Adaptive Signal Processing HW4

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Problem1

	$\mathbf{x}(0)$	$\mathbf{v}_1(0)$	$\mathbf{v}_2(0)$	$\mathbf{v}_1(1)$	$\mathbf{v}_2(1)$	$\mathbf{v}_1(2)$	$\mathbf{v}_2(2)$	$\mathbf{v}_1(3)$	$\mathbf{v}_2(3)$	Reason
$\mathbf{x}(1)$	✔	✔								(1) for $n=0$
$\mathbf{y}(1)$	✔	✔			✔					(2) for $n=1$ and the row of $\mathbf{x}(1)$
$\boldsymbol{\alpha}(1)$	✔	✔			✔					$\boldsymbol{\alpha}(1)=\mathbf{y}(1)$
$\mathbf{x}(2)$	✔	✔		✔						$x(2)=F(2,1)x(1)+v(1)$
$\mathbf{y}(2)$	✔	✔		✔			✔			$y(2)=C(2)x(2)+v_2(2)$
$\boldsymbol{\alpha}(2)$	✔	✔		✔	✔		✔			$\alpha(2)=y(2)-C(2)\hat{x}(2\|y_1)$
$\hat{\mathbf{x}}(2\|\mathscr{Y}_1)$	✔	✔			✔					$y_1=\text{span}\{\alpha_1\}$
$\hat{\mathbf{x}}(2\|\mathscr{Y}_2)$	✔	✔		✔	✔		✔			$y_2=\text{span}\{\alpha_1,\alpha_2\}$
$\mathbf{x}(3)$	✔	✔		✔		✔				$x(3)=F(3,2)x(2)+v(2)$
$\mathbf{y}(3)$	✔	✔		✔		✔			✔	$y(3)=C(3)x(3)+v_2(3)$
$\boldsymbol{\alpha}(3)$	✔	✔		✔	✔	✔	✔		✔	$\alpha(3)=y(3)-C(3)\hat{x}(3\|y_2)$
$\hat{\mathbf{x}}(3\|\mathscr{Y}_2)$	✔	✔		✔	✔		✔			$y_2=\text{span}(\alpha_1,\alpha_2)$
$\hat{\mathbf{x}}(3\|\mathscr{Y}_3)$	✔	✔		✔	✔	✔	✔		✔	$y_3=\text{span}\{\alpha_1,\alpha_2,\alpha_3\}$

Problem 2

(a) $\alpha(1) = x(1)$

$\alpha(2) = L_{21}\, x(1) + x(2)$

$$E[\alpha(2)\alpha^*(1)] = 0$$

$$= E[L_{21}\, x(1) x^*(1) + x(2) x^*(1)]$$

$$= L_{21} E[x(1) x^*(1)] + E[x(2) x^*(1)]$$

$$= L_{21} \cdot r_x(0) + r_x(1)$$

$$= L_{21} + \frac{1}{4} = 0 \Rightarrow L_{2,1} = \frac{-1}{4}.$$

(b). $R_x = \begin{bmatrix} r_x(0) & r_x(1) \\ r_x(-1) & r_x(0) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{4} \\ \frac{1}{4} & 1 \end{bmatrix}$

(c) $R_\alpha = E[\alpha\alpha^H] = E[(Lx)(Lx)^H]$

$$= E[L x x^H L^H]$$

$$= L \underbrace{E[x x^H]}_{= R_x} L^H$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{-1}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{4} \\ \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{-1}{4} \\ 0 & \frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & \frac{15}{16} \end{bmatrix}$$

(d) $R_\alpha = L R_x L^H$

$$R_x = L^{-1} R_\alpha (L^H)^{-1} = \begin{bmatrix} 1 & 0 \\ \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{15}{16} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{4} \\ 0 & 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{\sqrt{15}}{4} \end{bmatrix}}_{\mathcal{L}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{15}{16}} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{4} \\ 0 & 1 \end{bmatrix}}_{\mathcal{L}^H} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1}{4} & \frac{\sqrt{15}}{4} \end{bmatrix}}_{\mathcal{L}} \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1}{4} & \frac{\sqrt{15}}{4} \end{bmatrix}^H}_{\mathcal{L}^H}$$

