Robotics : Assignment 2

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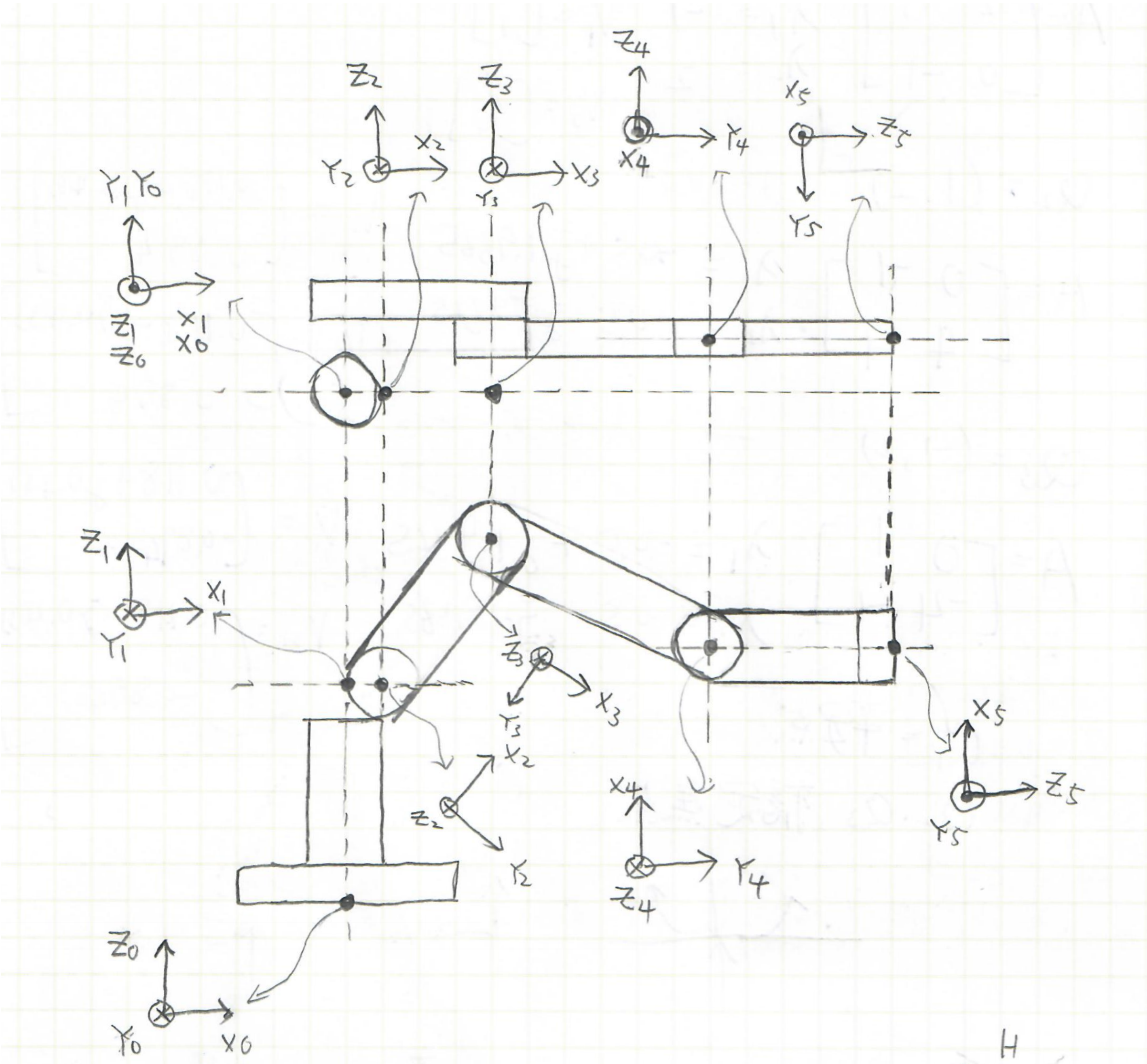
Part A

(1)



(2)

Joint	$\alpha_{i-1}(^\circ)$	$a_{i-1}(mm)$	$d_i(mm)$	θ_i
1	0	0	358.5	θ_1
2	-90°	50	0	θ_2
3	0	300	0	θ_3
4	0	350	35.3	θ_4
5	-90°	0	251	θ_5

PART B

Use the follower formula

$$ {}^{i-1}_{i}T = R_x(\alpha_{i-1})D_x(a_{i-1})R_z(\theta_i)D_z(d_i) $$

Among them, R_x is the rotation of the x axis. D_x is the displacement in the x direction, and so on.

