1 Chap9

1.1 THE WAVEFRONT RECONSTRUCTION PROBLEM

1.1.1 Recording Amplitude and Phase

Recording media respond to light intensity, using **interferometry** technique to record phase information.
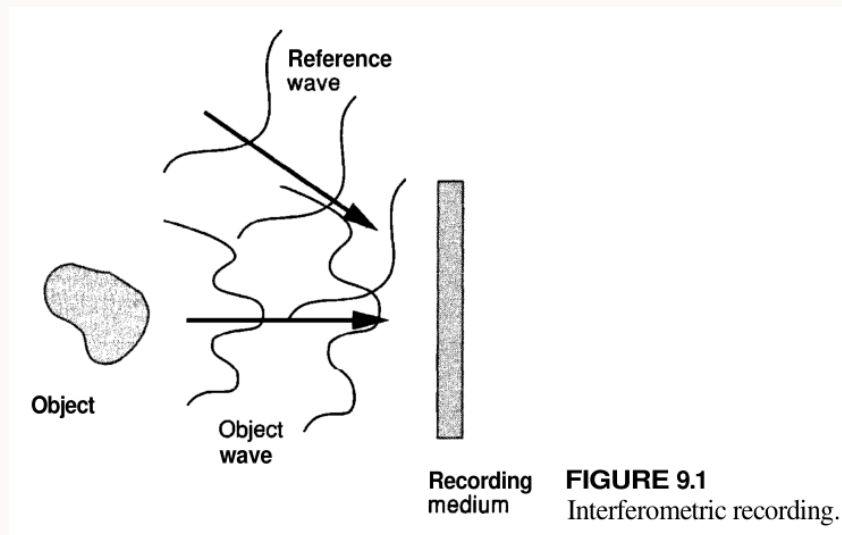
Two complex fields are:

$$\text{Unknown field:} \quad a(x,y) = |a(x,y)| \exp\left[-i\phi(x,y)\right], \tag{1}$$

$$\text{Reference field:} \quad A(x,y) = |A(x,y)| \exp\left[-i\psi(x,y)\right]. \tag{2}$$

The intensity of sum is

$$I(x,y) = |A(x,y)|^2 + |a(x,y)|^2 + 2|A(x,y)||a(x,y)|\cos\left[\psi(x,y) - \phi(x,y)\right]. \tag{3}$$

FIGURE 9.1
Interferometric recording.

1.1.2 The Recording Medium

Assume that the variations of exposure in the interference pattern remain within a linear region of the t_A vs. E curve.

$$t_A(x,y) = t_b + \beta' \left(|a|^2 + A^*a + Aa^* \right). \tag{4}$$

Here t_b is the uniform bias transmittance established by the constant reference exposure, and β' is the product of the slope β of the t_A vs. E curve at the bias point and the exposure time.

1.1.3 Reconstruction of the Original Wavefront

Suppose that the developed transparency is illuminated by a coherent reconstruction wave $B(x,y)$. The light transmitted by the transparency is evidently

$$\begin{aligned} B(x,y)\, t_A(x,y) &= t_b B + \beta' a a^* B + \beta' A^* B a + \beta' A B a^* \\ &= U_1 + U_2 + U_3 + U_4. \end{aligned} \tag{5}$$

- If B is the exact duplication of the original uniform reference wavefront A, the third term of this equation becomes

$$U_3(x,y) = \beta' |A|^2 a(x,y), \tag{6}$$

- If B is the conjugate of the original reference wave, i.e. as $A^*(x,y)$, the fourth term of the reconstructed field becomes

$$U_4(x,y) = \beta' |A|^2 a^*(x,y). \tag{7}$$

1.1.4 Linearity of the Holographic Process

The linear system concept plays no role in holography.