Problem3

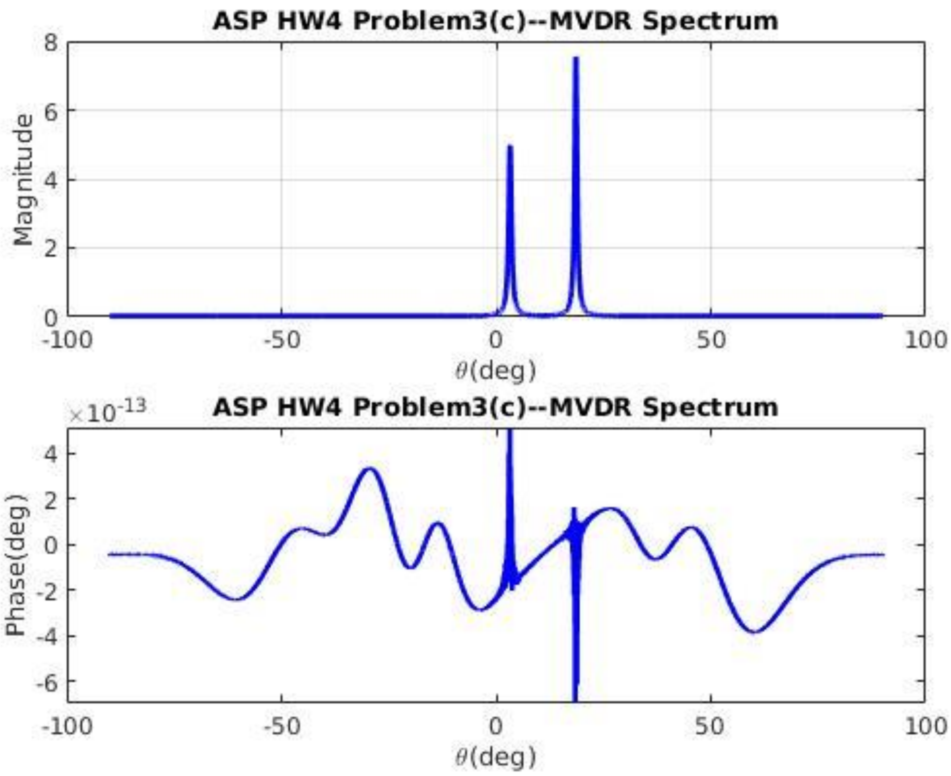
(a)R 矩陣

```
 Columns 1 through 8

 12.6627 + 0.0000i   8.9067 - 7.2179i   1.4652 - 8.5748i  -3.1576 - 3.5350i  -1.1632 + 2.4753i   5.2685 + 3.2840i   9.6392 - 2.2483i   7.1867 - 9.6166i
  8.9067 + 7.2179i  12.7772 - 0.0000i   8.9780 - 7.2449i   1.5867 - 8.6158i  -3.1957 - 3.6725i  -1.1971 + 2.3865i   5.2400 + 3.3963i   9.5660 - 2.2458i
  1.4652 + 8.5748i   8.9780 + 7.2449i  12.7536 + 0.0000i   9.0266 - 7.2066i   1.5315 - 8.6385i  -3.2225 - 3.6096i  -1.2323 + 2.5292i   5.2645 + 3.3905i
 -3.1576 + 3.5350i   1.5867 + 8.6158i   9.0266 + 7.2066i  12.9885 + 0.0000i   9.1677 - 7.3654i   1.5464 - 8.7842i  -3.2766 - 3.7380i  -1.1959 + 2.3856i
 -1.1632 - 2.4753i  -3.1957 + 3.6725i   1.5315 + 8.6385i   9.1677 + 7.3654i  12.9966 + 0.0000i   9.0430 - 7.4074i   1.5031 - 8.7465i  -3.2006 - 3.6116i
  5.2685 - 3.2840i  -1.1971 - 2.3865i  -3.2225 + 3.6096i   1.5464 + 8.7842i   9.0430 + 7.4074i  12.8675 + 0.0000i   9.0767 - 7.2336i   1.5766 - 8.5395i
  9.6392 + 2.2483i   5.2400 - 3.3963i  -1.2323 - 2.5292i  -3.2766 + 3.7380i   1.5031 + 8.7465i   9.0767 + 7.2336i  12.9048 - 0.0000i   8.9547 - 7.2551i
  7.1867 + 9.6166i   9.5660 + 2.2458i   5.2645 - 3.3905i  -1.1959 - 2.3856i  -3.2006 + 3.6116i   1.5766 + 8.5395i   8.9547 + 7.2551i  12.6692 - 0.0000i
 -0.3844 +12.2513i   7.2500 + 9.5898i   9.6261 + 2.1855i   5.2331 - 3.3270i  -1.1629 - 2.4811i  -3.1360 + 3.5506i   1.4697 + 8.5836i   8.8941 + 7.1293i
 -7.0559 + 8.0373i  -0.3831 +12.4459i   7.3474 + 9.7094i   9.7259 + 2.3158i   5.3801 - 3.4329i  -1.1314 - 2.5446i  -3.2754 + 3.5605i   1.4967 + 8.5843i
 -7.2718 + 0.6961i  -7.0464 + 8.0678i  -0.4059 +12.3869i   7.2626 + 9.8577i   9.7851 + 2.4012i   5.4355 - 3.3633i  -1.0836 - 2.5537i  -3.1487 + 3.5020i

 Columns 9 through 11

 -0.3844 -12.2513i  -7.0559 - 8.0373i  -7.2718 - 0.6961i
  7.2500 - 9.5898i  -0.3831 -12.4459i  -7.0464 - 8.0678i
  9.6261 - 2.1855i   7.3474 - 9.7094i  -0.4059 -12.3869i
  5.2331 + 3.3270i   9.7259 - 2.3158i   7.2626 - 9.8577i
 -1.1629 + 2.4811i   5.3801 + 3.4329i   9.7851 - 2.4012i
 -3.1360 - 3.5506i  -1.1314 + 2.5446i   5.4355 + 3.3633i
  1.4697 - 8.5836i  -3.2754 - 3.5605i  -1.0836 + 2.5537i
  8.8941 - 7.1293i   1.4967 - 8.5843i  -3.1487 - 3.5020i
 12.6074 - 0.0000i   8.9321 - 7.2588i   1.4552 - 8.5296i
  8.9321 + 7.2588i  12.8972 + 0.0000i   8.9978 - 7.3153i
  1.4552 + 8.5296i   8.9978 + 7.3153i  12.8479 + 0.0000i
```

