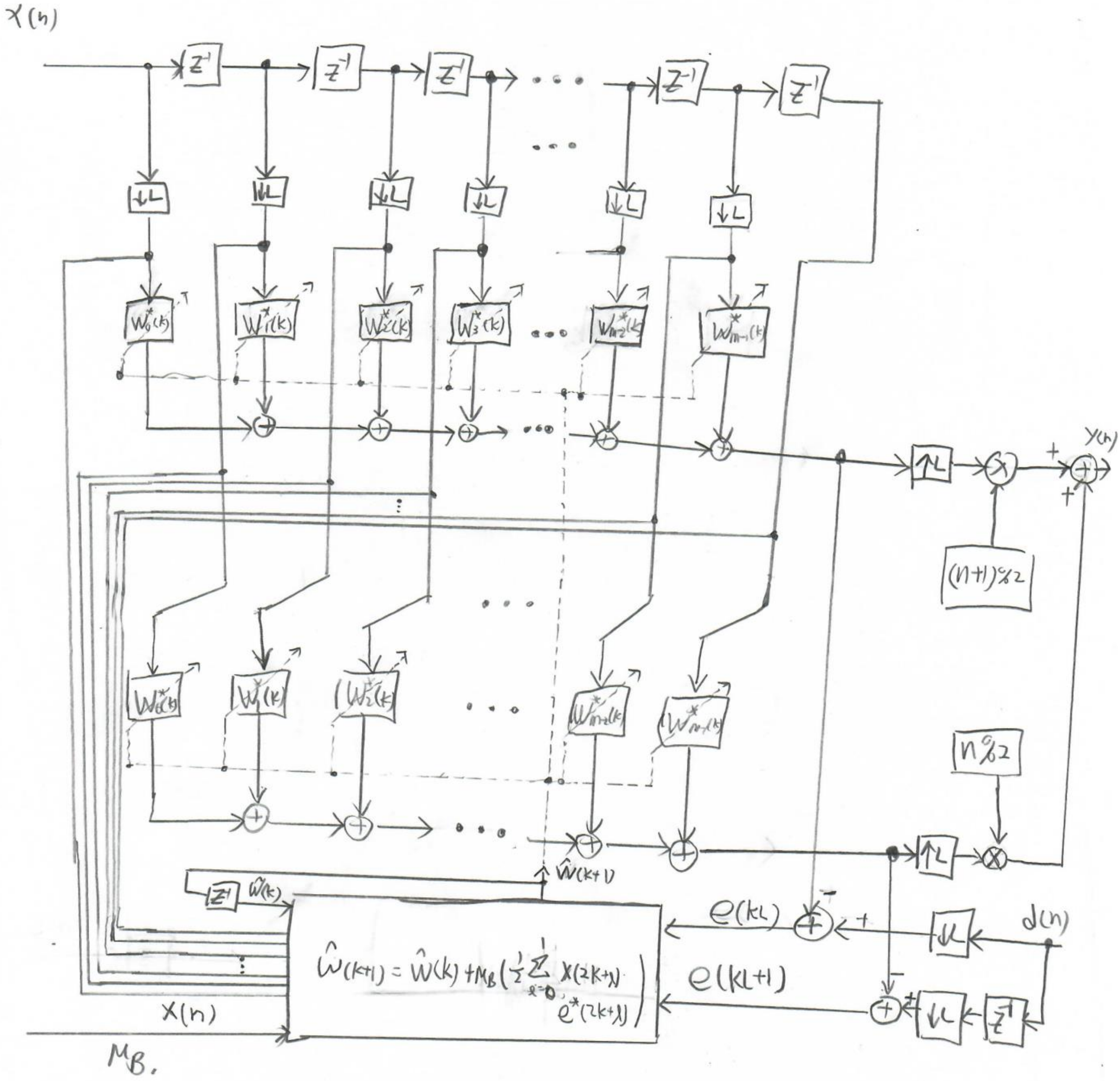
可適性訊號處理 HW3
R09921016 陳宇強

P1



P2

$$\sum_{l=M}^{N} y_{LS}(l)\, e^*_{LS}(l)$$

$$= [y(M) \; y(M+1) \; \ldots \; y(N)] \begin{bmatrix} e^*(M) \\ e^*(M+1) \\ \vdots \\ e^*(N) \end{bmatrix}$$