The nonlinearity of the detection process manifests itself in the generation of several output terms, but there is no nonlinear distortion of the one term of interest, assuming that the exposure variations remain in the linear region of the t_A vs. E curve.
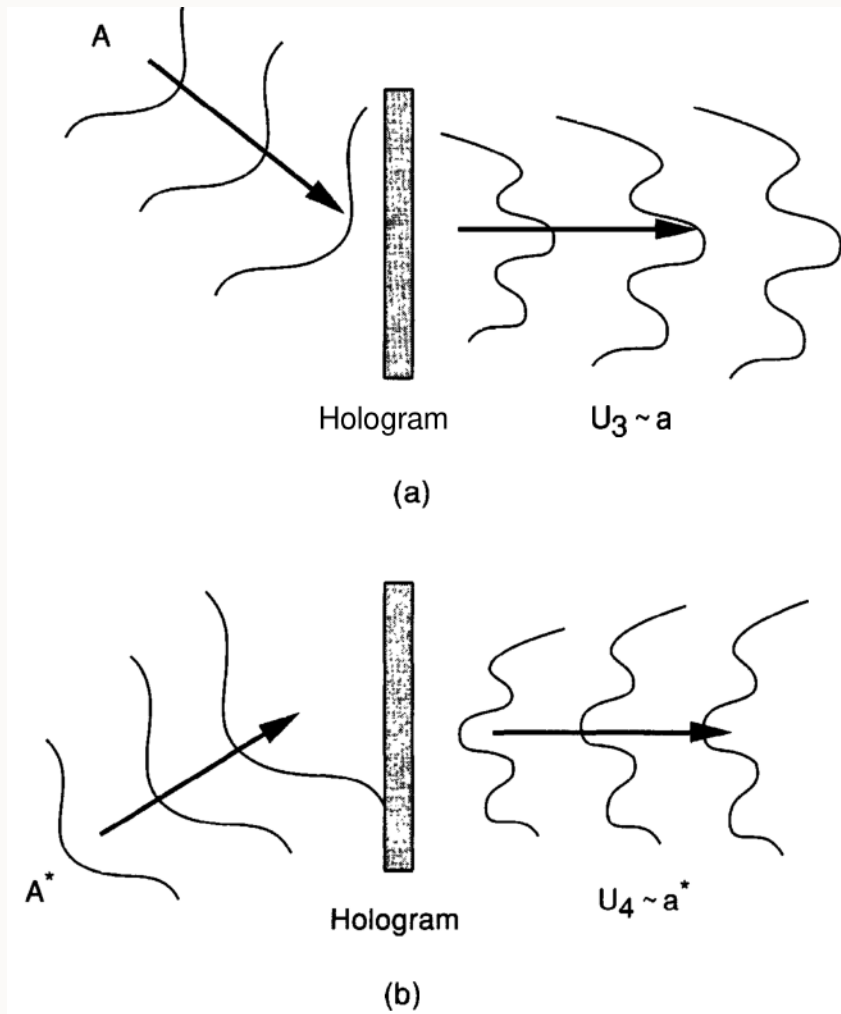
FIGURE 9.2
Wavefront reconstruction with (a) the original reference wave A as illumination, and (b) the conjugate reference wave A* as illumination.

1.1.5 Image Formation by Holography

Considered only the problem of reconstructing a wavefront which arrived at a recording medium from a coherently illuminated object

- In Figure3(b), we see that the wave U_3 under reconstruction field A can be seen as a **duplication** of the object and as a **virtual object**;
- In Figure 3(c), we see that the wave U_4 under reconstruction field A^* can generate a **real image**.

Example 1.1 *If we consider that the object field is*

$$a\left(x,y\right) = a_0 \exp\left[ik\sqrt{z_0^2 + \left(x - x_0\right)^2 + \left(y - y_0\right)^2}\right], \tag{8}$$

then we can see that under two reconstruction field U_3 and U_4 will be

$$U_3\left(x,y\right) = \beta'\left|A\right|^2 a_0 \exp\left[ik\sqrt{z_0^2 + \left(x - x_0\right)^2 + \left(y - y_0\right)^2}\right], \tag{9}$$

and

$$U_4\left(x,y\right) = \beta'\left|A\right|^2 a_0 \exp\left[-ik\sqrt{z_0^2 + \left(x - x_0\right)^2 + \left(y - y_0\right)^2}\right]. \tag{10}$$

1.2 THE GABOR HOLOGRAM

1.2.1 Origin of the Reference Wave

Object assumed to be highly transmissive, with an amplitude transmittance

$$t\left(x_o, y_o\right) = t_0 + \Delta t\left(x_o, y_o\right), \tag{11}$$

where the variation is smaller than the average.