(b)特徵值

```
 89.7903 + 0.0000i
 48.9009 - 0.0000i
  0.2957 + 0.0000i
  0.2130 - 0.0000i
  0.2269 - 0.0000i
  0.2818 + 0.0000i
  0.2348 - 0.0000i
  0.2465 + 0.0000i
  0.2502 + 0.0000i
  0.2702 - 0.0000i
  0.2620 - 0.0000i
```

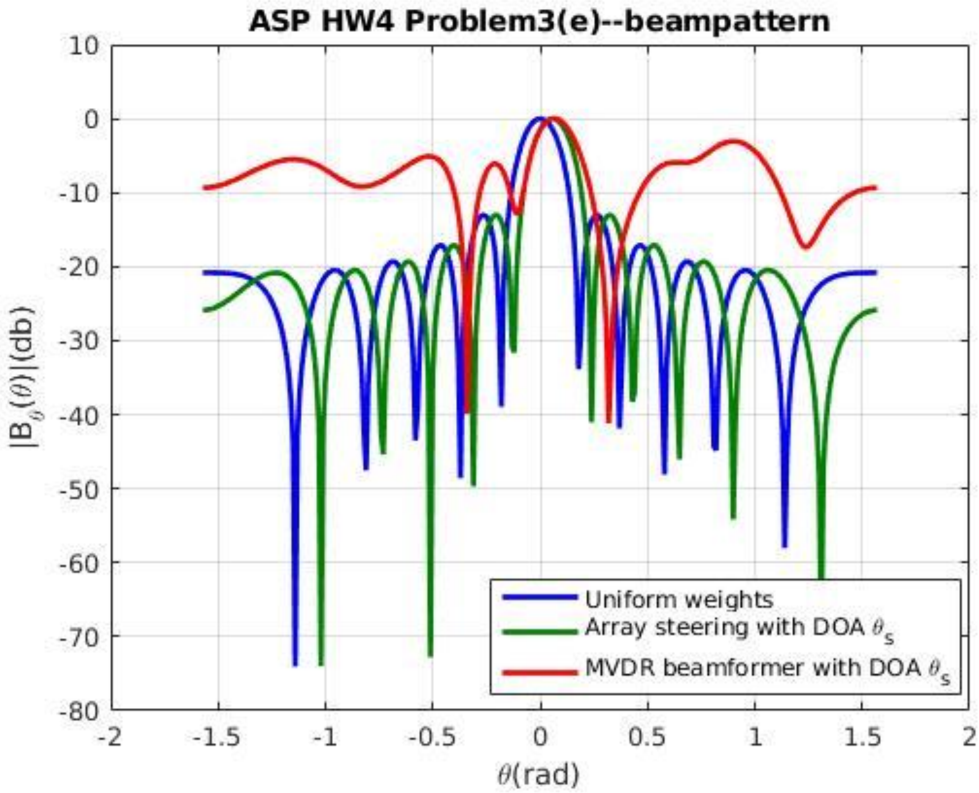
(c)



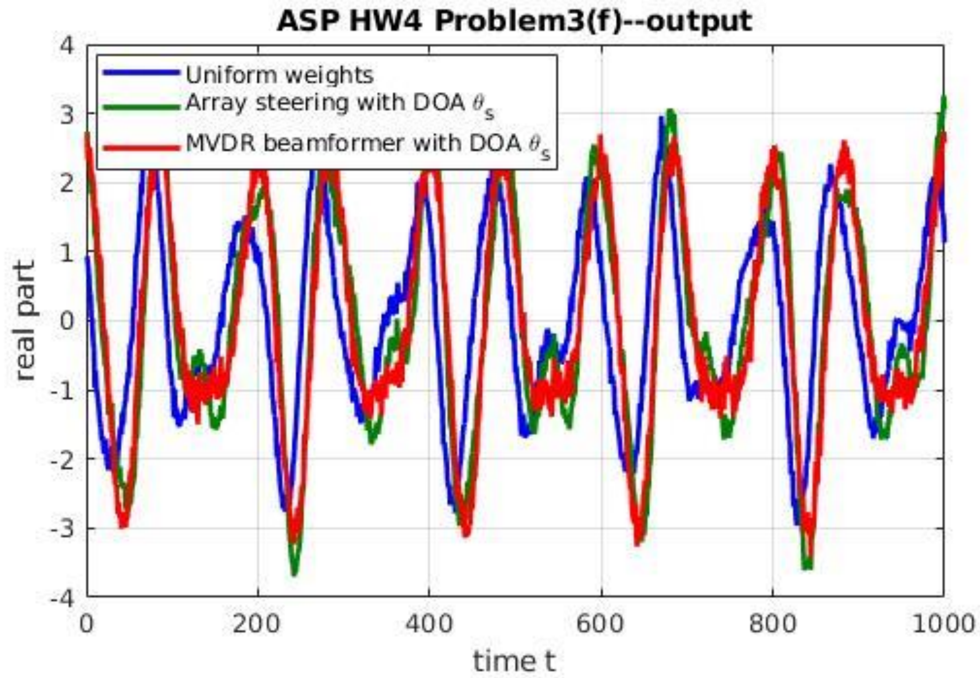
(d)