Regarding the base as joint 0, the following sets of matrices can be obtained, where s=sin,c=cos

$$ {}^{\text{BASE}}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 358.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & 50 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} c_3 & -s_3 & 0 & 300 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} c_4 & -s_4 & 0 & 350 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 35.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}_{5}T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & 1 & 251 \\ -s_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

So combine

$$ \begin{aligned} {}^{\text{BASE}}_{5}T &= {}^{\text{BASE}}_{1}T\, {}^{1}_{2}T\, {}^{2}_{3}T\, {}^{3}_{4}T\, {}^{4}_{5}T \\ &= \begin{bmatrix} s_1 s_5 + c_{234} c_1 c_5 & s_1 c_5 - c_{234} c_1 s_5 & -s_{234} c_1 & p_x \\ -c_1 s_5 + c_{234} s_1 c_5 & -c_1 c_5 - c_{234} s_1 s_5 & -s_{234} s_1 & p_y \\ -s_{234} c_5 & s_{234} s_5 & -c_{234} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} $$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} a_1c_1 - d_5(c_4(c_1c_2s_3 + c_1s_2c_3) - s_4(c_1s_2s_3 - c_1c_2c_3)) - a_3(c_1s_2s_3 - c_1c_2c_3) - d_4s_1 + a_2c_1c_2 \\ a_1s_1 - d_5(c_4(s_1c_2s_3 + s_1s_2c_3) - s_4(s_1s_2s_3 - s_1c_2c_3)) - a_3(s_1s_2s_3 - s_1c_2c_3) - d_4c_1 + a_2s_1c_2 \\ d_1 - a_3s_{23} - a_2s_2 - d_5c_{234} \end{bmatrix}$$

$${}^5_0T = {}_{\mathrm{BASE}}^{\ \ 5}T = ({}^{\mathrm{BASE}}_{\ \ 5}T)^{-1} = \begin{bmatrix} {}^5_0T_{11} & {}^5_0T_{12} & {}^5_0T_{13} & {}^5_0T_{14} \\ {}^5_0T_{21} & {}^5_0T_{22} & {}^5_0T_{23} & {}^5_0T_{24} \\ {}^5_0T_{31} & {}^5_0T_{32} & {}^5_0T_{33} & {}^5_0T_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
{}^5_0T_{11} &= s_1s_5 + c_1c_2c_3c_4c_5 - c_1c_2c_5s_3s_4 - c_1c_3c_5s_2s_4 - c_1c_4c_5s_2s_3 \\
{}^5_0T_{12} &= c_2c_3c_4c_5s_1 - c_1s_5 - c_2c_5s_1s_3s_4 - c_3c_5s_1s_2s_4 - c_4c_5s_1s_2s_3 \\
{}^5_0T_{13} &= -s_{234-5}/2 - s_{2345}/2 \\
{}^5_0T_{14} &= (d_1s_{2345})/2 - (a_1c_{2345})/2 - (a_2c_{34-5})/2 - (a_3c_{45})/2 \\
&\quad + d_4s_5 - (a_1c_{234-5})/2 + (d_1s_{234-5})/2 - (a_3c_{4-5})/2 - (a_2c_{345})/2 \\
{}^5_0T_{21} &= c_5s_1 - c_1c_2c_3c_4s_5 + c_1c_2s_3s_4s_5 + c_1c_3s_2s_4s_5 + c_1c_4s_2s_3s_5 \\
{}^5_0T_{22} &= c_2s_1s_3s_4s_5 - c_2c_3c_4s_1s_5 - c_1c_5 + c_3s_1s_2s_4s_5 + c_4s_1s_2s_3s_5 \\
{}^5_0T_{23} &= c_{234-5}/2 - c_{2345}/2 \\
{}^5_0T_{24} &= (d_1c_{2345})/2 - (a_2s_{34-5})/2 + (a_1s_{2345})/2 + (a_3s_{45})/2 \\
&\quad + d_4c_5 - (d_1c_{234-5})/2 - (a_1s_{234-5})/2 - (a_3s_{4-5})/2 + (a_2s_{345})/2 \\
{}^5_0T_{31} &= -s_{234-1}/2 – s_{1234}/2 \\
{}^5_0T_{32} &= c_{1234}/2 - c_{234-1}/2 \\
{}^5_0T_{33} &= -c_{234} \\
{}^5_0T_{34} &= a_2s_{34} - d_5 + a_3s_4 + d_1c_{234} + a_1s_{234}
\end{aligned}$$