When such an object is coherently illuminated by the collimated wave shown in Fig.9.4, the transmitted light consists of two components: (1) a strong uniform plane wave passed by the term t_0, and (2) a weak scattered wave generated by the transmittance variations $\Delta t\left(x_o, y_o\right)$.
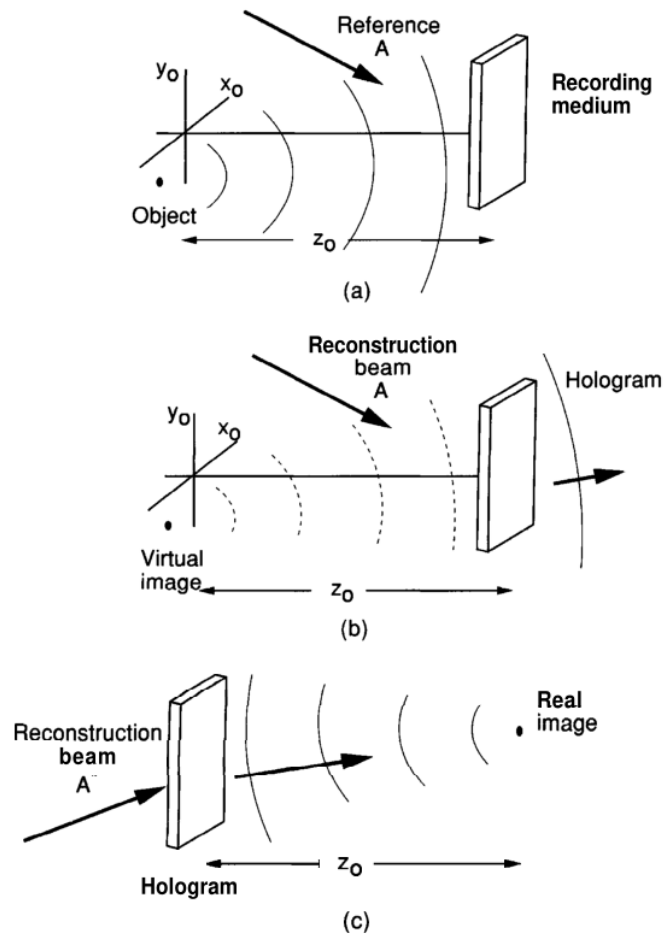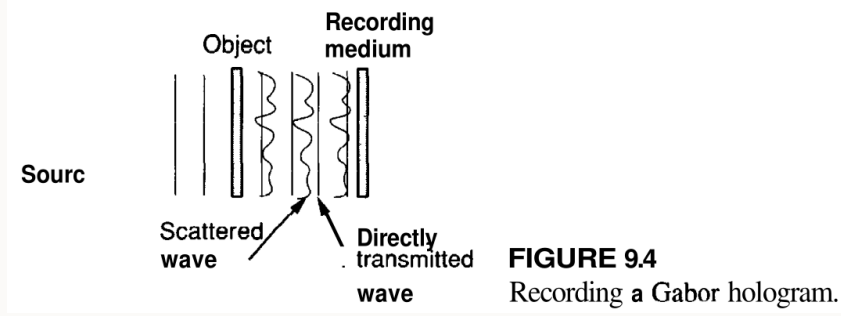
FIGURE 9.3
Imaging by wavefront reconstruction. (a) Recording the hologram of a point-source object; (b) generation of the virtual image; (c) generation of the real image.

Then the medium records the intensity as

$$I\left(x,y\right) = |A|^2 + |a\left(x,y\right)|^2 + A^*a\left(x,y\right) + Aa^*\left(x,y\right), \qquad (12)$$

Here A is the amplitude of the plane wave and a is the scattered field.

FIGURE 9.4
Recording a Gabor hologram.

