1 Probability Theory

1.1 Elemantary Probability

Definition 1.1.1 (Cardano's Principle)**.** A be a random outcome of an experiment that may proceed in various ways. Assume each of these ways is **equally likely**, then, probability $P[A]$ of outcome A is

$$P[A] = \frac{number\ of\ ways\ leading\ to\ outcome\ A}{number\ of\ ways\ the\ experiment\ can\ be\ proceeded} \tag{1}$$

1.1.1 Basic principles of Counting

- Permutation:
$$A_n^k = \frac{n!}{(n-k)!} \tag{2}$$
- Combination:
$$\tbinom{n}{k} = \frac{n!}{(n-k)!k!} \tag{3}$$
- Permutation of k **Indisguishable** Objects:
$$\frac{n!}{n_1!n_2!n_3!\cdots n_k!} = \tbinom{n}{n_1} \cdot \tbinom{n-n_1}{n_2} \cdots \tbinom{n-(n_1+n_2+\cdots+n_{k-1})}{n_k} \tag{4}$$
  Remark:
  In permutation of k indistinguishable objects, the elements has noorder within a group but are differnent from each other; the groups either has order or are different from each other.
  Example:
  Consider 10 balls, 5 red, 3 green and 2 blue. How many ways can they be arranged on a line?
$$\frac{10!}{5! \cdot 3! \cdot 2!} = 2520 \tag{5}$$

1.1.2 Sample Points, Sample Space and σ-Field

Definition 1.1.2 (Sample Points)**.** Mathematical objects are called sample points.

Definition 1.1.3 (Sample Space)**.** The sample space S is large enough to accommodate all the sample points.

Definition 1.1.4 (Event)**.** An outcome in the sense of Cardano's principle is interpreted as a subset A of a sample space S abd called an event.

Definition 1.1.5 (Mutually exclusive)**.** Two events A_1, A_2 are called mutual exclusive if $A_1 \cap A_2 = \emptyset$