PART C

(1)

To observe the matrix ${}^0_5T$

$$\begin{aligned}
{}^{\mathrm{BASE}}_{\ \ 5}T &= {}^{\mathrm{BASE}}_{\ \ 1}T\, {}^1_2T\, {}^2_3T\, {}^3_4T\, {}^4_5T \\
&= \begin{bmatrix} s_1s_5 + c_{234}c_1c_5 & s_1c_5 - c_{234}c_1s_5 & -s_{234}c_1 & p_x \\ -c_1s_5 + c_{234}s_1c_5 & -c_1c_5 - c_{234}s_1s_5 & -s_{234}s_1 & p_y \\ -s_{234}c_5 & s_{234}s_5 & -c_{234} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

Can find that throuhglt r_{13}, r_{23} and r_{33} can get angle of $(\theta_2 + \theta_3 + \theta_4)$

$$\begin{aligned}
r_{13}^2 + r_{23}^2 &= s_{234}^2c_2^2 + s_{234}^2s_1^2 = s_{234}^2 \\
s_{234} &= \pm\sqrt{r_{13}^2 + r_{23}^2} \\
c_{234} &= -r_{33} \\
\theta_{234} &= \arctan 2(s_{234}, c_{234})
\end{aligned}$$

There exist 2 possible solution

And than θ_1 and θ_5 can be get

$$\theta_1 = \arctan 2(\frac{-r_{23}}{s_{234}}, \frac{-r_{13}}{s_{234}})$$

$$\theta_5 = \arctan 2(\frac{r_{32}}{s_{234}}, \frac{-r_{31}}{s_{234}})$$

Since θ_1 and θ_5 have been found, separate them from the original ${}^0_5T$

$$\begin{aligned}
{}^1_4T &= {}^1_0 T\, {}^0_5T\, {}^5_4T \\
&= ({}^0_1T)^{-1}\, {}^0_5T\, ({}^4_5T)^{-1} \\
&= \begin{bmatrix} c_{234} & -s_{234} & 0 & a_1 + a_3c_{23} + a_2c_2 \\ 0 & 0 & 1 & d_4 \\ -s_{234} & -c_{234} & 0 & -a_3s_{23} - a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} r'_{11} & r'_{12} & r'_{13} & p'_x \\ r'_{21} & r'_{22} & r'_{23} & p'_y \\ r'_{31} & r'_{32} & r'_{33} & p'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

Throuhglt p'_x and p'_z can get $\cos(\theta_3)$

$$\begin{aligned}
(p'_x - a_1)^2 + (p'_z)^2 - a_2^2 - a_3^2 &= (a_3c_{23} + a_2c_2)^2 + (-a_3s_{23} - a_2s_2)^2 - a_2^2 - a_3^2 \\
&= a_3^2c_{23}^2 + 2a_2a_3c_{23}c_2 + a_2^2c_2^2 \\
&+ a_3^2s_{23}^2 + 2a_2a_3s_{23}s_2 + a_2^2s_s^2 - a_2^2 - a_3^2 \\
&= 2a_2a_3(c_{23}c_2 + s_{23}s_2) \\
&= 2a_2a_3c_3
\end{aligned}$$

$$c_3 = \frac{(p'_x - a_1)^2 + (p'_z)^2 - a_2^2 - a_3^2}{2a_2a_3}$$

$$\theta_3 = \arctan 2(\pm\sqrt{1 - c_3^2}, c_3)$$

There exist 2 possible solution

Throuhglt p'_x, p'_z and θ_3 can get θ_2

$$\begin{aligned}
p'_x - a_1 &= a_3c_{23} + a_2c_2 \\
&= a_3(c_2c_3 - s_2s_3) + a_2c_2 \\
&= (-a_3s_3)s_2 + (a_3c_3 + a_2)c_2 \\
p'_z &= -a_3s_{23} - a_2s_2 \\
&= -a_3(s_2c_3 + c_2s_3) - a_2s_2 \\
&= (-a_2 - a_3c_3)s_2 + (-a_3s_3)c_2
\end{aligned}$$

