MINI PROJECT 1

Part 1

My best guess for parameters:

$$\rho = 1.2250(kg/m^3)$$
$$C_d = 0.35$$
$$C_r = 0.01$$
$$m = 1000(kg)$$
$$g = 9.8(m/s^2)$$
$$I_w = \frac{1}{2}m_w \cdot R^2 = 0.5 \cdot 15 \cdot 0.3^2 = 0.675(kg/m^2)$$
$$M_t = \frac{(mR^2 + I_w)R_g}{R} = \frac{(1000 \cdot 0.3^2 + 0.675) \cdot 3.5}{0.3} = 1057.875(kg/m)$$

Part 2

a): Design a linear controller by linearizing the system

1.

Total torque: $T = \frac{T_e}{M_t} - \frac{T_b R_b}{M_t}$

Speed error: $e(t) = \overline{v} - v(t)$

Then, the dynamical system becomes

$$-\dot{e} = -ae^2 + 2a\overline{v}e + T - (a\overline{v}^2 + c)$$
$$\dot{e} = ae^2 - 2a\overline{v}e - T + a\overline{v}^2 + c$$

where

$$a = \frac{\rho C_d}{2M_t} = 2.03E - 04$$
$$c = \frac{C_r mg}{M_t} = 0.093$$

2.

In the neighborhood of the origin, $x^2 = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n = 0 + 0 + x^2 + 0 + \ldots\ldots$

Thus, the system can be linearized as:

$$\dot{e} = -2a\overline{v}e - T + (a\overline{v}^2 + c)$$

3.

Distance error: $y(t) = \overline{x}(t) - x(t)$

$T = k_1 y + k_2 e + a\overline{v}^2 + c$

4.

Plug in

$$\begin{aligned}\dot{e} &= -2a\overline{v}e - k_1 y - k_2 e \\ &= -(2a\overline{v} + k_2)e - k_1 y\end{aligned}$$

Reformulate ($e = \overline{v} - v$)

$$\begin{aligned}-\dot{v} &= -(2a\overline{v} + k_2)(\overline{v} - v) - k_1 y \\ -\dot{v} &= -(2a\overline{v} + k_2)(-v) - k_1 y - 2a\overline{v}^2 - k_2\overline{v} \\ \dot{v} &= -(2a\overline{v} + k_2)v + k_1 y + 2a\overline{v}^2 + k_2\overline{v}\end{aligned}$$

5.

$$\begin{aligned}v[t+1] &= v[t] + [-(2a\overline{v} + k_2)v[t] + k_1(\overline{x}[t] - x[t]) + 2a\overline{v}^2 + k_2\overline{v}]\delta \\ &= v[t] + [-(2a\overline{v} + k_2)v[t] + k_1\overline{v}t - k_1 x[t] + 2a\overline{v}^2 + k_2\overline{v}]\delta \\ &= v[t] + [-(2a\overline{v} + k_2)v[t] - k_1 x[t] + 2a\overline{v}^2 + (k_2 + k_1 t)\overline{v}]\delta\end{aligned}$$

$$x[t+1] = x[t] + v[t]\delta$$

6.

Take $\sigma = 0.05$, $\mu = 0$ for $w[t]$

b): Directly design a controller for the nonlinear system

1.

$T = ae^2 + k_1 y + k_2 e + a\overline{v}^2 + c$

2.

Plug in

$$\dot{e} = -(2a\overline{v} + k_2)e - k_1 y$$

Reformulate

$$\begin{aligned}-\dot{v} &= -(2a\overline{v} + k_2)(\overline{v} - v) - k_1 y \\ \dot{v} &= -(2a\overline{v} + k_2)v + k_1 y + 2a\overline{v}^2 + k_2\overline{v}\end{aligned}$$

3.

