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1. $a_0 = 1$

$P_0 = r(0) = 0.1482$

$\Delta_0 = r^*(1) = 0.05$

$$k_1 = -\frac{\Delta_0}{P_0} = \frac{-0.05}{0.1482} = -0.3374$$

$$a_1 = \begin{bmatrix} a_0 \\ 0 \end{bmatrix} + (-0.3374)\begin{bmatrix} 0 \\ a_0^{B*} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.3374 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.3374 \end{bmatrix}$$

$$P_1 = P_0 \times (1 - |k_1|^2) = 0.1482(1 - 0.3374^2)$$
$$= 0.1313$$

$$\Delta_1 = r_2^{BT} a_1 = \begin{bmatrix} r(-2) \\ r(-1) \end{bmatrix}^T \begin{bmatrix} 1 \\ -0.3374 \end{bmatrix}$$

$$= 0.017 + 0.05(-0.3374)$$

$$= 0.00013$$

$$k_2 = -\frac{\Delta_1}{P_1} = -\frac{0.00013}{0.1313} = -0.001$$

$$a_2 = \begin{bmatrix} a_1 \\ 0 \end{bmatrix} + (-0.001)\begin{bmatrix} 0 \\ a_1^{B*} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -0.3374 \\ 0 \end{bmatrix} + (-0.001)\begin{bmatrix} 0 \\ -0.3374 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -0.337 \\ -0.001 \end{bmatrix}$$

$$P_2 = P_1 \times (1 - |k_2|^2) = 0.1313 \times (1 - (0.001)^2)$$
$$= 0.1313$$

$$\Delta_2 = r_3^{BT} a_2 = \begin{bmatrix} r(-3) \\ r(-2) \\ r(-1) \end{bmatrix}^T \begin{bmatrix} 1 \\ -0.337 \\ -0.001 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0323 \\ 0.017 \\ 0.05 \end{bmatrix}^T \begin{bmatrix} 1 \\ -0.337 \\ -0.001 \end{bmatrix}$$

$$= -0.0381$$

$$k_3 = \frac{-\Delta_2}{P_2} = \frac{-0.0381}{-0.1313} = 0.2901$$

$$a_3 = \begin{bmatrix} a_2 \\ 0 \end{bmatrix} + k_3 \begin{bmatrix} 0 \\ a_2^{B*} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -0.337 \\ -0.001 \\ 0 \end{bmatrix} + (0.2901) \begin{bmatrix} 0 \\ -0.001 \\ -0.337 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -0.3372 \\ -0.099 \\ 0.2901 \end{bmatrix}$$

$$P_3 = P_2 \times (1 - (0.2901)^2) = 0.1203$$

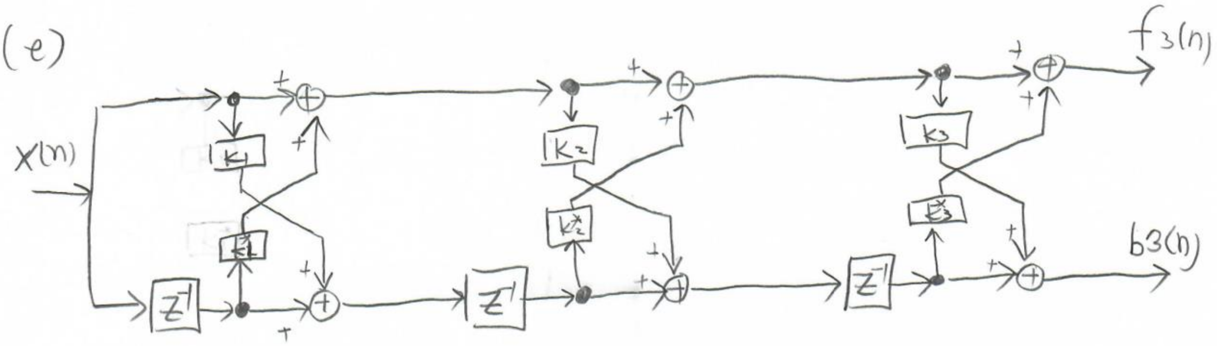
1.(a) $k_1 = -0.3374 \quad k_2 = -0.001 \quad k_3 = 0.2901$