$$\begin{bmatrix} \sin(\theta_2) \\ \cos(\theta_2) \end{bmatrix} = \begin{bmatrix} -a_3s_3 & a_3c_3 + a_2 \\ -a_2 - a_3c_3 & -a_3s_3 \end{bmatrix}^{-1} \begin{bmatrix} p'_x - a_1 \\ p'_z \end{bmatrix}$$

$$\theta_2 = \arctan 2(\sin(\theta_2), \cos(\theta_2))$$

We have $\theta_2, \theta_3 \text{ and } (\theta_2 + \theta_3 + \theta_4)$ so that can get θ_4

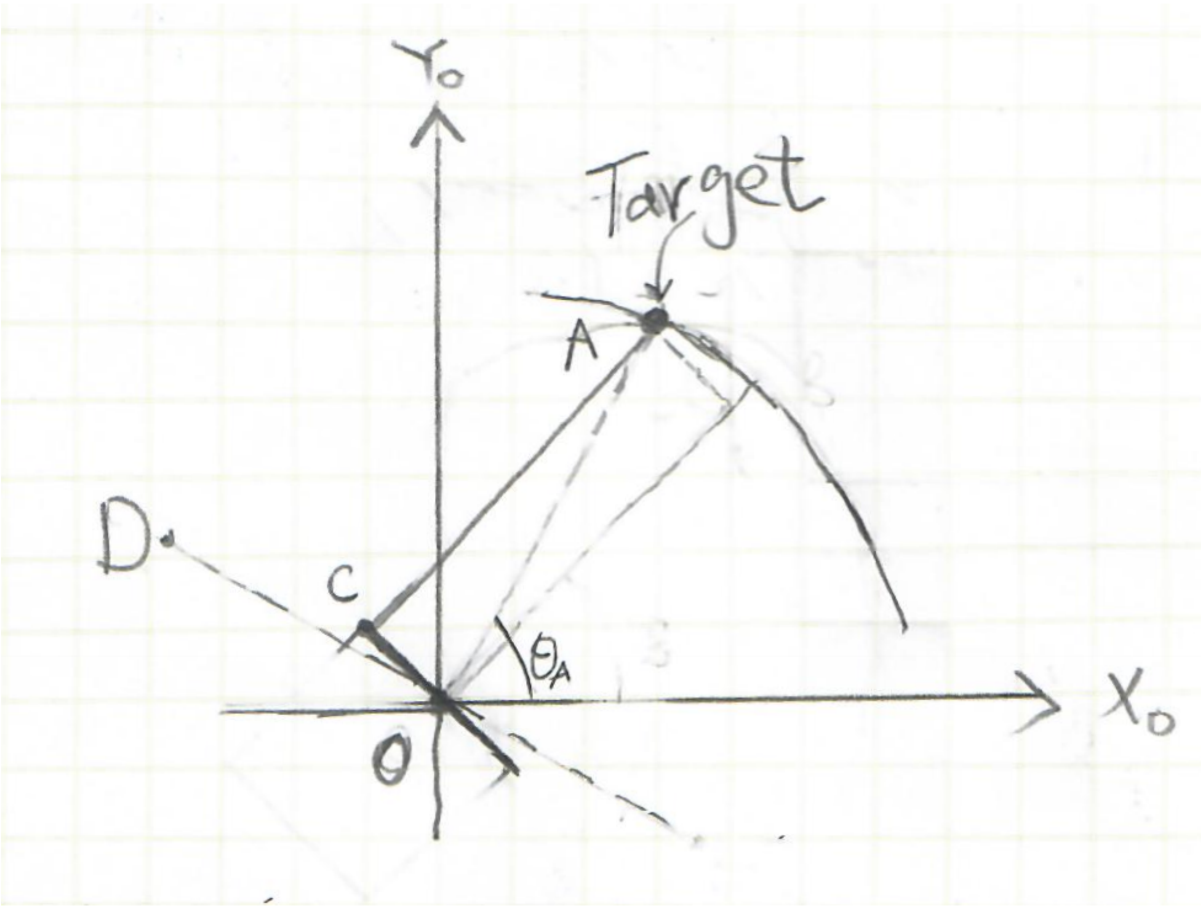
$$\theta_4 = \theta_{234} - \theta_2 - \theta_3$$

So that all the theta have been found!

