Homework 5: Design in State-Space Model

- Problem 5-1:
  - Given the system:

$$\mathbf{x}(k+1) = \begin{bmatrix} 1.0 & 0.1 \\ 0.5 & 0.1 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{x}(k)$$

  - Determine a linear state-feedback controller:

$$u(k) = -K\mathbf{x}(k)$$

  - such that the closed-loop poles are in 0.1 and 0.25.

Homework 5: Design in State-Space Model

- Problem 5-2:
  - Consider the continuous-time system:

$$\begin{aligned} \frac{d}{dt}\mathbf{x} &= \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} \end{aligned}$$

  - Sampling the system with h = 0.2 gives:

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.55 & 0.12 \\ 0 & 0.67 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0.01 \\ 0.16 \end{bmatrix} u(k)$$

a) Determine a linear state-feedback controller such that the closed-loop characteristic polynomial is:

$$(z^2 - 0.63z + 0.21)$$

b) Simulate the closed-loop system when $x(0)$ = [1 0]^T and plot all the signals (states, input, output) of the systems.
Note that in your simulation the plant should be the CT model.

Homework 5: Design in State-Space Model

- Problem 5-3:
  - Given the system:

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.25 & 0.5 \\ 1 & 2 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ 4 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k)$$

a) Determine the linear state-feedback controller:

$$u(k) = -K\mathbf{x}(k) + K_r r(k)$$

such that the states are brought to the origin in two sampling intervals.

b) Is it possible to determine a linear state-feedback controller that can take the system from the origin to x(k) = [2 8]^T.

c) Determine an observer that estimates the states such that the observer has the desired characteristic polynomial: (λ - 0.2)^2.