(b) $\Delta_0 = 0.05 \quad \Delta_1 = 0.00013 \quad \Delta_2 = -0.0381$

(c) $a_0 = 1 \quad a_1 = \begin{bmatrix} 1 \\ -0.3374 \end{bmatrix} \quad a_2 = \begin{bmatrix} 1 \\ -0.337 \\ -0.001 \end{bmatrix} \quad a_3 = \begin{bmatrix} 1 \\ -0.3372 \\ -0.099 \\ 0.2901 \end{bmatrix}$

(d) $P_0 = 0.1482, P_1 = 0.1313, P_2 = 0.1313, P_4 = 0.1203$

1、(e)



2. $x(n) = \alpha x(n-1) + v(n)$

$X(z) = \alpha X(z) z^{-1} + V(z)$

$X(z)(1-\alpha z^{-1}) = V(z)$

$\frac{X(z)}{V(z)} = H(z) = \frac{1}{1-\alpha z^{-1}}$

$S_x = |H(z)|^2 S_v(z) \Big|_{z=e^{j2\pi f}}$

$= |H(z)|^2 \Big|_{z=e^{j2\pi f}}$

$= \left(\frac{1}{1-\alpha z^{-1}}\right)\left(\frac{1}{1-\alpha z^{-1}}\right)^* \Big|_{z=e^{j2\pi f}} = \frac{1}{1-\alpha z^{-1}} \frac{1}{1-\alpha^*(z^{-1})^*} \Big|_{z=e^{j2\pi f}}$

$= \frac{1}{1-\alpha z^{-1} - \alpha^*(z^{-1})^* + |\alpha|^2 |z^{-1}|^2} \Big|_{z=e^{j2\pi f}}$

$= \frac{1}{1+|\alpha|^2 - \alpha^* e^{-j2\pi f} - \alpha e^{j2\pi f}}$

$P_m \geq \exp\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \log S_x(e^{j2\pi f})\, df\right)$

$= \exp\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \log\left(\left(\frac{1}{1-\alpha e^{j2\pi f}}\right)\left(\frac{1}{1-\alpha^* e^{j2\pi f}}\right)\right) df\right)$

$$\exp\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \log\left(\frac{1}{1-\alpha e^{-j2\pi f}}\right) df + \int_{-\frac{1}{2}}^{\frac{1}{2}} \log\left(\frac{1}{1-\alpha^* e^{j2\pi f}}\right) df \right)$$

$$= \exp\left(-\int_{-\frac{1}{2}}^{\frac{1}{2}} \log(1-\alpha e^{-j2\pi f})\, df - \int_{-\frac{1}{2}}^{\frac{1}{2}} \log(1-\alpha^* e^{j2\pi f})\, df \right)$$

$$= \exp\left(-\int_{-\frac{1}{2}}^{\frac{1}{2}} \sum_{n=1}^{\infty} \frac{-(\alpha e^{-j2\pi f})^n}{n}\, df - \int_{-\frac{1}{2}}^{\frac{1}{2}} \sum_{n=1}^{\infty} \frac{-(\alpha^* e^{j2\pi f})^n}{n}\, df \right)$$

$$= \exp\left(-\sum_{n=1}^{\infty} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{-(\alpha e^{-j2\pi f})^n}{n}\, df - \sum_{n=1}^{\infty} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{-(\alpha^* e^{j2\pi f})^n}{n}\, df \right)$$

$$= \exp\left(\sum_{n=1}^{\infty} \left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\alpha^n}{n} e^{-j2\pi f n}\, df + \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{(\alpha^*)^n}{n} e^{j2\pi f n}\, df \right) \right)$$

$$= \exp\left(\sum_{n=1}^{\infty} \left(\frac{\alpha^n}{n} \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j2\pi f n}\, df + \frac{(\alpha^*)^n}{n} \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{j2\pi f n}\, df \right) \right)$$

$$= \exp\left(\sum_{n=1}^{\infty} \left(\frac{\alpha^n}{n} \frac{1}{-j2\pi n} e^{-j2\pi f n}\Big|_{f=-\frac{1}{2}}^{\frac{1}{2}} + \frac{(\alpha^*)^n}{n} \frac{1}{j2\pi n} e^{j2\pi f n}\Big|_{f=-\frac{1}{2}}^{\frac{1}{2}} \right) \right)$$