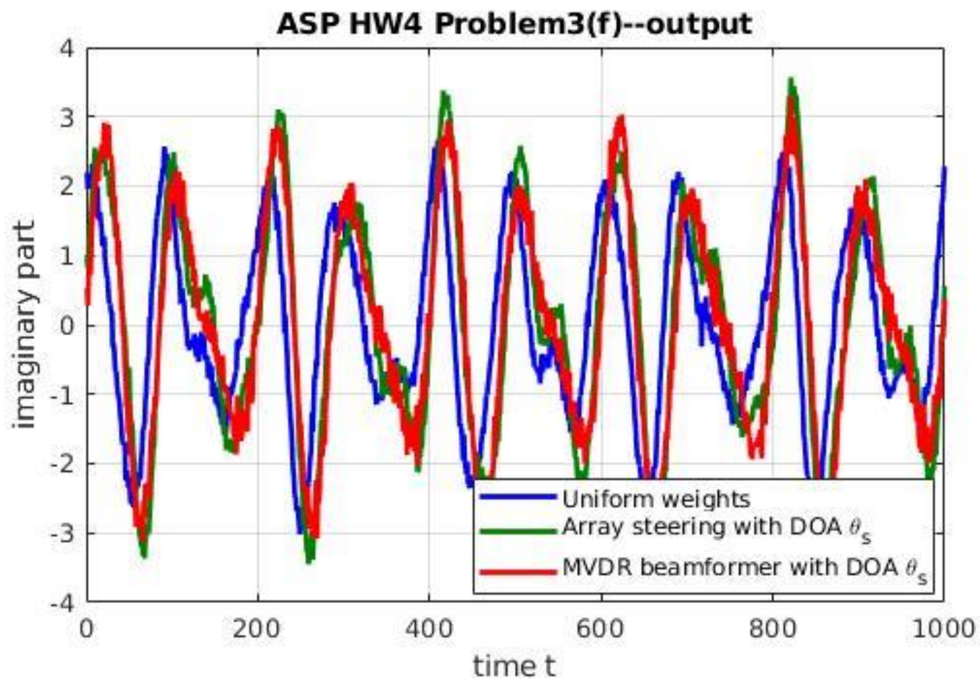
可以看出來兩個峰值在 3.25 度與 18.57 度，因為題目說訊號源的角度 DOA $0° \le \theta_s \le 10°$，所以選擇前者 3.25 度

(e)



（f）


ASP HW4 Problem3(f)--output
Uniform weights
Array steering with DOA θ_s
MVDR beamformer with DOA θ_s
real part
time t
4
3
2
1
0
-1
-2
-3
-4
0
200
400
600
800
1000


ASP HW4 Problem3(f)--output
Uniform weights
Array steering with DOA θ_s
MVDR beamformer with DOA θ_s
imaginary part
time t
4
3
2
1
0
-1
-2
-3
-4
0
200
400
600
800
1000

Problem 4

(a)參考上課關於 Subspace Metheds 中的 MUSIC 演算法，首先，先用測量的向量$\tilde{x}$估計$\tilde{R}$協方差矩陣

$$\hat{R} = \frac{1}{K}\sum_{k=1}^{K} \tilde{x}(k)\tilde{x}^H(k)$$

將$\tilde{R}$做特徵值分解

$$\tilde{R} = \tilde{Q}\tilde{\Lambda}\tilde{Q}^T$$

可以將這些特徵值、特徵向量分成訊號部分$\tilde{Q}_D$、$\tilde{\Lambda}_D$以及雜訊部分$\tilde{Q}_N$、$\tilde{\Lambda}_N$之後可以得到 MUSIC 的 spectrum

