Contents

1 Signal & Systems (Fundamental)

1.1 Signal Definition

Definition 1.1.1 (Signal)**.** Any "physical" quantity that varies with time or space (or other independetn variables).

Example 1.1 (Ambulance Siren:)**.**

$$s(t) = (1+t)sin(2\pi[1000t + 10t^2 + 300sin(2\pi t/2)]) \tag{1}$$

- $(1+t)$: amplitude term represents incresing loudness as ambulance approaches
- $1000t$: represents 1kHz siren oscillatione
- $10t^2$: increasing pitch due to the *Doppler effect* as the ambulance approaches.
- $300sim(2\pi 2t)$: the eeh-ooh-eeh-ohh periodic variation in pitch.

Definition 1.1.2 (Systems)**.** A physical "devicec" that performs an operation on a signal.

Definition 1.1.3 (Signal Processing)**.** Take some input signals and produce some related output signals.

Example 1.2 (audio amplifier)**.**

$$S_{out}(t) = aS_{in}(t) \tag{2}$$

Emphasize on continuous-time of analog signals.

1.2 Classification of Signals

1.2.1 Dimensionality

- By the domain of the function, i.e. how many arguments a function has.
- By the dimension of the range of the function, i.e. the space of values the funciton can take.

1.2.2 Time characteristics

Definition 1.2.1 (Continuous-time Signal)**.** A function defined for all times $t \in (-\infty, \infty)$, or at least some interval (a, b).

Classify signals by time characteristics

- Continuous-11:10 signals or analog signals
- Discrete-time signals

1.2.3 Value Characteristics

Definition 1.2.2 (continuous-valued signal or continuous-amplitude signal)**.** Can take any value in some continuous interval.

Definition 1.2.3 (discrete-valued signal or discrete-amplitude signal)**.** Only takes values from a discrete set of possible values.

1.2.4 Determinstic vs Random Signals

Definition 1.2.4 (Determinstic signals)**.** Can be described by an explicit mathematical representation.

Definition 1.2.5 (Random signals)**.** Evolve over time in an unpredictable manner.

1.3 Transformation of CT Signals

1.3.1 Transformations

- Time transformations
  - Folding/reflecting/time-reversal

$$y(t) = x(-t) \tag{3}$$

  - Time-scaling

$$y(t) = x(at) \tag{4}$$

  - Time-shifting

$$y(t) = x(t - t_0) \tag{5}$$

- General time transformations
  Involves all three of the above time transformations.

$$y(t) = x(at - b) = x(\frac{t - t_0}{w}) \tag{6}$$

  where $t_0 = b/a$, $w = 1/a$

- Amplitude transformations
  - reverse

$$y(t) = -x(t) \tag{7}$$

  - scaling

$$y(t) = ax(t) \tag{8}$$

  - shifting

$$y(t) = x(t) + b \tag{9}$$

- Differentiator

$$y(t) = \frac{d}{dt}x(t) \tag{10}$$

  Example 1.3.

$$y(t) = -RC\frac{d}{dt}x(t) \tag{11}$$

- Integrator

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau \tag{12}$$

  Example 1.4.

$$y(t) = -\frac{1}{RC}\int_{-\infty}^{t} x(\tau)d\tau \tag{13}$$

- Operation with two signals
  Sum or product at any point

1.4 Signal Characteristic

1.4.1 Periodic signals

$$x(t + T) = x(t)\forall t \tag{14}$$

if no T exists, called aperiodic.

Definition 1.4.1 (Fundamental Period)**.** Smallest T_0 of T.

Theorem 1.4.1. With period $T > 0$,

$$x(t + nT) = x(t) \tag{15}$$

Sum of two periodic signals
Suppose a value $T > 0$ satisfies $T = n_1 T_1$ and $T = n_2 T_2$
then, x(t) is periodic with period T.

Theorem 1.4.2. A sum of two periodic signals is period iff the ratio od their periods is rational.

1.4.2 Even and Odd Symmetry

Definition 1.4.2 (Even Symmetry)**.** iff $x(-t) = x(t) \forall t$

Definition 1.4.3 (Odd Symmetry)**.** iff $x(-t) = x(t) \forall t$

Even and Odd Components
We can decompose any signal into even and odd components:

$$x(t) = x_e(t) + x_o(t) \tag{16}$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)], \quad x_o(t) = \frac{1}{2}[x(t) - x(-t)] \tag{17}$$

1.4.3 Average value and energy

Definition 1.4.4 (Average Value)**.**

$$A = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) dt \tag{18}$$

Definition 1.4.5 (Energy)**.**

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \tag{19}$$

Definition 1.4.6 (Average Power)**.**

$$E = \lim_{T \to \infty} \frac{1}{2T} \int_{-\infty}^{\infty} |x(t)|^2 dt \tag{20}$$

Definition 1.4.7 (Energy Signal)**.** If E is finite, then $x(t)$ is called energy signal and $P = 0$.

Definition 1.4.8 (Power Signal)**.** If E is infinite and P is finite and nonzero, $x(t)$ is called power signal.