$$= A^* w^* (d^T - w^T A^T)$$

$$= A^* W^* (d^T - ((A^H A)^{-1} A^H d)^T A^T)$$

$$= A^* w^* (d^T - (d^T A^* (A^T A^*)^{-1}) A^T)$$

$$= A^* w^* d^T (I - A^* (A^T A^*)^{-1} A^T)$$

$$= A^* w^* d^T [I - A^* (A^T A^*)^{-1} A^T (A^* A^T)(A^* A^T)^{-1}]^T$$

$$= A^* w^* d^T [I - A^* (A^T A^*)^{-1} (A^T A^*) A^T (A^* A^T)^{-1}]$$

$$= A^* w^* d^T [I - A^* A^T (A^* A^T)^{-1}]$$

$$= A^* w^* d^T [I - I] = 0$$

P3(a) 令 $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ $M^{-1} = \begin{bmatrix} W & X \\ Y & Z \end{bmatrix}$

$$MM^{-1} = I = \begin{bmatrix} AW+BY & AX+BZ \\ CW+DY & CX+DZ \end{bmatrix}$$

$AW+BY = I$	$CW+DY = 0$
$\Rightarrow W + A^{-1}BY = A^{-1}$	$\Rightarrow D^{-1}CW + Y = 0$
$AX+BZ = 0$	$CX+DZ = I$
$\Rightarrow X + A^{-1}BZ = 0$	$\Rightarrow D^{-1}CX + Z = D^{-1}$

$Y = -D^{-1}CW$

$X = -A^{-1}BZ$

$$W + A^{-1}B(-D^{-1}CW) = A^{-1}$$
$$D^{-1}C(-A^{-1}BZ) + Z = D^{-1}$$
$$\Rightarrow \quad (I - A^{-1}BD^{-1}C)W = A^{-1}$$
$$(I - D^{-1}CA^{-1}B)Z = D^{-1}$$

$$W = (I - A^{-1}BD^{-1}C)^{-1}A^{-1} = (A - AA^{-1}BD^{-1}C)^{-1} = (A - BD^{-1}C)^{-1}$$

$$Z = (I - D^{-1}CA^{-1}B)^{-1}D^{-1} = (D - DD^{-1}CA^{-1}B)^{-1} = (D - CA^{-1}B)^{-1}$$

$$M^{-1}M = I = \begin{bmatrix} WA+XC & WB+XD \\ YA+ZC & YB+ZD \end{bmatrix}$$

$$\begin{cases} YA+ZC=0 \\ WB+XD=0 \end{cases} \quad \begin{cases} Y+ZCA^{-1}=0 \\ WBD^{-1}+X=0 \end{cases} \quad \begin{cases} Y = -ZCA^{-1} \\ X = -WBD^{-1} \end{cases}$$

$Y = -ZCA^{-1}, \quad Y = -D^{-1}CW$

$ZCA^{-1} = D^{-1}CW$

$W = (A - BD^{-1}C)^{-1}$

$Z = (D - CA^{-1}B)^{-1}$

$(D - CA^{-1}B)^{-1}CA^{-1} = D^{-1}C(A - BD^{-1}C)^{-1}$

$X = -A^{-1}BZ, \quad X = -WBD^{-1}$

$A^{-1}BZ = WBD^{-1}$

$A^{-1}B(D - CA^{-1}B)^{-1} = (A - BD^{-1}C)^{-1}BD^{-1}$

$A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1} = (A - BD^{-1}C)^{-1}BD^{-1}CA^{-1}$

$(A - BD^{-1}C)(-A^{-1}) = BD^{-1}CA^{-1} - I \Rightarrow BD^{-1}CA^{-1} = I + (A - BD^{-1}C)(A^{-1})$

$(A - BD^{-1}C)^{-1}(I + (A - BD^{-1}C)(-A^{-1})) = A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1}$

$(A - BD^{-1}C)^{-1} + I(-A^{-1}) = A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1}$