BUT! When the $\theta_2 + \theta_3 + \theta_4 = 0$ than solve θ_1, θ_5 a Huge problems will happen, it have a Zero denominator. On the other hand, the pose designed for this problem always puts the end down, causing the 234-axis angle sum to be 0.

So that the previous methods are not applicable in this problem. 😭

So I thought of another method based on geometry.



First, by observing the xy plane, we can find that by rotating the first axis to align the x_1 axis with the target, we will get θ_A

$$\theta_A = \arctan 2(p_x, p_y)$$

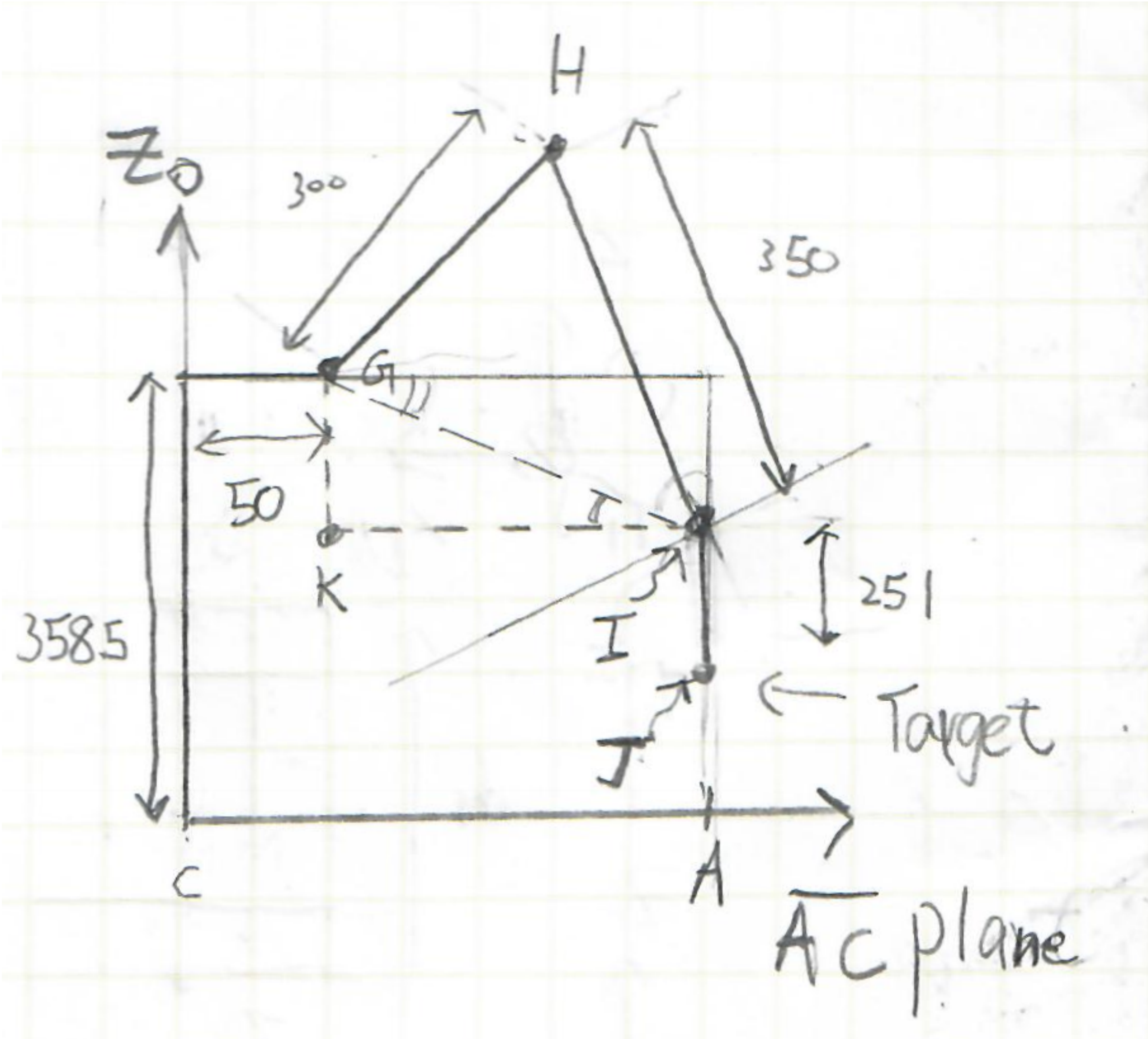
Then you can know that the coordinate system of the first axis and the fourth axis has a known offset in the y_1 direction (35.3mm) $\overline{OC}$ and the distance to the target $\overline{AO}$ can get the distance of $\overline{AC}$