$$\begin{aligned}v[t+1] &= v[t] + [-(2a\overline{v} + k_2)v[t] + k_1(\overline{x}[t] - x[t]) + 2a\overline{v}^2 + k_2\overline{v}]\delta \\ &= v[t] + [-(2a\overline{v} + k_2)v[t] - k_1 x[t] + 2a\overline{v}^2 + (k_2 + k_1 t)\overline{v}]\delta\end{aligned}$$

$$x[t+1] = x[t] + v[t]\delta$$

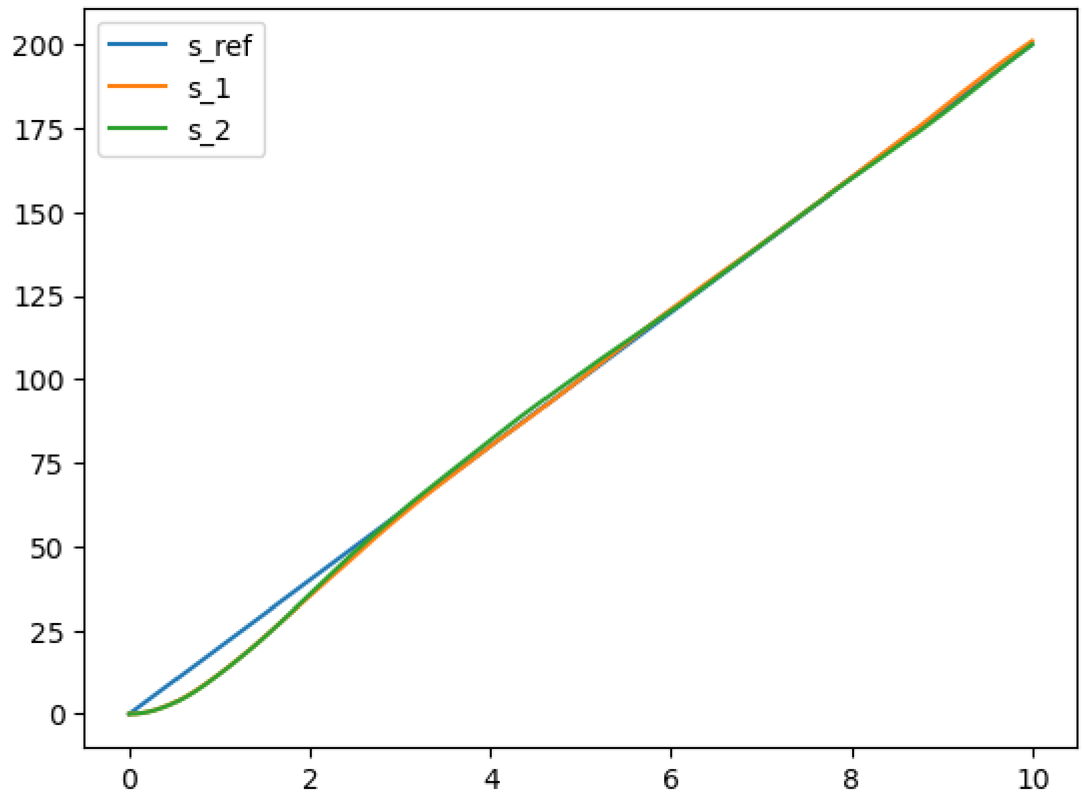
4.