$$= \exp\left(\sum_{n=1}^{\infty} \left(\frac{\alpha^n}{n} \frac{1}{-j2\pi n} \left(e^{-j\pi n} - e^{j\pi n}\right) + \frac{(\alpha^*)^n}{n} \frac{1}{j2\pi n} \left(e^{j\pi n} - e^{-j\pi n}\right) \right) \right)$$

$$= \exp\left(\sum_{n=1}^{\infty} \left(\frac{\alpha^n}{n} \frac{1}{\pi n} \left(\frac{e^{j\pi n} - e^{-j\pi n}}{j2}\right) + \frac{(\alpha^*)^n}{n} \frac{1}{\pi n} \left(\frac{e^{j\pi n} - e^{-j\pi n}}{j2}\right) \right) \right)$$

$$= \exp\left(\sum_{n=1}^{\infty} \frac{\alpha^n}{\pi n^2} \sin(\pi n) + \frac{(\alpha^*)^n}{\pi n^2} \sin(\pi n) \right)$$

$$= \exp(0) = 1$$

$$\gamma_X^2 = \frac{\exp\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \log S_X(e^{j2\pi f})\,df\right)}{\int_{-\frac{1}{2}}^{\frac{1}{2}} S_X(e^{j2\pi f})\,df} = \frac{1}{\int_{-\frac{1}{2}}^{\frac{1}{2}} S_X(e^{j2\pi f})\,df}$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} S_X(e^{j2\pi f})\,df = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+|\alpha|^2 - \alpha^* e^{-j2\pi f} - \alpha e^{j2\pi f}}\,df$$

令 $\alpha = A + jB$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+A^2+B^2-(A-jB)e^{-j2\pi f}-(A+jB)e^{j2\pi f}}\,df$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+A^2+B^2 - A\,(e^{j2\pi f}+e^{-j2\pi f}) + jB\,(e^{-j2\pi f}-e^{j2\pi f})}\,df$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+A^2+B^2-2A\cos(2\pi f)-2B\sin(2\pi f)}\,df$$

令 $x = 2\pi f \quad \frac{dx}{df} = 2\pi \quad df = \frac{1}{2\pi}\,dx$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{1+A^2+B^2-2A\cos(x)-2B\sin(x)}\,dx$$

令 $u = \tan\left(\frac{x}{2}\right) \quad \frac{du}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right) \quad \sin x = \frac{2u}{u^2+1} \quad \cos x = \frac{1-u^2}{u^2+1}$

$dx = \frac{2}{u^2+1}\,du$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{1+A^2+B^2-2A\frac{1-u^2}{1+u^2}-2B\frac{2u}{1+u^2}}\;\frac{2}{u^2+1}\,du$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{(1+A^2+B^2)(u^2+1)-2A(1-u^2)-2B2u}\,du.$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{u^2(1+A^2+B^2+2A)-4Bu+1+A^2+B^2-2A}\,du$$

$$= \frac{1}{\pi}\,\frac{1}{1+A^2+B^2+2A}\int_{-\infty}^{\infty}\frac{1}{u^2+\frac{-4B}{1+A^2+B^2+2A}u+\frac{1+A^2+B^2-2A}{1+A^2+B^2+2A}}\,du$$

$$= \frac{1}{(1+A^2+B^2+2A)\,\pi}\int_{-\infty}^{\infty}\frac{1}{u^2+\frac{-4B}{1+A^2+B^2+2A}u+\left(\frac{-2B}{1+A^2+B^2+2A}\right)^2-\left(\frac{-2B}{1+A^2+B^2+2A}\right)^2+\frac{1+A^2+B^2-2A}{1+A^2+B^2+2A}}\,du$$

$$= \frac{1}{(1+A^2+B^2+2A)\,\pi}\int_{-\infty}^{\infty}\frac{1}{\left(u-\frac{2B}{1+A^2+B^2+2A}\right)^2-\frac{4B^2}{(1+A^2+B^2+2A)^2}+\frac{1+A^2+B^2-2A}{1+A^2+B^2+2A}}\,du$$