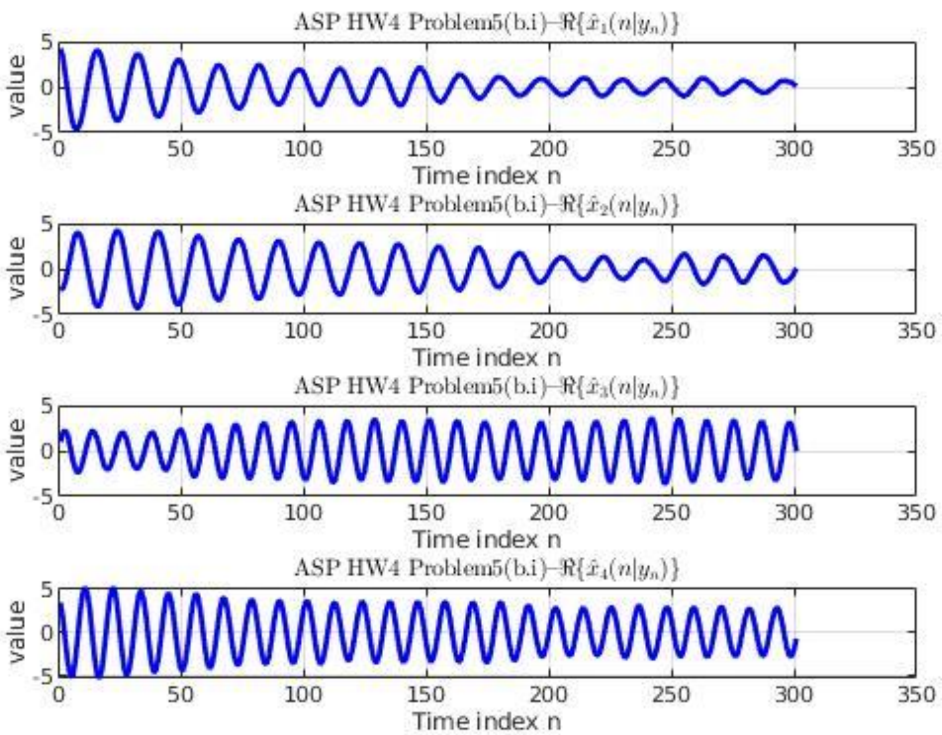
$$\mathrm{P_{MUSIC}}(\theta) = \frac{1}{\left\|\tilde{Q}_n^H a(\theta)\right\|_2^2}$$