Take $\sigma = 0.05$, $\mu = 0$ for $w[t]$

Part 3

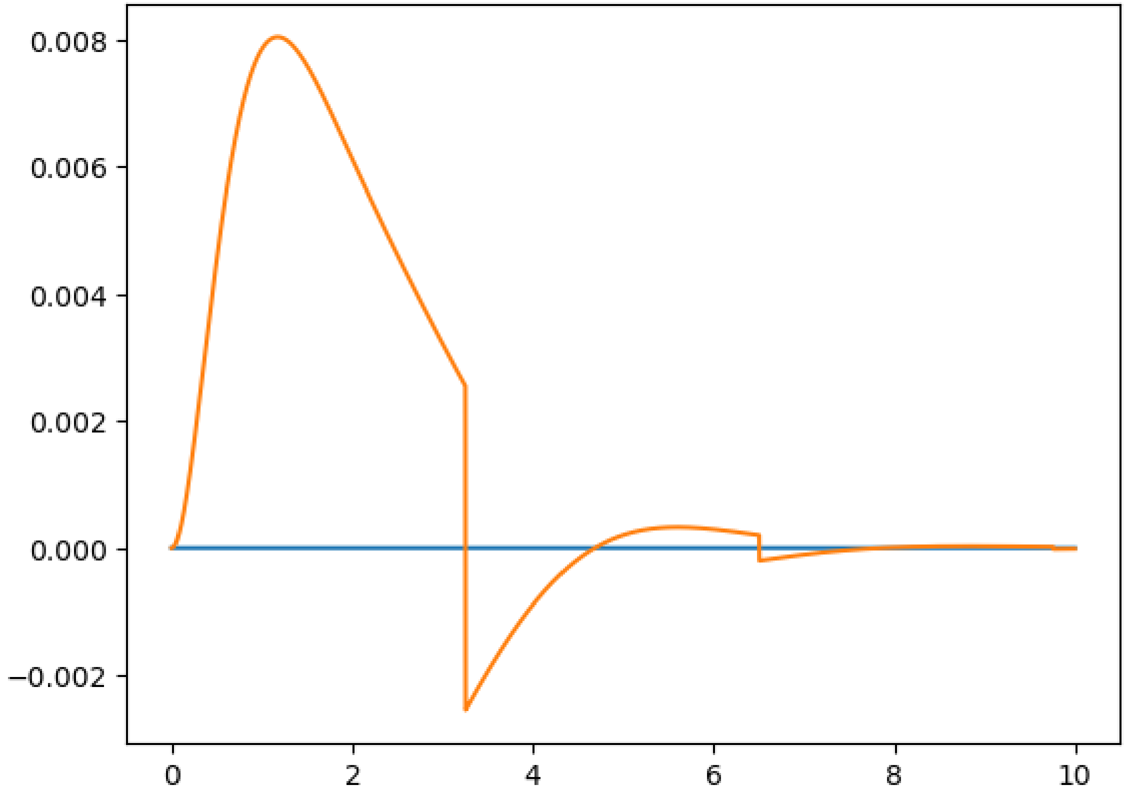
a)

Delta time: $0.001s$
Totally 10000 iterations
Choose $\overline{v} = 20m/s$
For the two policy, $k_1 = 1.5$, $k_2 = 0.8$



b)

Use $z[t] = ||s_{ref}[t] - s_1[t]|| - ||s_{ref}[t] - s_2[t]||$ (with no noise) to visualize the performance of two controllers:



When $z[t] > 0$, it means that that controller 2 behaves better than controller 1 at time t

And from $\frac{\sum_t |s_{ref}[t]-s_1[t]|}{\sum_t |s_{ref}[t]-s_2[t]|} = 1.0010$, controller 2 is slightly better than controller 1, which means that controller 1, as a linear controller, is already good enough.

Appendix

```
import matplotlib.pyplot as plt
import random
import numpy as np
```

```
# some const
a = 2.03*10**(-4)
c = 0.093

# some predetermined para
v_bar = 20
tMax = 10000
tDelta = 0.001
sigma = 0.05

# coeff for controller
k1 = 1.5
k2 = 0.8
```

```
# generate reference trajectory
x_bar = [v_bar*t*tDelta for t in range(tMax)]
T = [t*tDelta for t in range(tMax)]
```