令 $c = \frac{-4B^2}{(1+A^2+B^2+2A)^2}+\frac{1+A^2+B^2-2A}{1+A^2+B^2+2A} = \frac{(1-A^2-B^2)^2}{(1+A^2+B^2+2A)^2}$

$$= \frac{1}{(1+A^2+B^2+2A)\,\pi}\int_{-\infty}^{\infty}\frac{1}{\left(u-\frac{2B}{1+A^2+B^2+2A}\right)^2+(\sqrt{c})^2}\,du$$

$\boxed{\int\frac{1}{x^2+a^2}dx=\frac{1}{a}\arctan\left(\frac{x}{a}\right)+c}$

$$= \frac{1}{(1+A^2+B^2+2A)\,\pi}\,\frac{1}{\sqrt{c}}\,\arctan\left(\frac{u-\frac{2B}{1+A^2+B^2+2A}}{\sqrt{c}}\right)\Bigg|_{u=-\infty}^{\infty}$$

$$= \frac{1}{(1+A^2+B^2+2A)\,\pi}\,\frac{1}{\sqrt{c}}\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right)$$

$$= \frac{\cancel{\pi}}{\cancel{\pi}\,(1+A^2+B^2+2A)}\,\frac{1+A^2+B^2+2A}{1-A^2-B^2} = \frac{1}{(1-(A^2+B^2))}$$

$$= \frac{1}{(1-|\alpha|^2)}$$

$$r_X^2 = \frac{1}{\frac{1}{1-|\alpha|^2}} = \underline{(1-|\alpha|^2)}$$

3 (a)

$$e(0) = d(0) - y(0)$$
$$= d(0) - \hat{W}^H(0)\,x(0)$$
$$= 1 - 0.1 = 1$$

3(b) $\hat{W}(1) = \hat{w}(0) + \mu\, x(0)\, e^*(0)$

$$= 0 + \mu\, 1 \cdot 1 = \mu$$

3(c) $\hat{W}(2) = \hat{W}(1) + \mu\, x(1)\, e^*(1)$

$$= \mu + \mu\, e^{j2\pi f_1} \cdot \left(d(1) - \hat{w}(1)^H x(1) \right)^*$$
$$= \mu + \mu\, e^{j2\pi f_1} \left(e^{j2\pi f_2} - \mu e^{j2\pi f_1} \right)^*$$
$$= \mu + \mu e^{j2\pi(f_1 - f_2)} - \mu^2$$

3.(d) $\hat{w}(n) = \hat{w}(n-1) + \mu\, x(n-1) \left(d(n-1) - \hat{w}^H(n-1)\, x(n-1) \right)^*$

$$= \hat{w}(n-1) + \mu\, x(n-1) \left(d^*(n-1) - x^H(n-1)\, \hat{w}(n-1) \right)$$
$$= \hat{w}(n-1) + \mu\, x(n-1)\, d^*(n-1) - \mu\, x(n-1)\, x^H(n-1)\, \hat{w}(n-1)$$
$$= \hat{w}(n-1) + \mu P(n-1) - \mu R(n-1)\, \hat{w}(n-1)$$
$$= \left(I - \mu R(n-1) \right) \hat{w}(n-1) + \mu P(n-1)$$
$$= \left(I - \mu R(n-1) \right) \left(\left(I - \mu R(n-2) \right) \hat{w}(n-2) + \mu P(n-2) \right) + \mu P(n-1)$$

$$= (I-\mu R(n-1))(I-\mu R(n-2))\hat{w}(n-2)$$
$$+\mu (I-\mu R(n-1))P(n-2)$$
$$+\mu P(n-1)$$

$$= (I-\mu R(n-1))(I-\mu R(n-2))(I-\mu R(n-3))\hat{w}(n-3)$$
$$+\mu (I-\mu R(n-1))(I-\mu R(n-2))P(n-3)$$
$$+\mu (I-\mu R(n-1))P(n-2)$$
$$+\mu P(n-1)$$

$$= \sum_{i=1}^{n} \left[\mu (I-\mu R(n-1))(I-\mu R(n-2))\cdots(I-\mu R(n-i+1))P(n-i)\right]$$
$$+\mu (I-\mu R(n-1))(I-\mu R(n-2))\cdots(I-\mu R(0))\cancelto{0}{\hat{w}(0)}$$

$$= \mu \sum_{i=1}^{n} \left[(I-\mu R(n-1))(I-\mu R(n-2))\cdots(I-\mu R(n-i+1))P(n-i)\right]$$