之後從 spectrum 找到峰值，就是估計的 DOA

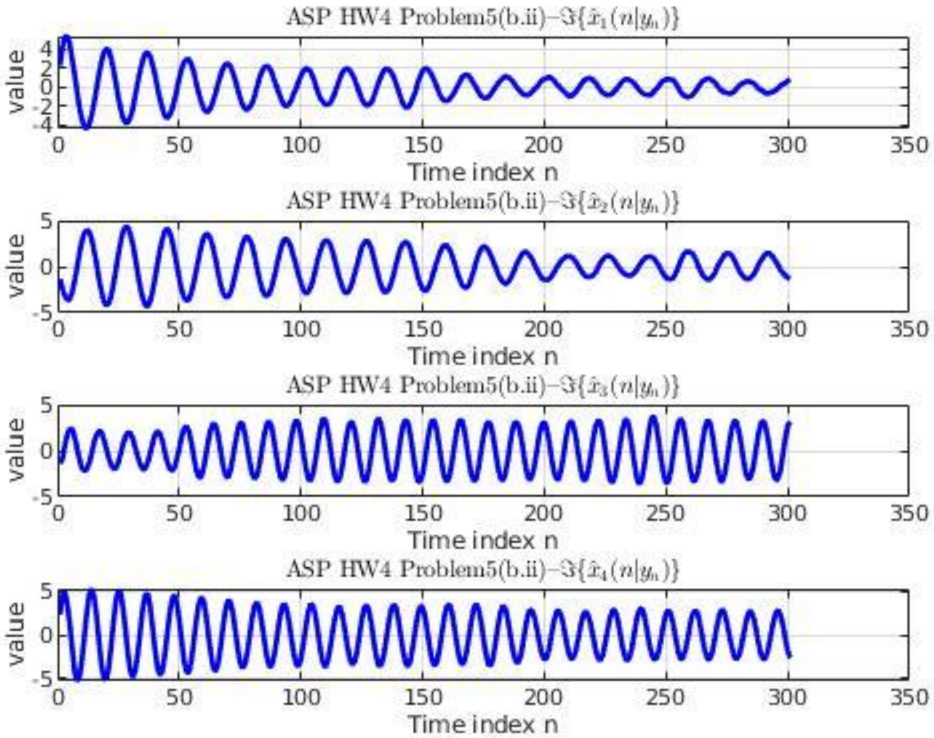
Problem 5

(a)　M=4、N=9、L=301