$$\overline{CO} = 35.3$$
$$\overline{AO} = \sqrt{p_x^2 + p_y^2}$$
$$\overline{AC} = \sqrt{\overline{AO}^2 - \overline{CO}^2}$$

In the triangle $\triangle ACO$, the length of the three sides is known. Through the law of cosines, three angles and the angle of the first axis can be obtained.

$$\theta_1 = \theta_A - \angle CAO = \theta_A - \arccos(\frac{\overline{AO}^2 + \overline{AC}^2 - \overline{CO}^2}{2\overline{AO}\,\overline{AC}})$$

Then observe the AC plane and z_0 axis



After observing the AC plane and the z_0 axis, you can intuitively get many side lengths

$$\overline{GK} = 358.5 - p_z - 251$$
$$\overline{IK} = \overline{AC} - 50$$
$$\overline{GH} = 300$$
$$\overline{HI} = 350$$
$$\overline{GI} = \sqrt{\overline{GK}^2 + \overline{IK}^2}$$

From the upper half of the triangle $\triangle IGH$, the angle of the third axis after $\angle GHI$ can be obtained

$$\theta_3 = \pi - \angle GHI = \pi - \arccos(\frac{\overline{GH}^2 + \overline{HI}^2 - \overline{GI}^2}{2\overline{GH}\,\overline{HI}})$$
$$\angle IGH = \arccos(\frac{\overline{GH}^2 + \overline{GI}^2 - \overline{HI}^2}{2\overline{GH}\,\overline{GI}})$$

From the lower half triangle cna get

$$\angle GIK = \arccos(\frac{\overline{GI}^2 + \overline{IK}^2 - \overline{GK}^2}{2\overline{GI}\,\overline{IK}})$$

$$\theta_2 = -\angle IGH + \angle GIK$$

$$\angle GIH = \arccos(\frac{\overline{GI}^2 + \overline{HI}^2 - \overline{GH}^2}{2\overline{GI}\,\overline{HI}})$$

$$\theta_3 = -(\angle GIK + \angle GIH)$$

Finally, consider the fifth axis. Since the problem is limited to the gripper facing downwards, (ϕ, θ, ψ) only needs to consider one rotation ϕ , which can be found when written as a rotation matrix

$$^0_5R = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ -\sin(\phi) & -\cos(\phi) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The overall rotation angle can be obtained. In this posture, only the first and fifth axis can rotate in thez_0 axis direction, and the angle of the first axis has been calculated so

$$\theta_5 = \arctan 2(-^0_5R_{21}, ^0_5R_{11}) + \theta_1$$

All angles have been found.

(2.a)

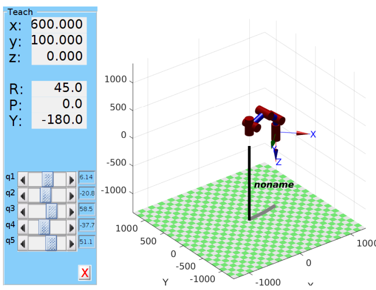
$$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = (6.1354°, -20.8224°, 58.5447°, -37.7222°, 51.1354°)$$

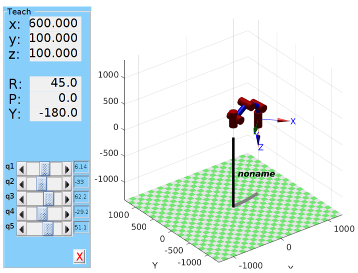
(2.b)