$(A - BD^{-1}C)^{-1} = A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1}$

(題目) $(A + UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}$

$(-B)$換成U，D^{-1}換C　C換V

P3 (b)

$$k(n) = \frac{\lambda^{-1} P(n-1)\, x(n)}{1+\lambda^{-1} x^H(n) P(n-1) x(n)}$$

$$k(n) + k(n)\lambda^{-1} x^H(n) P(n-1) x(n) = \lambda^{-1} P(n-1) x(n)$$

$$k(n) = \lambda^{-1} P(n-1) x(n) - k(n)\lambda^{-1} x^H(n) P(n-1) x(n)$$

$$= \left[\lambda^{-1} P(n-1) - k(n)\lambda^{-1} x^H(n) P(n-1)\right] x(n)$$

其中、$P(n) = \lambda^{-1} P(n-1) - \lambda^{-1} k(n) x^H(n) P(n-1)$

故 $k(n) = P(n)\, x(n)$

P4

$$X(t) = A s(t) + n(t)$$

$$A = \begin{bmatrix} 1 \\ e^{j\pi \sin\theta} \\ e^{j2\pi \sin\theta} \\ \vdots \\ e^{j(N-1)\pi \sin\theta} \end{bmatrix}$$

$(1 \quad -j \quad -ij \quad -3j$

$$R_x = E[X(t)X^H(t)]$$

$$= E[(As(t) + n(t))(As(t) + n(t))^H]$$

$$= E[s(t)s^H(t)]AA^H + E[n(t)n^H(t)]$$

$$= P_1 \begin{bmatrix} 1 & e^{-j\pi \sin\theta} & e^{-j2\pi \sin\theta} & \cdots \\ e^{j\pi \sin\theta} & 1 & e^{-j\pi \sin\theta} & \\ e^{j2\pi \sin\theta} & e^{j\pi \sin\theta} & 1 & \\ \vdots & \vdots & & \ddots \quad 1 \end{bmatrix} + P_n I$$

$$= P_1 AA^H + P_n I$$

P4(b)

$$P_{MVDR}(\theta) = \frac{1}{A^H(\theta) R_x^{-1} A(\theta)}$$

$$= \frac{1}{A^H(\theta)\left(P_1 A(\theta_1) A^H(\theta_1) + P_n I\right)^{-1} A(\theta)}$$

$$= \frac{1}{A^H(\theta) P_n \left(I + \frac{P_1}{P_n} A(\theta_1) A^H(\theta_1)\right)^{-1} A(\theta)}$$

$$\left(I + \frac{P_1}{P_n} A(\theta_1) A^H(\theta_1)\right)^{-1} = I - A(\theta_1)\left(\frac{P_n}{P_1} + A^H(\theta_1) A(\theta_1)\right)^{-1} A^H(\theta_1)$$

$$(A + UCV)^{-1} = A^{-1} - A^{-1} U (C^{-1} + V A^{-1} U)^{-1} V A^{-1}$$

$$P_{MVDR}(\theta) = \frac{1}{P_n A^H(\theta)\left(I - A(\theta_1)\left(\frac{P_n}{P_1} + A^H(\theta_1) A(\theta_1)\right)^{-1} A^H(\theta_1)\right) A(\theta)}$$

$$= \frac{1}{P_n A^H(\theta) A(\theta) - P_n A^H(\theta) A(\theta_1)\left(\frac{P_n}{P_1} + A^H(\theta_1) A(\theta_1)\right)^{-1} A^H(\theta_1) A(\theta)}$$

$$= \frac{1}{P_n A^H(\theta) A(\theta) - P_n\left(\frac{P_n}{P_1} A^H(\theta) A(\theta_1)\right)^{-1} \left(A^H(\theta_1) A(\theta)\right)^H \left(A^H(\theta_1) A(\theta)\right)}$$

其中 $A^H(\theta_1) A(\theta)$

$$= \begin{bmatrix} 1 & e^{-j\pi \sin\theta_1} & e^{-j2\pi \sin\theta_1} & \cdots & e^{-j(N-1)\pi \sin\theta_1} \end{bmatrix} \begin{bmatrix} 1 \\ e^{j\pi \sin\theta} \\ e^{j2\pi \sin\theta} \\ \vdots \\ e^{j(N-1)\sin\theta\pi} \end{bmatrix}$$

$$= 1 + e^{j\pi(\sin\theta - \sin\theta_1)} + e^{j2\pi(\sin\theta - \sin\theta_1)} + \cdots +$$
$$e^{j(N-1)\pi(\sin\theta - \sin\theta_1)}$$

$$= \sum_{i=0}^{N-1} e^{ji\pi(\sin\theta - \sin\theta_1)}$$

當 $\theta = \theta_1$ 時有 $(A^H(\theta_1)A(\theta))^H(A^H(\theta_1)A(\theta))$ 有最大值.