$R(n) = x(n)x^H(n) \quad P(n) = x(n)d^*(n)$

4 (a)

$$\hat{w}_{\varepsilon\text{-NLMS}}(n+1) = \underset{w}{\arg\min} \| w - \hat{w}_{\varepsilon\text{-NLMS}}(n) \|_2^2$$

$$\text{subject to } \left(d(n) - w^H x(n) \right) - \left(1 - \frac{\tilde{\mu}}{\varepsilon + \|x(n)\|_2^2} \|x(n)\|_2^2 \right) \left(d(n) - \hat{w}^H_{\varepsilon\text{-NLMS}}(n) x(n) \right) = 0$$

(b)

$$\mathcal{L}\{w, w^H, \lambda\} = \| w - \hat{w}_{\varepsilon\text{-NLMS}}(n) \|_2^2 + \mathrm{Re}\left[\lambda^* \left((d(n) - w^H x(n)) - \left(1 - \frac{\tilde{\mu}}{\varepsilon + \|x(n)\|_2^2} \|x(n)\|_2^2 \right) \left(d(n) - \hat{w}^H_{\varepsilon\text{-NLMS}}(n) x(n) \right) \right) \right]$$

$$= \left(w^H - \hat{w}^H_{\varepsilon\text{-NLMS}}(n) \right) \left(w - \hat{w}_{\varepsilon\text{-NLMS}}(n) \right) + \frac{\lambda^*}{2} \left[(d(n) - w^H x(n)) - \left(1 - \frac{\tilde{\mu}}{\varepsilon + \|x(n)\|_2^2} \|x(n)\|_2^2 \right) \left(d(n) - \hat{w}^H_{\varepsilon\text{-NLMS}}(n) x(n) \right) \right] + \frac{\lambda}{2} \left[(d^*(n) - x^H(n) w) - \left(1 - \frac{\tilde{\mu}}{\varepsilon + \|x(n)\|_2^2} \|x(n)\|_2^2 \right) \left(d^*(n) - x^H(n) \hat{w}_{\varepsilon\text{-NLMS}}(n) \right) \right]$$

$$\frac{\partial \mathcal{L}}{\partial w^H} = \left(w - \hat{w}_{\varepsilon\text{-NLMS}}(n) \right) - \frac{\lambda^*}{2} x(n) = 0$$

$$w = \hat{w}_{\varepsilon\text{-NLMS}}(n) + \frac{\lambda^*}{2} x(n)$$

$$d(n) - \left(\hat{w}_{\varepsilon\text{-NLMS}}(n) + \frac{\lambda^*}{2} x(n) \right)^H x(n) - \left(1 - \frac{\tilde{\mu}}{\varepsilon - \|x(n)\|_2^2} \|x(n)\|_2^2 \right) \left(d(n) - \hat{w}^H_{\varepsilon\text{-NLMS}}(n) x(n) \right) = 0$$

$$\cancel{d(n)} - \hat{W}^H_{\varepsilon\text{-NLMS}}(n)\,x(n) - \frac{\lambda}{2}x^H(n)x(n)$$

$$- d(n) + \frac{\tilde{\mu}}{\varepsilon - \|x(n)\|_2^2}\|x(n)\|_2^2\, d(n) + \cancel{\hat{W}^H_{\varepsilon\text{-NLMS}}(n)\,x(n)}$$

$$- \|x(n)\|_2^2 \frac{\tilde{\mu}}{\varepsilon - \|x(n)\|_2^2}\hat{W}^H_{\varepsilon\text{-NLMS}}(n)\,x(n) = 0$$