1.2.2 The Twin Images

Using the plane wave B to illuminate the medium, then the field will be

$$B\left(x,y\right)t_A\left(x,y\right) = Bt_b + \beta'B|a\left(x,y\right)|^2 + \beta'A^*Ba\left(x,y\right) + \beta'ABa^*\left(x,y\right).$$
$$(13)$$

- The first term is a plane wave which passes directly through the transparency, suffering uniform attenuation but without scattering.
- The second term may be dropped as negligible by virtue of our assumption $|\Delta t| < |t_0|$, which implies that

$$|a\left(x,y\right)| \ll A. \qquad (14)$$

- The third term represents a field component that is proportional to the original scattered wave $a(x,y)$. This wave appears to originate from a **virtual image** of the original object located at distance z_o from the transparency.
- The fourth term is proportional to $a^*(x,y)$ and, in accord with our earlier discussions, leads to the formation of a **real image** at distance z_o on the opposite side of the transparency from the virtual image.

FIGURE 9.5
Formation of twin images from a **Gabor** hologram.
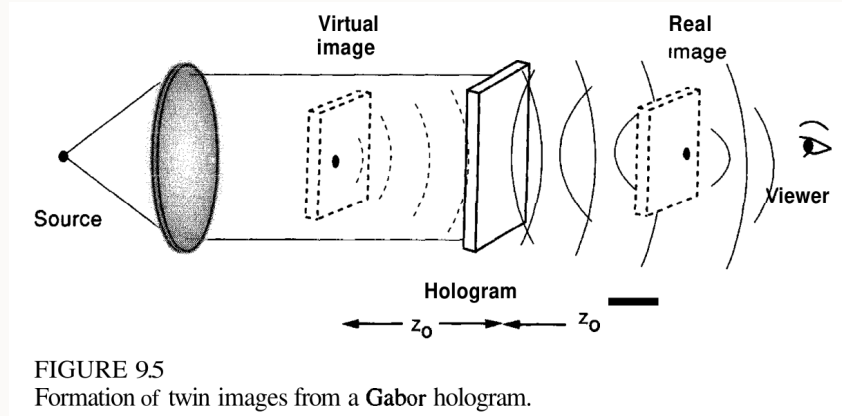
1.2.3 Limitations of the Gabor Hologram

- The most important limitation is inherent in the assumption of a highly transparent object and the consequent conclusion $|a(x,y)| \ll A$ that followed. $\rightarrow$ Then the second term will remain

$$U_2 = \beta' B |a(x,y)|^2. \tag{15}$$

Low average transmittance lead to U_2 to be the largest contributing term. So Gabor Hologram can be applied with opaque letters on a transparent background, but not with not transparent letters on an opaque background.

- A second serious limitation lies in the generation of **overlapping twin images**, rather than a single image. When the real image is brought to focus, it is always accompanied by an out-of-focus virtual image. Likewise an observer viewing the virtual image sees simultaneously a defocused image arising from the real-image term. Thus, even for highly transparent objects, the quality of the images is reduced by the twin image problem.

1.3 THE LEITH-UPATNIEKS HOLOGRAM

The major change between this type of hologram and the Gabor hologram is that, rather than depending on the light directly transmitted by the object to serve as a reference wave, a **separate and distinct reference** wave is introduced. Furthermore the reference is introduced at an offset angle, rather than being collinear with the object-film axis.
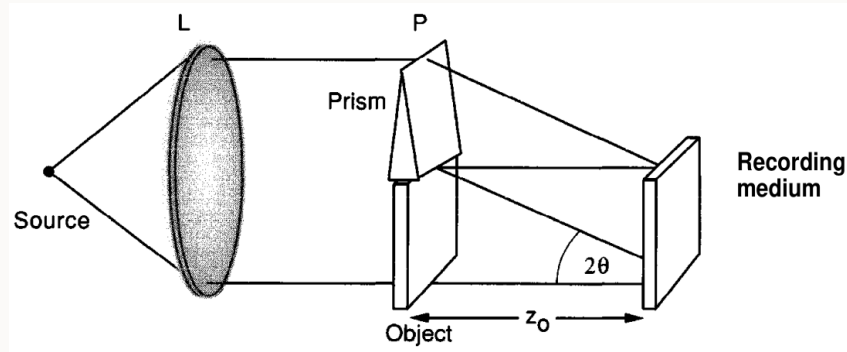
1.3.1 Recording the Hologram

FIGURE 9.6
Recording a Leith-Upatnieks hologram.