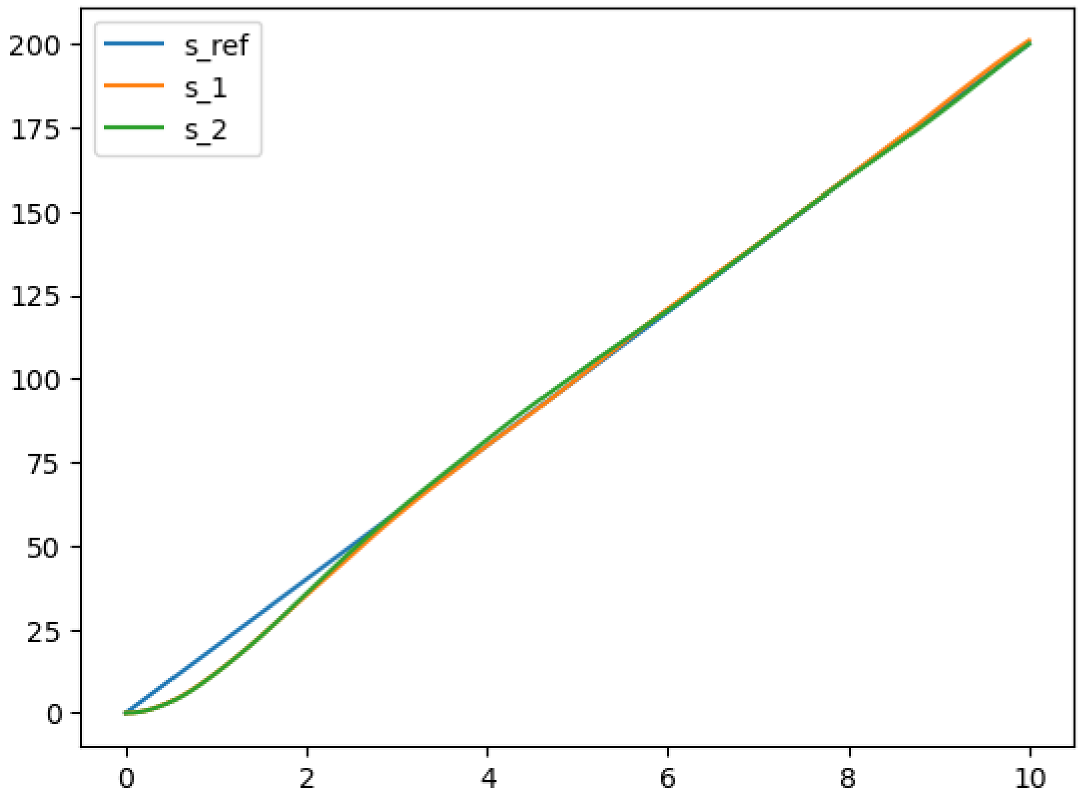
```
# policy 1
def policy1 (x_err, v_err):
    return k1*x_err+k1*v_err+a*v_bar**2+c
# policy 2
def policy2 (x_err, v_err):
    return a*v_err**2+k1*x_err+k1*v_err+a*v_bar**2+c
```

```
# generate trajectory for policy 1
accel1 = [0 for t in range(tMax)]
x1 = [0 for t in range(tMax)]
v1 = [0 for t in range(tMax)]
for t in range(tMax-1):
    accel1[t] = -a*v1[t]**2+policy1((x_bar[t]-x1[t]), (v_bar-v1[t]))-c
    v1[t+1] = v1[t]+accel1[t]*tDelta+random.gauss(0,sigma)
    x1[t+1] = x1[t]+v1[t]*tDelta

# generate trajectory for policy 2
accel2 = [0 for t in range(tMax)]
x2 = [0 for t in range(tMax)]
v2 = [0 for t in range(tMax)]
for t in range(tMax-1):
    accel2[t] = -a*v2[t]**2+policy2((x_bar[t]-x2[t]), (v_bar-v2[t]))-c
    v2[t+1] = v2[t]+accel2[t]*tDelta+random.gauss(0,sigma)
    x2[t+1] = x2[t]+v2[t]*tDelta
```

```
plt.plot(T, x_bar)
plt.plot(T,x1)
plt.plot(T,x2)
plt.legend(["s_ref", "s_1","s_2"])
```

```
<matplotlib.legend.Legend at 0x7fa60b770c70>
```