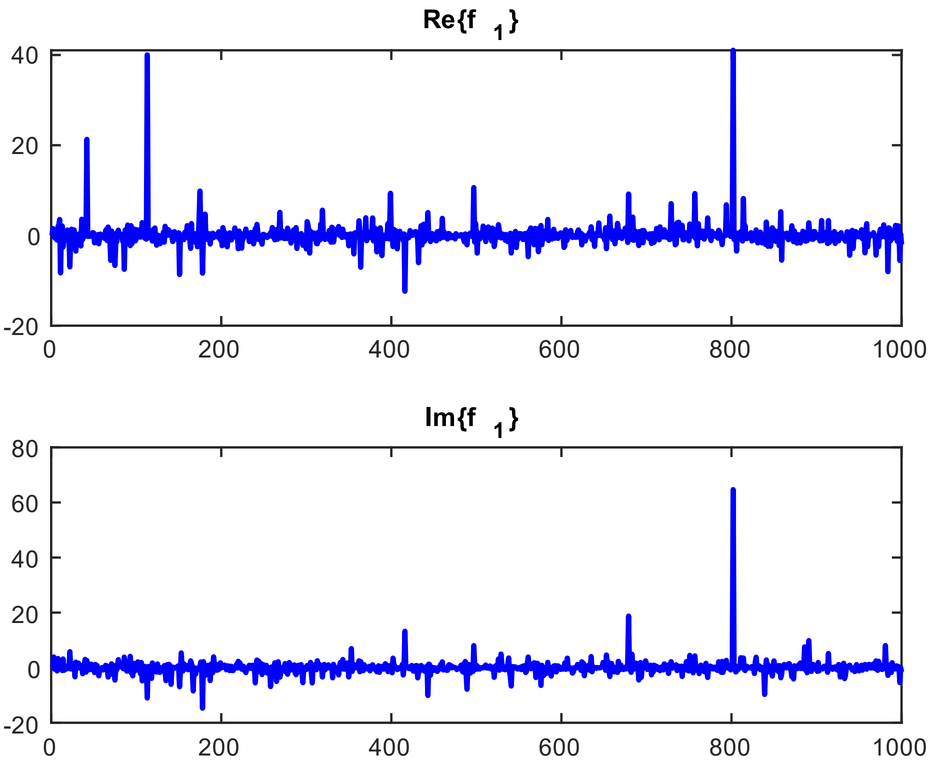
$$\frac{\lambda}{2}\cancel{\|x(n)\|_2^2} = \cancel{\|x(n)\|_2^2}\left(\frac{\tilde{\mu}}{\varepsilon - \|x(n)\|_2^2} - \frac{\tilde{\mu}}{\varepsilon - \|x(n)\|_2^2}\hat{W}^H_{\varepsilon\text{-NLMS}}(n)\,x(n)\right)$$

$$\lambda = 2\frac{\tilde{\mu}}{\varepsilon - \|x(n)\|_2^2}\left(1 - \hat{W}^H_{\varepsilon\text{-NLMS}}(n)\,x(n)\right)$$

$$W = \hat{W}_{\varepsilon\text{-NLMS}}(n) + \frac{\tilde{\mu}}{\varepsilon - \|x(n)\|_2^2}\left(1 - x^H(n)\,\hat{W}_{\varepsilon\text{-NLMS}}(n)\right)x(n)$$

