Contents

1 Probability Theory

1.1 Elemantary Probability

Definition 1.1.1 (Cardano's Principle)**.** A be a random outcome of an experiment that may proceed in various ways. Assume each of these ways is **equally likely**, then, probability $P[A]$ of outcome A is

$$P[A] = \frac{number\ of\ ways\ leading\ to\ outcome\ A}{number\ of\ ways\ the\ experiment\ can\ be\ proceeded} \tag{1}$$

1.1.1 Basic principles of Counting

- Permutation:
$$A_n^k = \frac{n!}{(n-k)!} \tag{2}$$

- Combination:
$$\tbinom{n}{k} = \frac{n!}{(n-k)!k!} \tag{3}$$

- Permutation of k **Indisguishable** Objects:
$$\frac{n!}{n_1!n_2!n_3!\cdots n_k!} = \tbinom{n}{n_1} \cdot \tbinom{n-n_1}{n_2} \cdots \tbinom{n-(n_1+n_2+\cdots+n_{k-1})}{n_k} \tag{4}$$

  Remark:
  In permutation of k indistinguishable objects, the elements has noorder within a group but are differnent from each other; the groups either has order or are different from each other.
  Example:
  Consider 10 balls, 5 red, 3 green and 2 blue. How many ways can they be arranged on a line?
$$\frac{10!}{5! \cdot 3! \cdot 2!} = 2520 \tag{5}$$

1.1.2 Sample Points, Sample Space and σ-Field

Definition 1.1.2 (Sample Points)**.** Mathematical objects are called sample points.

Definition 1.1.3 (Sample Space)**.** The sample space S is large enough to accommodate all the sample points.

Definition 1.1.4 (Event)**.** An outcome in the sense of Cardano's principle is interpreted as a subset A of a sample space S abd called an event.

Definition 1.1.5 (Mutually exclusive)**.** Two events A_1, A_2 are called mutual exclusive if $A_1 \cap A_2 = \emptyset$

Definition 1.1.6 (σ-Field)**.** Suppose that a non-empty set S is given. A *σ-field* $\mathcal{F}$ on S is a family of subsets of S such that:

- $\emptyset \in \mathcal{F}$
- If $A \in \mathcal{F}$, then $S \backslash A \in \mathcal{F}$
- If $A_1, A_2, \cdots \in \mathcal{F}$ is a finit sequence of subsets, then the union $\cup_k A_k \in \mathcal{F}$

1.1.3 Probability Measures and Spaces

Definition 1.1.7 (Probability Measure)**.** Let S be the sample space and $\mathcal{F}$ be a σ-field. Then a function

$$P : \mathcal{F} \to [0, 1], A \to P[A], \tag{6}$$

is called *probability measure / probability function* on S if

- $P[S] = 1$
- For any set of events $A_k \subset mathcal F$ such that $A_j \cap A_k = \emptyset$ for $j \neq k$,

$$P[\cup_k A_k] = \sum_k a_n P[A_k] \tag{7}$$

Theorem 1.1.1 (Basic Properties)**.** $P[A_1 \cup A_2] = p[A_1] + P[A_2] - P[A_1 \cap A_2]$

1.2 Conditional Probability

Definition 1.2.1 (Conditional Probability)**.** B occurs given that A has occured

$$P[B \mid A] := \frac{P[A \cap B]}{P[A]} \tag{8}$$

1.2.1 Independence of Events

Definition 1.2.2 (independent)**.** Two events are *independent* if

$$P[A \cap B] = P[A]P[B] \tag{9}$$

equivalent to

$$P[A \mid B] = P[A], P[B] \neq 0 \tag{10}$$

Definition 1.2.3 (Total Probability)**.**

$$P[B] = P[B \mid A_1] * P[A_1] + \cdots + P[B \mid A_n]P[A_n] = \sum_{k=1}^{n} P[B \mid A_k] \cdot P[A_k] \tag{11}$$

is called total propability formula for $P[B]$

1.2.2 Bayes' Theorem

Theorem 1.2.1 (Bayes's Theoremm)**.** Let $A_1, \cdots, A_n \subset S$ be a set of pairwise mutually exclusive events whose union is S and who each have non zero probability of occuring. Let $B \subset S$ be any events such that $P[B] \neq 0$. Then for any $A_k, k = 1, \cdots, n$

$$P[A_k \mid B] = \frac{P[B \cap A_k]}{P[B]} = \frac{P[B \mid A_k] \cdot P[A_k]}{\sum_{j=1}^{n} P[B \mid A_j] \cdot P[A_j]} \tag{12}$$