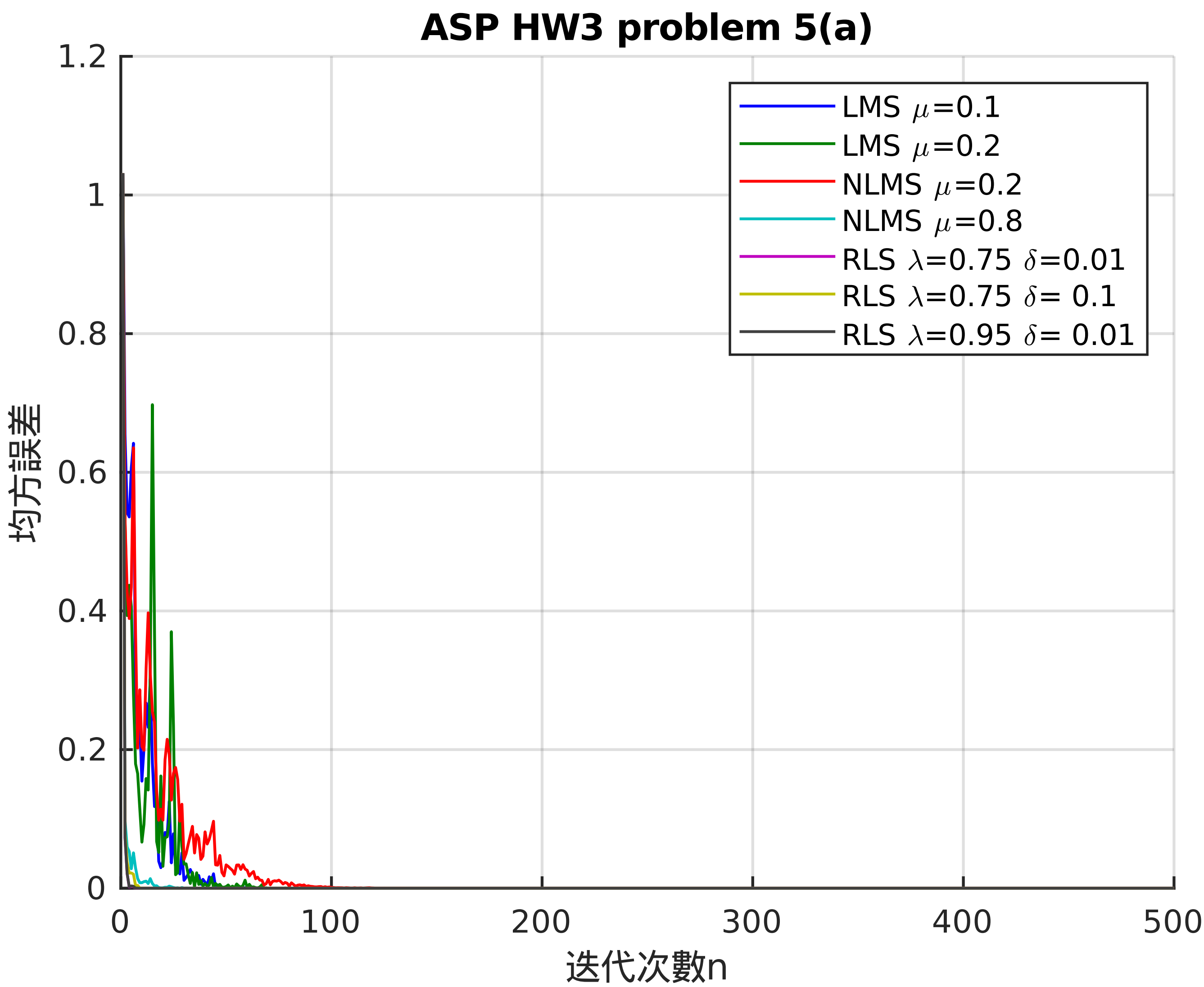
$$\sum_{i=1}^{N-1} e^{ji\pi(\sin\theta_1 - \sin\theta_1)} = N$$

$P_{MVDR}(\theta)$ 的分母在 $\theta = \theta_1$ 時有最小值.