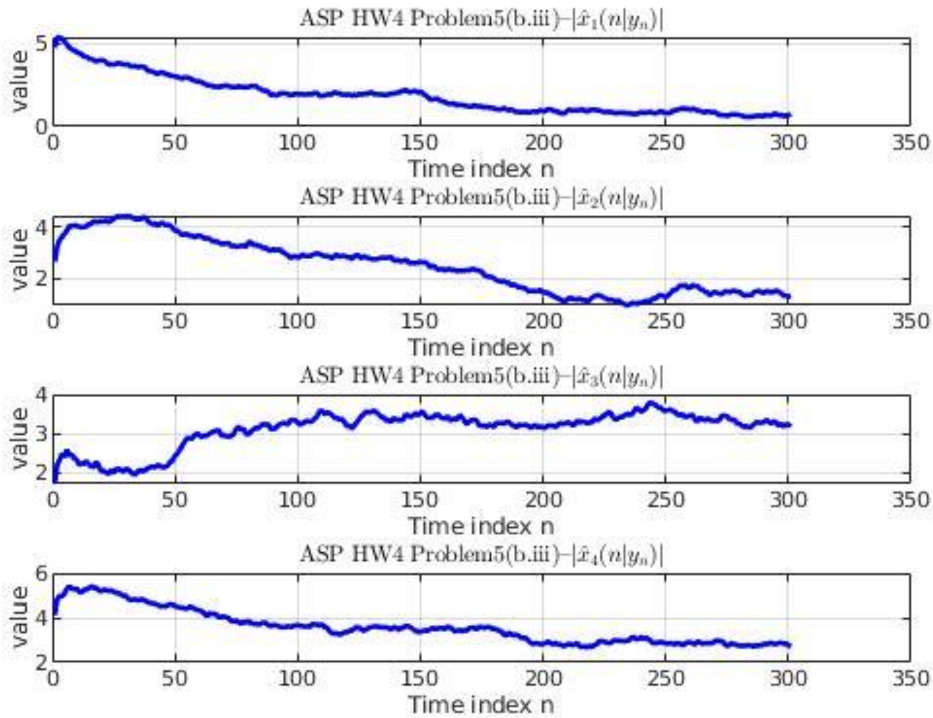
(b.i)



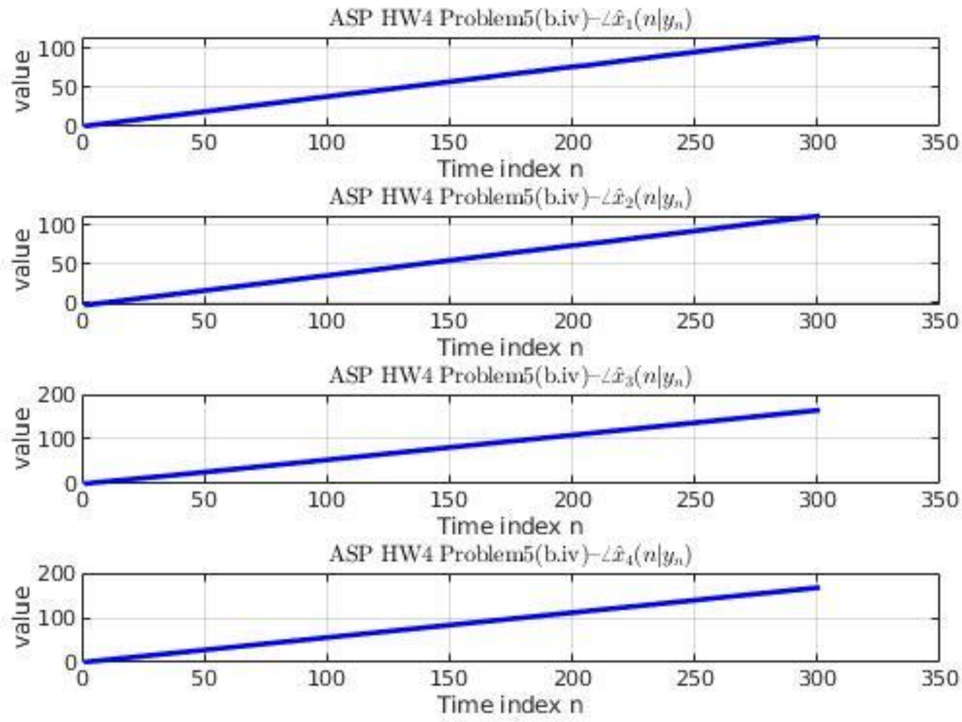
(b. ii)


