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1 Signal & Systems (Fundamental)

1.1 Signal Definition

Definition 1.1.1 (Signal). Any "physical" quantity that varies with time or space (or other independent variables).

Example 1.1 (Ambulance Siren:).

$$s(t) = (1 + t)\sin(2\pi[1000t + 10t^2 + 300\sin(2\pi t/2)]) \quad (1)$$

- $(1 + t)$: amplitude term represents increasing loudness as ambulance approaches
- $1000t$: represents 1kHz siren oscillation
- $10t^2$: increasing pitch due to the *Doppler effect* as the ambulance approaches.
- $300\sin(2\pi t)$: the eeh-oooh-eeh-ohh periodic variation in pitch.

Definition 1.1.2 (Systems). A physical "device" that performs an operation on a signal.

Definition 1.1.3 (Signal Processing). Take some input signals and produce some related output signals.

Example 1.2 (audio amplifier).

$$S_{out}(t) = aS_{in}(t) \quad (2)$$

Emphasize on continuous-time of analog signals.

1.2 Classification of Signals

1.2.1 Dimensionality

- By the domain of the function, i.e. how many arguments a function has.
- By the dimension of the range of the function, i.e. the space of values the function can take.

1.2.2 Time characteristics

Definition 1.2.1 (Continuous-time Signal). A function defined for all times $t \in (-\infty, \infty)$, or at least some interval (a, b) .

Classify signals by time characteristics

- Continuous-time signals or analog signals
- Discrete-time signals

1.2.3 Value Characteristics

Definition 1.2.2 (continuous-valued signal or continuous-amplitude signal). Can take any value in some continuous interval.

Definition 1.2.3 (discrete-valued signal or discrete-amplitude signal). Only takes values from a discrete set of possible values.

1.2.4 Deterministic vs Random Signals

Definition 1.2.4 (Deterministic signals). Can be described by an explicit mathematical representation.

Definition 1.2.5 (Random signals). Evolve over time in an unpredictable manner.

1.3 Transformation of CT Signals

1.3.1 Transformations

- Time transformations
 - Folding/reflecting/time-reversal

$$y(t) = x(-t) \tag{3}$$

- Time-scaling

$$y(t) = x(at) \tag{4}$$

- Time-shifting

$$y(t) = x(t - t_0) \tag{5}$$

- General time transformations
Involves all three of the above time transformations.

$$y(t) = x(at - b) = x\left(\frac{t - t_0}{w}\right) \quad (6)$$

where $t_0 = b/a$, $w = 1/a$

- Amplitude transformations

- reverse

$$y(t) = -x(t) \quad (7)$$

- scaling

$$y(t) = ax(t) \quad (8)$$

- shifting

$$y(t) = x(t) + b \quad (9)$$

- Differentiator

$$y(t) = \frac{d}{dt}x(t) \quad (10)$$

Example 1.3.

$$y(t) = -RC \frac{d}{dt}x(t) \quad (11)$$

- Integrator

$$y(t) = \int_{-\infty}^t x(\tau) d\tau \quad (12)$$

Example 1.4.

$$y(t) = -\frac{1}{RC} \int_{-\infty}^t x(\tau) d\tau \quad (13)$$

- Operation with two signals
Sum or product at any point

1.4 Signal Characteristic

1.4.1 Periodic signals

$$x(t + T) = x(t) \forall t \quad (14)$$

if no T exists, called aperiodic.

Definition 1.4.1 (Fundamental Period). Smallest T_0 of T .

Theorem 1.4.1. With period $T > 0$,

$$x(t + nT) = x(t) \quad (15)$$

Sum of two periodic signals
 Suppose a value $T > 0$ satisfies $T = n_1 T_1$ and $T = n_2 T_2$
 then, $x(t)$ is periodic with period T .

Theorem 1.4.2. A sum of two periodic signals is periodic iff the ratio of their periods is rational.

1.4.2 Even and Odd Symmetry

Definition 1.4.2 (Even Symmetry). iff $x(-t) = x(t) \forall t$

Definition 1.4.3 (Odd Symmetry). iff $x(-t) = -x(t) \forall t$

Even and Odd Components
 We can decompose any signal into even and odd components:

$$x(t) = x_e(t) + x_o(t) \quad (16)$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)], \quad x_o(t) = \frac{1}{2}[x(t) - x(-t)] \quad (17)$$

1.4.3 Average value and energy

Definition 1.4.4 (Average Value).

$$A = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) dt \quad (18)$$

Definition 1.4.5 (Energy).

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (19)$$

Definition 1.4.6 (Average Power).

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad (20)$$

Definition 1.4.7 (Energy Signal). If E is finite, then $x(t)$ is called energy signal and $P = 0$.

Definition 1.4.8 (Power Signal). If E is infinite and P is finite and nonzero, $x(t)$ is called power signal.

1.5 Exponential signals

To be done

1.6 Continuous-time system

Definition 1.6.1 (Continuous-time(CT) System). a device that transforms input CT signal into another output CT signal.

$$y(\cdot) = \mathcal{T}(x(\cdot)) \quad (21)$$

Definition 1.6.2 (Input-output Relationship). Precisely defines how the output signal is related to the input signal.

- Series connection

$$x(t) \rightarrow \boxed{\mathcal{T}_1} \rightarrow \boxed{\mathcal{T}_2} \rightarrow y(t) \quad (22)$$

$$y(t) = \mathcal{T}_2[\mathcal{T}_1[x(t)]] \quad (23)$$

- Parallel connection

$$y(t) = \mathcal{T}_1[x(t)] + \mathcal{T}_2[x(t)] \quad (24)$$

1.6.1 Classification of CT Systems

- Amplitude properties
 - A-1 linearity

$$\mathcal{T}[a_1x_1(t) + a_2x_2(t)] = a_1\mathcal{T}[x_1(t)] + a_2\mathcal{T}[x_2(t)] \quad (25)$$

Property

$$\mathcal{T}[ax(t)] = a\mathcal{T}[x(t)] \quad (26)$$

superposition property

$$\mathcal{T}\left[\sum_{k=1}^K \mathcal{T}[x_k(t)]\right] \quad (27)$$

$$\mathcal{T}\left[\int x(t;v)dv\right] = \int \mathcal{T}[x(t;v)]dv \quad (28)$$

- A-2 stability
Satisfy BIBO

Definition 1.6.3 (Bounded-input Bounded-output (BIBO) stable). Every bounded input produces a bounded output

Triangle Inequality

$$\left|\sum_n a_n\right| \leq \sum_n |a_n| \quad (29)$$

- invertibility

Definition 1.6.4 (invertible). each output signal is the response to only one input signal.

Property

$$\mathcal{T}^{-1}[\mathcal{T}[x(t)]] = x(t) \quad (30)$$

- Time properties

- T-1 causality

Definition 1.6.5 (Casual System). The output $y(t)$ at time t only depends on the present and (possibly) past inputs, not on future inputs.

Noncasual systems arise often when t is other variables than time, such as space.

- T-2 memory

Definition 1.6.6 (Static system or Memoryless System). The output $y(t)$ at time t only depends on the current input $x(t)$. Otherwise is a dynamic system.

- T-3 time-invariance Systems whose input-output behavior does not change with time

Definition 1.6.7 (Time Invariant).

$$x(t) \xrightarrow{\mathcal{T}} y(t) \quad \text{implies that} \quad x(t - t_0) \xrightarrow{\mathcal{T}} y(t - t_0) \quad (31)$$