On the recording plane

$$U(x, y) = A \exp(-j2\pi\alpha y) + a(x, y), \tag{16}$$

where

$$\alpha = \frac{\sin 2\theta}{\lambda}. \tag{17}$$

Then the intensity distribution across the recording plane is evidently

$$I(x, y) = |A|^2 + |a(x, y)|^2 + A^* a(x, y) \exp(j2\pi\alpha y) + A a^*(x, y) \exp(-j2\pi\alpha y). \tag{18}$$

If the object field is expressed by the modula and the phase

$$a(x, y) = |a(x, y)| \exp[-j\phi(x, y)], \tag{19}$$

then the intensity will be

$$I(x, y) = |A|^2 + |a(x, y)|^2 + 2|A| |a(x, y)| \cos[2\pi\alpha y - \phi(x, y)]. \tag{20}$$

This expression demonstrates that the amplitude and phase of the light arriving from the object have been recorded, respectively, as amplitude and phase modulations of a spatial carrier of frequency α. If the carrier frequency is sufficiently high, the amplitude and phase distributions can be unambiguously recovered from this pattern of interference.

1.3.2 Obtaining the Reconstructed Images

Like before, we find that the transmittance is

$$t_A (x, y) = t_b + \beta' \left[|a (x, y)|^2 + A^* a (x, y) \exp (j2\pi\alpha y) + Aa^* (x, y) \exp (-j2\pi\alpha y) \right].$$
(21)

The four terms can be represented as

$$t_1 = t_b, \qquad\qquad t_2 = \beta' |a (x, y)|^2,$$
$$t_3 = \beta' A^* a (x, y) \exp (j2\pi\alpha y), \quad t_4 = \beta' Aa^* (x, y) \exp (-j2\pi\alpha y).$$
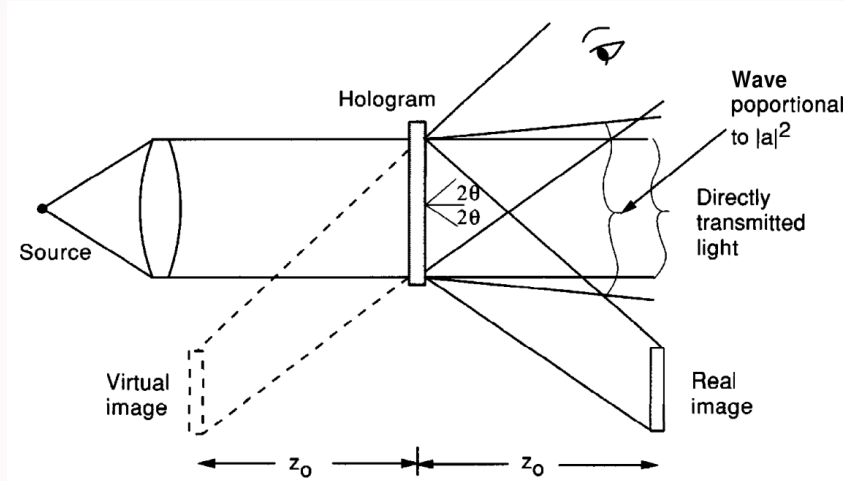(22)

FIGURE **9.7**
Reconstruction of images from a Leith-Upatnieks hologram.

Considering a normal incident light plane wave B, the transimitted wave will be

$$U_1 = t_b B, \qquad\qquad U_2 = \beta' B |a (x, y)|^2,$$
$$U_3 = \beta' BA^* a (x, y) \exp (j2\pi\alpha y), \quad U_4 = \beta' BAa^* (x, y) \exp (-j2\pi\alpha y).$$
(23)

- U_1 is an attenuated version of the incident reconstruction illumination;
- U_2 is spatially varying and therefore has plane wave components traveling at various angles with respect to the optical axis;
- U_3 is proportional to the original object wavefront a multiplied by a linear exponential factor. This term generates a **virtual image** of the object at distance z_o to the left of the transparency, while the linear exponential factor

$\exp(j2\pi\alpha y)$ indicates that this image is deflected away from the optical axis at angle 2θ, as shown in Fig. 9.7.