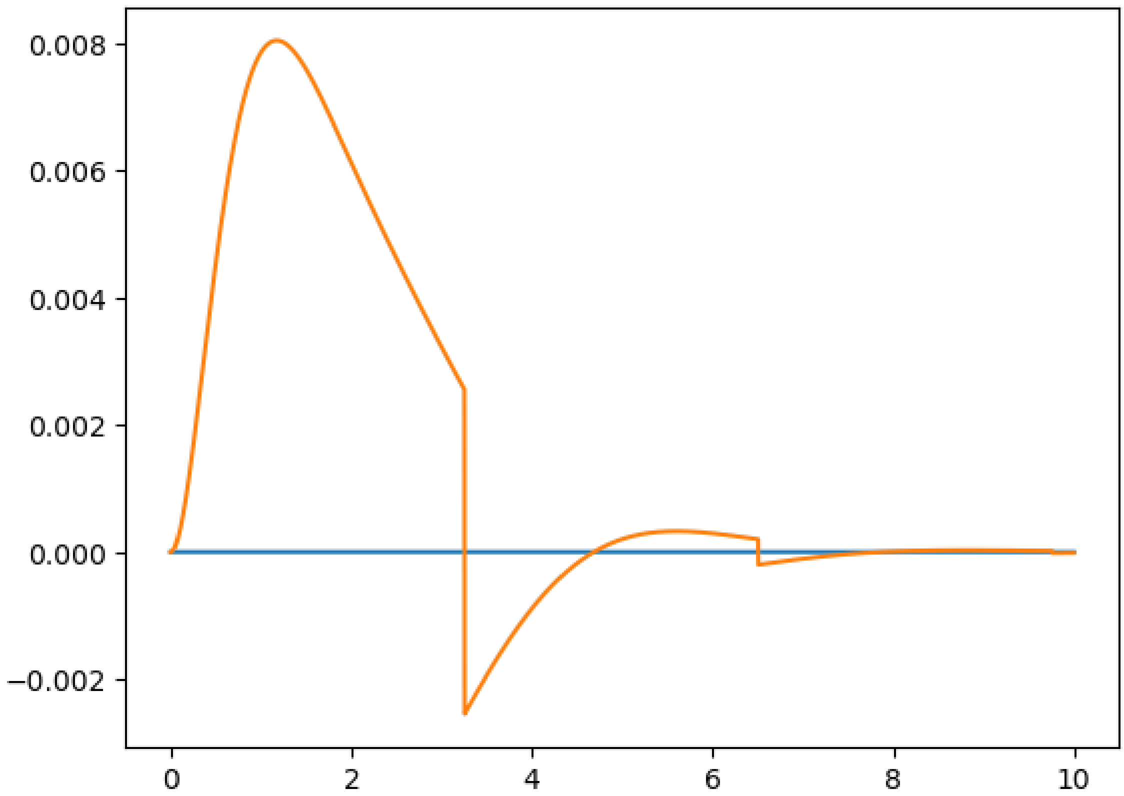



```
# regenerate trajectory for policy 1 with no noise
accel1 = [0 for t in range(tMax)]
x1 = [0 for t in range(tMax)]
v1 = [0 for t in range(tMax)]
for t in range(tMax-1):
    accel1[t] = -a*v1[t]**2+policy1((x_bar[t]-x1[t]), (v_bar-v1[t]))-c
    v1[t+1] = v1[t]+accel1[t]*tDelta
    x1[t+1] = x1[t]+v1[t]*tDelta

# generate trajectory for policy 2 with no noise
accel2 = [0 for t in range(tMax)]
x2 = [0 for t in range(tMax)]
v2 = [0 for t in range(tMax)]
for t in range(tMax-1):
    accel2[t] = -a*v2[t]**2+policy2((x_bar[t]-x2[t]), (v_bar-v2[t]))-c
    v2[t+1] = v2[t]+accel2[t]*tDelta
    x2[t+1] = x2[t]+v2[t]*tDelta
x1 = np.array(x1)
x2 = np.array(x2)
x_bar = np.array(x_bar)
```

```
# for policy evaluation
plt.plot(T, [0 for t in range(tMax)])
plt.plot(T, np.absolute(x_bar-x1)-np.absolute(x_bar-x2))
```

```
[<matplotlib.lines.Line2D at 0x7fa60b7316a0>]
```




```
np.sum(np.absolute(x_bar-x1))/np.sum(np.absolute(x_bar-x2))
```

```
1.0010031571576983
```