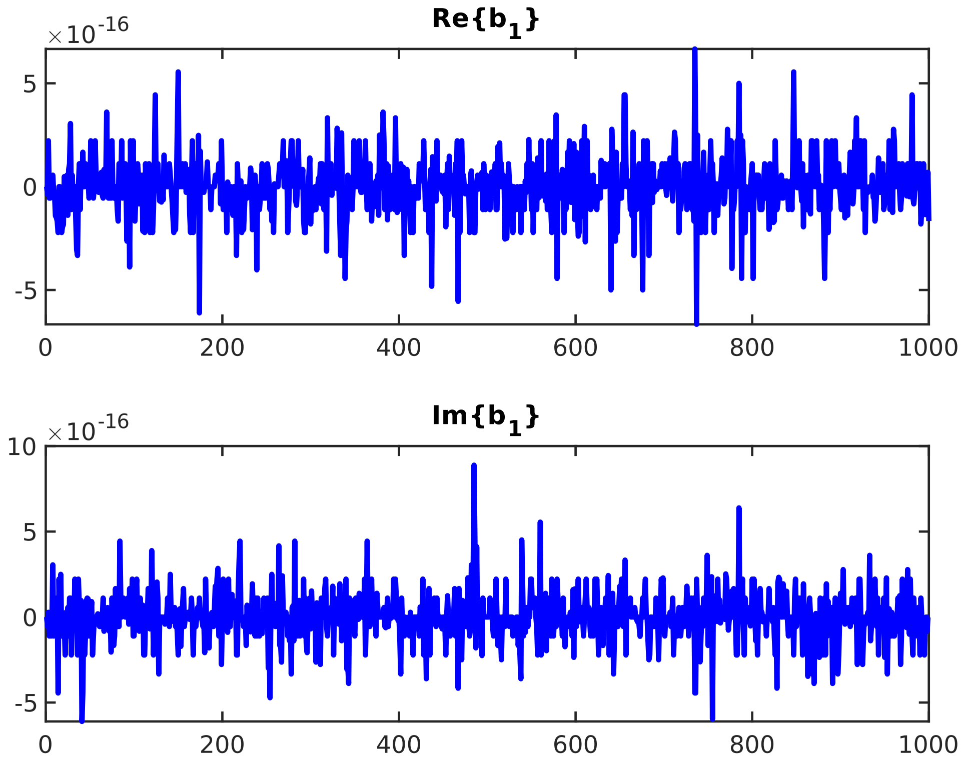
$$= \hat{W}_{\varepsilon\text{-NLMS}}(n) + \frac{\tilde{\mu}}{\varepsilon - \|x(n)\|_2^2}\,e^*(n)\,x(n)$$

5(b)



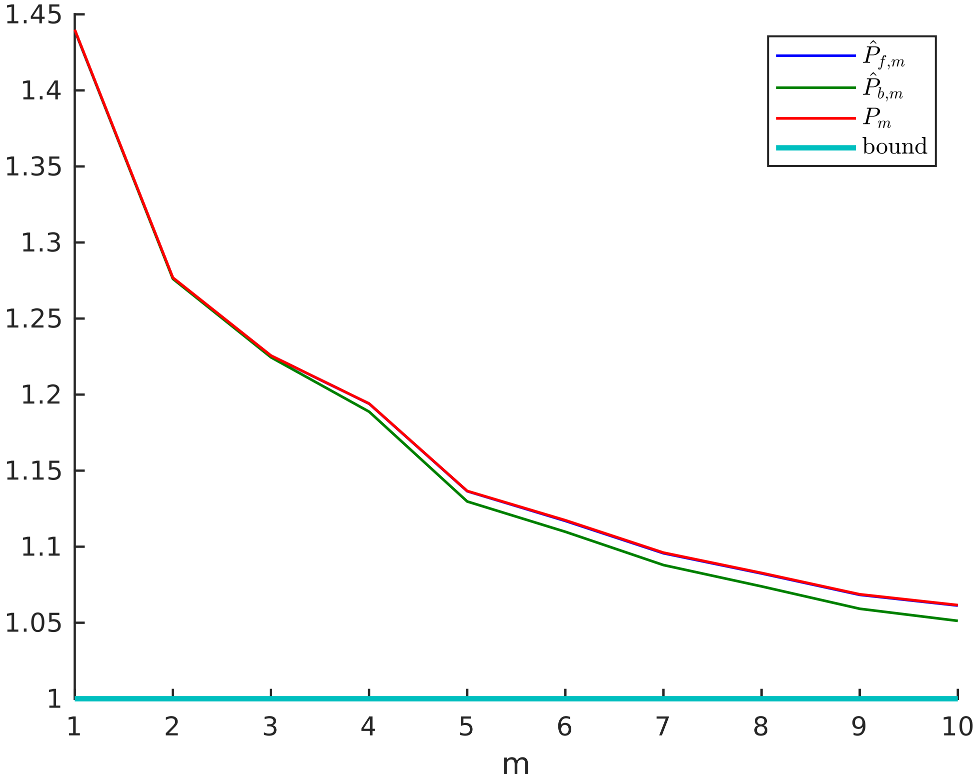
pf1 = 14.3140

5(c)



pb1 = 3.6316e-32

5(d)



1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
1
2
3
4
5
6
7
8
9
10
m
$\hat{P}_{f,m}$
$\hat{P}_{b,m}$
P_m
bound