- U_4 is proportional to the conjugate wavefront a^*, which indicated a **real image** at distance z_o to the right of the transparency. The presence of the linear exponential factor $\exp(-j2\pi\alpha y)$ indicates that the real image is deflected at angle -2θ from the optical axis, as again can be seen in Fig. 9.7.

1.4 Device Options

To make the notes more comfortable to read, we designed four output options (of different sizes) that correspond to different reading devices: Pad (default), Kindle, PC and A4paper.

New: For the convenience of notes presentation, version 2.20 offers a new option for device, i.e. `device`=screen, which is similar to the size of MS Powerpoint with ratio aspect of 4:3 (2019/12/06).

The options of output for different devices are

```
\documentclass[device=pad]{elegantnote}    % ipad screen size
\documentclass[device=kindle]{elegantnote} % kindle screen
   size
\documentclass[device=pc]{elegantnote}     % double pages for
   pc
\documentclass[device=normal]{elegantnote} % a4 normal page
\documentclass[device=screen]{elegantnote} % 4:3 PPT size
```

Note You can also select the device by using a direct assignment method, such as:

```
\documentclass[pad]{elegantnote}
\documentclass[kindle]{elegantnote}
\documentclass[pc]{elegantnote}
\documentclass[normal]{elegantnote}
\documentclass[screen]{elegantnote}
```

Note To get a normal A4paper size PDF, please select `device`=normal.

1.5   Math Fonts

This template defines a new option (`math`), with three options:

1. `math`=cm (default), use L^AT_EX default math font (recommended).
2. `math`=`newtx`, use `newtxmath` math font (may bring about bugs).
3. `math`=mtpro2, use mtpro2 package to set math font.

1.6   Color Themes[1]

This template contains 5 color themes, green, cyan, blue(default), sakura and black. If you don't need color, you can choose black theme. The color theme is enabled in the same way as before:

```
\documentclass[green]{elegantnote}
\documentclass[color=green]{elegantnote}
....
\documentclass[black]{elegantnote}
\documentclass[color=black]{elegantnote}
```

1.7   Languages

This template contains two sets of language environments, changing the language environment will change the title of table/figure (figure, table), article structure words (such as the table of contents, references, etc.), and the environment Introductory words (such as Theorem, Lemma, etc.). The different language modes are enabled as follows:

```
\documentclass[cn]{elegantnote}
\documentclass[lang=cn]{elegantnote}
\documentclass[en]{elegantnote}
\documentclass[lang=en]{elegantnote}
```

Note Chinese characters are allowed in Chinese mode only. To type in Chinese

---

[1]Test for chapter footnote.

characters in English mode, please include `ctex`[2] or `xeCJK` package.

1.8   Theorem Class Environments

This template used the `amsthm` to create theorems, there are 4 types of theorem environments

- **Theorem-Class**: theorem, lemma, proposition, corollary;
- **Definition-Class**: definition, conjecture, example;
- **Remark-Class**: remark, note, case;
- **Proof-Class**: proof.

Remark With the option `lang=cn` , the introductory words of the theorem class environments will be changed to Chinese.

2   Writing Sample

We will define the integral of a measurable function in three steps. First, we define the integral of a nonnegative simple function. Let E be the measurable set in $\mathcal{R}^N$.

Definition 2.1 (Left Coset) *Let H be a subgroup of a group G. A left coset of H in G is a subset of G that is of the form xH, where $x \in G$ and $xH = \{xh : h \in H\}$. Similarly a right coset of H in G is a subset of G that is of the form Hx, where $Hx = \{hx : h \in H\}$*

Note that a subgroup H of a group G is itself a left coset of H in G.