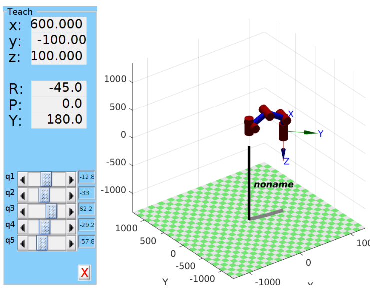
$$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = (6.1354°, -32.9612°, 62.1556°, -29.1944°, 51.1354°)$$

(2.c)

$$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = (-12.7892°, -32.9612°, 62.1556°, -29.1944°, -57.7892°)$$

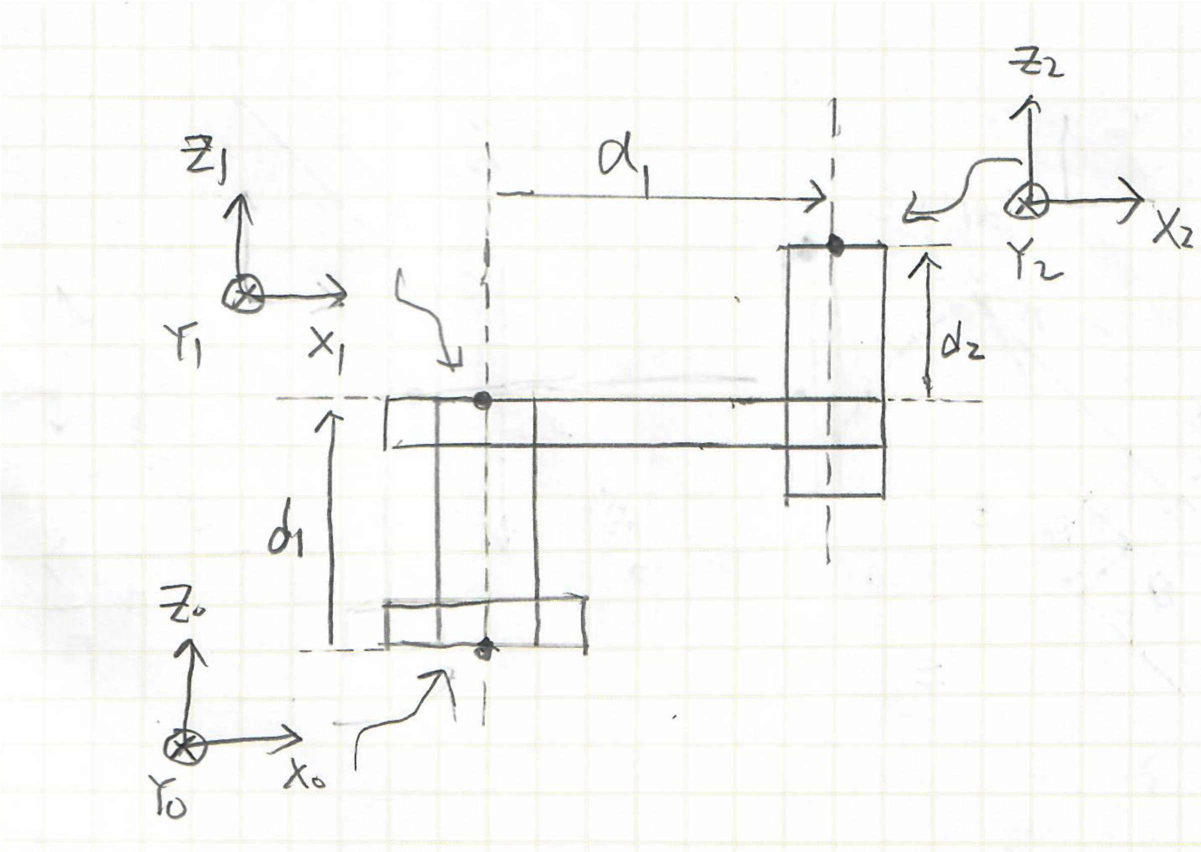






PART D

(1)



Joint	$\alpha_{i-1}(^\circ)$	$a_{i-1}(mm)$	$d_i(mm)$	θ_i
1	0	0	d_1	θ_1
2	0	a_1	d_2	0

(2)

$$\theta_1, d_2$$