ASP HW4 Problem5(b.ii)–$\Im\{\hat{x}_1(n|y_n)\}$
ASP HW4 Problem5(b.ii)–$\Im\{\hat{x}_2(n|y_n)\}$
ASP HW4 Problem5(b.ii)–$\Im\{\hat{x}_3(n|y_n)\}$
ASP HW4 Problem5(b.ii)–$\Im\{\hat{x}_4(n|y_n)\}$
value
Time index n

(b. iii)


ASP HW4 Problem5(b.iii)–$|\hat{x}_1(n|y_n)|$
ASP HW4 Problem5(b.iii)–$|\hat{x}_2(n|y_n)|$
ASP HW4 Problem5(b.iii)–$|\hat{x}_3(n|y_n)|$
ASP HW4 Problem5(b.iii)–$|\hat{x}_4(n|y_n)|$
value
Time index n

(b. iv)



(c)

做一個卡爾曼濾波器，將前面題目(iv)部分做好的角度作為感測資料，狀態變數設定角度與角速度

狀態變數

$$x = \begin{bmatrix} \theta_1 \\ \omega_1 \\ \theta_2 \\ \omega_2 \\ \theta_3 \\ \omega_3 \\ \theta_4 \\ \omega_4 \end{bmatrix}$$

系統，考慮等速模型

$$F(n+1,n) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

感測資料，就是上面算出來的(iv)

$$y = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

感測矩陣

$$C(n) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

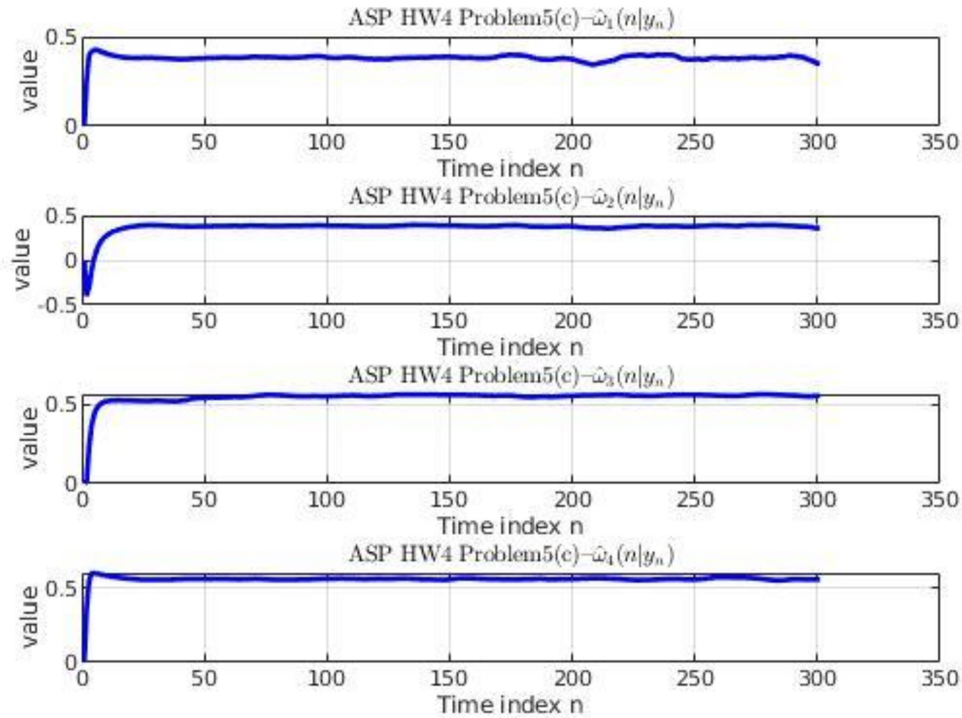
程序的雜訊的協方差矩陣，設定小一點

$$Q_1 = 0.001 * I_8$$

感測的雜訊的協方差矩陣

$$Q_2 = I_4$$

估計出來的角速度



考慮第 50 筆到最後一筆的平均值

$$\omega_1 = 0.3816, \quad \omega_2 = 0.3814, \quad \omega_3 = 0.5561, \quad \omega_4 = 0.5574$$