Lemma 2.1 (Size Of Left Coset) *Let H be a finite subgroup of a group G. Then each left coset of H in G has the same number of elements as H.*

Theorem 2.2 (Lagrange's Theorem) *Let G be a finite group, and let H be a subgroup of G. Then the order of H divides the order of G.*

---

[2]Please use `scheme=plain` to retain headlines in English.

Proof. Let z be some element of $xH \cap yH$. Then $z = xa$ for some $a \in H$, and $z = yb$ for some $b \in H$. If h is any element of H then $ah \in H$ and $a^{-1}h \in H$, since H is a subgroup of G. But $zh = x(ah)$ and $xh = z(a^{-1}h)$ for all $h \in H$. Therefore $zH \subset xH$ and $xH \subset zH$, and thus $xH = zH$. Similarly $yH = zH$, and thus $xH = yH$, as required.
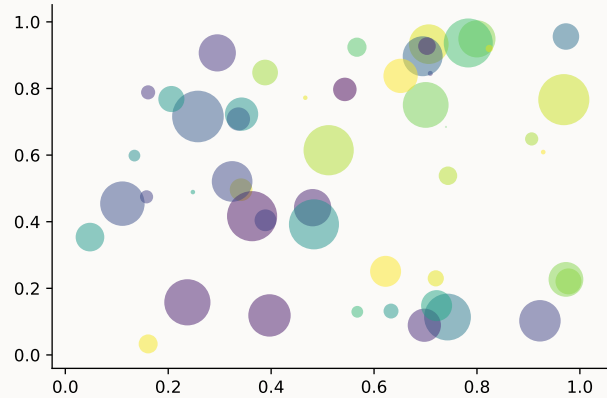
Figure 1: Matplotlib: Scatter Plot Example

Regression analysis is a powerful statistical method that allows you to examine the relationship between two or more variables of interest. While there are many types of regression analysis, at their core they all examine the influence of one or more independent variables on a dependent variable. The process of performing a regression allows you to confidently determine which factors matter most, which factors can be ignored, and how these factors influence each other.

Let's continue using our application training example. In this case, we'd want to measure the historical levels of satisfaction with the events from the past three years or so, as well as any information possible in regards to the independent variables.

- Routing and resource discovery;
    - Language Models
    - Vector Space Models
- Resilient and scalable computer networks;

Table 1: Auto MPG and Price

	(1)	(2)
mpg	-238.90***	-49.51
	(53.08)	(86.16)
weight		1.75***
		(0.641)
constant	11,253***	1,946
	(1,171)	(3,597)
obs	74	74
R^2	0.220	0.293

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

- Distributed storage and search.

3 Recruit Support Members

Recruit support members for ElegantLATEX to translate template official guide, maintain wiki entries, update Wechat articles. No deadline for this recruitment.

So far, ElegantLATEX has four support members:
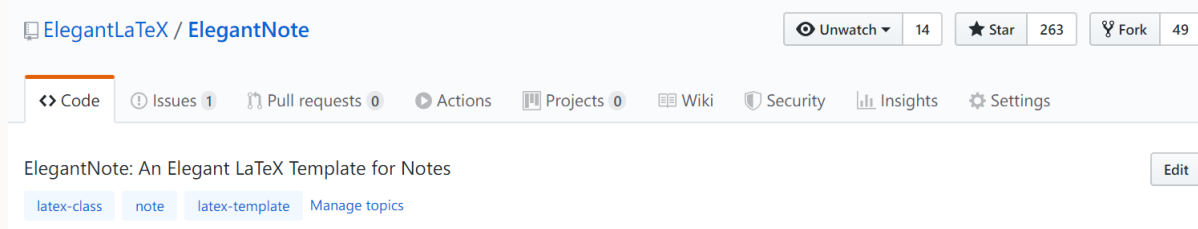
- OG Translator: YPY;
- Wiki Maintainer: Ingo Zinngo, Xiaohao890809;
- QQ Group Manager: Sikouhjw.

Thank them all!!!

4 Acknowledgement

The number of stars on GitHub for ElegantPaper reached 176 on April 12, 2020 at the release of ElegantNote v2.20. Thank ChinaTEX and LATEX studio for their promotion.

If you like our templates, star on GitHub.