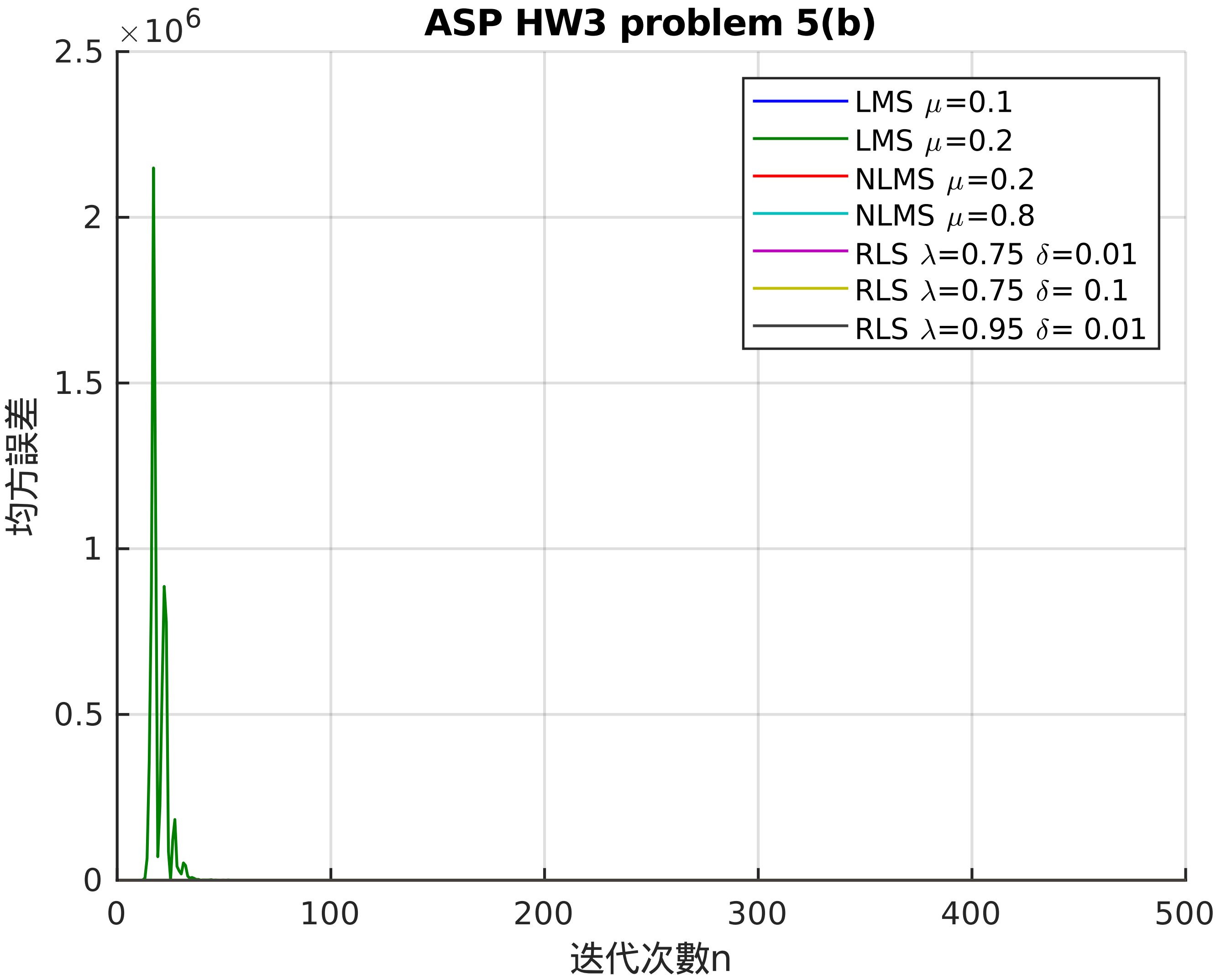
$P_{MVDR}(\theta)$ 在 $\theta = \theta_1$ 時有最大值.

故 $P_{MVDR}(\theta_1) \geq P_{MVDR}(\theta)$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

