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}.$$

(a) Obtain parametric models for the plant in the form of SPM and DPM when $\theta^* = [b_2, b_1, b_0, a_2, a_1, a_0]^T$.

(b) If a_0, a_1, and a_2 are known, i.e., $a_0 = 2$, $a_1 = 1$, and $a_2 = 3$, obtain a parametric model for the plant in terms of $\theta^* = [b_2, b_1, b_0]^T$.

(c) If b_0, b_1, and b_2 are known, i.e., $b_0 = 2$, $b_1 = b_2 = 0$, obtain a parametric model in terms of $\theta^* = [a_2, a_1, a_0]^T$.

2. (The example shown in **Lecture 2-6, use Simulink instead of the Adaptive Control Toolbox**)

Consider the mass–spring–dashpot system of Figure 2.1 described by (2.5) and the SPM with $\theta^* = [M, f, k]^T$ presented in Example 2.1.

(a) Generate the signals z, ϕ of the parametric model using the Adaptive Control Toolbox for $M = 100$ kg, $f = 0.15$ kg/sec, $k = 7$ kg/sec^2, $u(t) = 1 + \cos(\frac{\pi}{3}t)$, and $0 \leq t \leq 25$ sec.

(b) The SPM in (a) is based on the assumption that M, f, k are unknown. Assume that M is known. Use the Adaptive Control Toolbox to generate the signals of the reduced SPM for the same values of M, f, k, $u(t) = 1 + \cos(\frac{\pi}{3}t)$, and $0 \leq t \leq 25$ sec.

3.

Consider the second-order stable system

$$\dot{x} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 0 \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u,$$

where x, u are available for measurement, $u \in \mathcal{L}_\infty$, and a_{11}, a_{12}, a_{21}, b_1, b_2 are unknown parameters. Design an online estimator to estimate the unknown parameters. Simulate your scheme using $a_{11} = -0.25$, $a_{12} = 3$, $a_{21} = -5$, $b_1 = 1$, $b_2 = 2.2$, and $u = 10 \sin 2t$. Repeat the simulation when $u = 10 \sin 2t + 7 \cos 3.6t$. Comment on your results.

4. (Problem 4. (c) and its simulation are left for HW 2)

Consider the mass-spring-damper system shown in Figure 4.11.

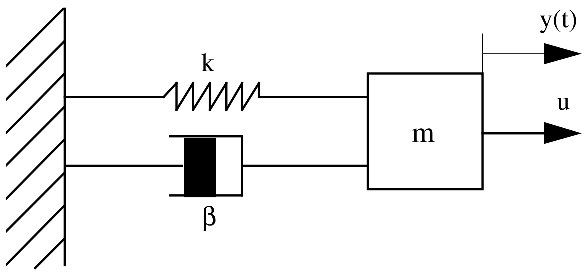

Figure 4.11 The mass-spring-damper system for Problem 4.9.

where β is the damping coefficient, k is the spring constant, u is the external force, and $y(t)$ is the displacement of the mass m resulting from the force u.

(a) Verify that the equations of the motion that describe the dynamic behavior of the system under small displacements are

$$m\ddot{y} + \beta\dot{y} + ky = u$$

(b) Design a gradient algorithm to estimate the constants m, β, k when y, u can be measured at each time t.

(c) Repeat (b) for a least squares algorithm.

(d) Simulate your algorithms in (b) and (c) on a digital computer by assuming that $m = 20$ kg, $\beta = 0.1$ kg/sec, $k = 5$ kg/sec^2 and inputs u of your choice.

- We also provide a simulation for **Example 4.3.1** in the book (*Robust Adaptive Control* by P. Ioannou. Please refer to **pp. 178** of the textbook for details.)