Figure 2: Twinkle, Twinkle, Little Star

5 Donation

To express your love for our templates and/or our developers, please do not hesitate to tip us.

The explanation right of the tip usage belongs to ElegantLATEX with no supervision. Feel free to tip us. Those who donate more than 10 RMB will be recorded in the donation list and will receive a donation certificate. Thank all the tippers!

6 FAQ

1). *How to remove the information of version?*
   Please comment `\version{x.xx}`.
2). *How to remove the information of date?*
   Please type in `\date{}`.
3). *How to add several authors?*
   Use `\and` in `\author` and use `\\` to start a new line.

Table 2: Donation List

Tipper	Amount	Date	Channel	Tipper	Amount	Date	Channel
Lerh	10 RMB	2019/05/15	Wechat	yueguodipingxian	10 RMB	2019/05/15	Wechat
yinsang	20 RMB	2019/05/27	Wechat	*kong	10 RMB	2019/05/30	Wechat
latexstudio.net	666 RMB	2019/06/05	Alipay	A*n	40 RMB	2019/06/15	Wechat
* xia	22 RMB	2019/06/15	Wechat	* qian	21 RMB	2019/06/15	Wechat
Cassis	11 RMB	2019/06/30	Wechat	* jun	10 RMB	2019/07/23	Wechat
P*u	50 RMB	2019/07/30	Wechat	* meng	19 RMB	2019/08/28	Wechat
Qu Doudou	10 RMB	2019/08/28	Wechat	Li Bo	100 RMB	2019/10/06	Wechat
Njustsll	10 RMB	2019/10/11	Wechat	Liu Zhikuo	99.99 RMB	2019/10/15	Alipay
* tao	16 RMB	2019/10/17	Wechat	Chini	12 RMB	2019/10/17	Alipay
yuanfengjing	10 RMB	2019/10/28	Wechat	Guo Deliang	88 RMB	2019/11/03	Wechat
ziqiangbuxi	20 RMB	2019/11/04	Alipay	dushuzhichong	20 RMB	2019/11/18	Wechat
* deng	10 RMB	2019/11/18	Wechat	* zhe	20 RMB	2019/11/18	Wechat
anonymous	10 RMB	2019/11/24	Wechat	Jiye Qian	66 RMB	2019/12/04	Wechat
* yang	20 RMB	2019/12/05	Wechat	Catcher	11 RMB	2019/12/08	Alipay
xierbotementu	10 RMB	2019/12/09	Alipay	* wei	10 RMB	2019/12/09	Wechat
Simon	20 RMB	2019/12/11	Alipay	liushangqianyi	66.60 RMB	2019/12/18	Alipay
yu	10 RMB	2019/12/20	Alipay	*chen	15 RMB	2019/12/20	Wechat
suifeng	20 RMB	2019/12/27	Alipay	Ws	23.30 RMB	2019/12/28	Wechat
chuba	100 RMB	2020/01/02	Alipay	p*e	20 RMB	2020/01/03	Wechat
Shunmx	100 RMB	2020/01/03	Wechat	hj	10 RMB	2020/01/03	Wechat
F*5	10 RMB	2020/01/03	Wechat	S*m	20.20 RMB	2020/01/03	Wechat
erdaiqingzhi	13 RMB	2020/01/14	Alipay	*?	66 RMB	2020/01/15	Wechat
Mr. Xiong	20 RMB	2020/01/17	Wechat	*bo	15 RMB	2020/01/18	Wechat
*Zhe	10 RMB	2020/02/02	Wechat	Jackie	88.80 RMB	2020/02/09	Wechat
Henry_Sun	50 RMB	2020/02/14	Alipay	* Qiao	50 RMB	2020/02/21	Wechat
YunLian	10 RMB	2020/03/02	Alipay	S*y	10 RMB	2020/03/15	Wechat
* Ge	66.66 RMB	2020/03/17	Wechat	K*e	30 RMB	2020/03/30	Wechat
* Yang	20 RMB	2020/04/02	Wechat	Shi*n	30 RMB	2020/04/11	Wechat

```latex
\author{author 1\\ org. 1 \and author 2 \\ org. 2 }
```