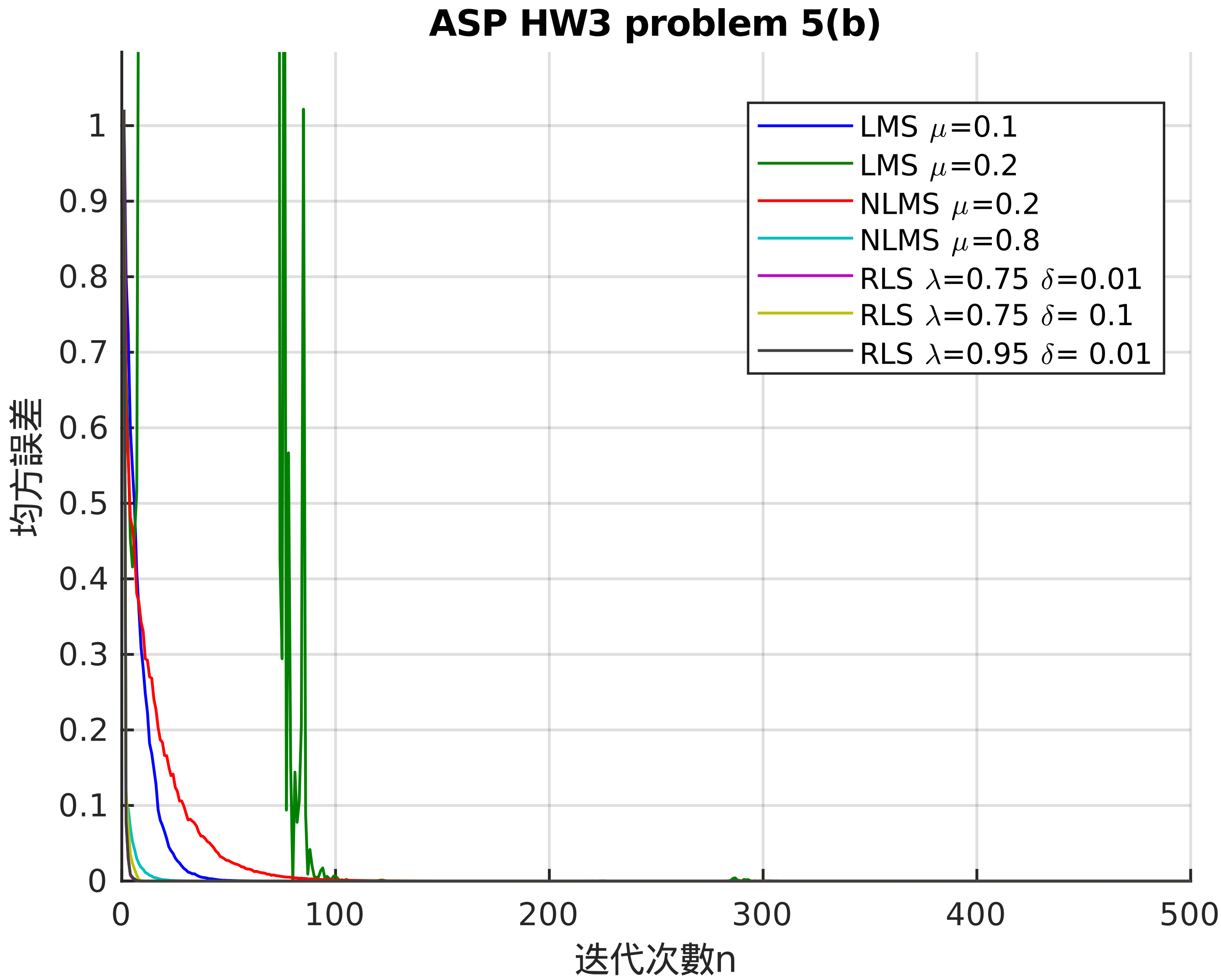
Problem5(a)


ASP HW3 problem 5(a)
LMS μ=0.1
LMS μ=0.2
NLMS μ=0.2
NLMS μ=0.8
RLS λ=0.75 δ=0.01
RLS λ=0.75 δ= 0.1
RLS λ=0.95 δ= 0.01
均方誤差
迭代次數n
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500

Problem5(b)


ASP HW3 problem 5(b)
×10^6
2.5
2
1.5
1
0.5
0
均方誤差
0
100
200
300
400
500
迭代次數n
LMS μ=0.1
LMS μ=0.2
NLMS μ=0.2
NLMS μ=0.8
RLS λ=0.75 δ=0.01
RLS λ=0.75 δ= 0.1
RLS λ=0.95 δ= 0.01

若把下面放大來看


ASP HW3 problem 5(b)
LMS μ=0.1
LMS μ=0.2
NLMS μ=0.2
NLMS μ=0.8
RLS λ=0.75 δ=0.01
RLS λ=0.75 δ= 0.1
RLS λ=0.95 δ= 0.01
均方誤差
迭代次數n
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
100
200
300
400
500

Problem6

考慮 LMS 的第二條線為什麼會爆炸，在講義中提到 LMS 演算法中的μ是有上界的

$$0 < \mu < \min_{1 \le n \le N} \frac{1}{\|x(n)\|_2^2}$$

把不等式畫出來如圖，μ不符合他的上界