HITsz Beamer Presentation

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July 1, 2024

Contents

Outline

Introduction

- The main purpose is to illustrate how to use Beamer syntax to organize content and to demonstrate some basic syntax for future reference.
- We share an open-source LaTeX implementation of this template at https://github.com/Chen861368/HITsz-Academic-Beamer-Template.

Header 1	**Header 2**	**Header 3**
Row 1, Col 1	Row 1, Col 2	Row 1, Col 3
Row 2, Col 1	Row 2, Col 2	Row 2, Col 3
Row 3, Col 1	Row 3, Col 2	Row 3, Col 3

Document Overview

- **Introduction:** Overview of the research problem and objectives.
- **Mathematical Estimation Problem:** Detailed explanation of the estimation problem and its formulation.
- **Kalman Filter:** Introduction to the Kalman filter and its application in state estimation.
- **Numerical Example:** Demonstration of the methodology using a Multi-Degree-Of-Freedom (MDOF) system.
- **Hamilton's Principle:** Explanation of the principle and its role in deriving equations of motion.
- **Conclusion:** Summary of findings and future work.

Outline

The Mathematical Estimation problem

The estimation problem is mathematically formulated as determining parameters θ from a discrete dataset $\{x_0, x_1, ..., x_N\}$, associated with a signal following a stochastic model $x \sim f(x, \theta)$[2, 3].

- The Mathematical Estimation Problem encompasses the challenge of inferring unknown parameters or system states from observational data.
- Digital computers allow us to analyze sampled data, leading to the problem of estimating parameters from a discrete dataset.
- Parameter θ is deterministic in classical and random in Bayesian estimation.

The aim is to find θ or construct an estimator $\hat{\theta}$ using the function g, expressed as:

$$\hat{\theta} = g(x_0, x_1, ..., x_N)$$

Here, the function g is designed to estimate the parameters θ from the data.

The Mathematical Estimation problem

Often, the main trick is finding the right mathematical formulation of your estimation problem

- **Function** $x \sim f(x, \theta)$: Identify the function that best represents the data or the system.
- **Metric** $L(\theta, \hat{\theta})$: Choose a loss or cost function L that quantifies the error or difference between the estimated parameters $\hat{\theta}$ and the true parameters θ.
- **Constraints** $R(\theta)$: Define any constraints R that the parameters θ must satisfy. These could be physical constraints, regulatory requirements, or computational limitations.

Algorithm Selection

- Once the function, metric, and constraints are defined, the next step is to choose the best algorithms that exploit these definitions to solve the estimation problem effectively.

Kalman Filter

- State Space (White box)
- Prediction-Correction (Recursion)

The Kalman filter model assumes the true state at time $n+1$ is evolved from the state at n according to

$$x_{n+1} = A_n x_n + v_n$$
$$y_n = C_n x_n + w_n$$

- Based on observational data $y_1, \ldots, y_{n+1}$, the Kalman filter aims to infer the state x_{n+1}.
- Optimal estimation, as per the Bayesian Minimum Mean Square Error, is given by the projection:

$$\hat{x}_{n+1|n+1} = E(x_{n+1}|y_1, \ldots, y_{n+1}) = \mathrm{Proj}_{y_1,\ldots,y_{n+1}} x_{n+1}$$

where:

$$\mathrm{Proj}_y x = E(xy^T)[E(yy^T)]^{-1} Y$$

Kalman Filter

We assume that we already have the estimate $\hat{x}_{n|n}$ and the new data y_{n+1}, and seek to compute $\hat{x}_{n+1|n+1}$.

$$
\begin{aligned}
\hat{x}_{n+1|n+1} &= \mathrm{Proj}_{y_1,\ldots,y_n,y_{n+1}} x_{n+1} \\
&= \mathrm{Proj}_{y_1,\ldots,y_n} x_{n+1} + \mathrm{Proj}_{\overline{y_{n+1}}} x_{n+1} \\
&= \hat{x}_{n+1|n} + E\left(x_{n+1}\overline{y_{n+1}}^T\right) E\left(\overline{y_{n+1}y_{n+1}}^T\right)^{-1} \overline{y_{n+1}} \\
&= A_n \hat{x}_{n|n} + K_{n+1}(y_{n+1} - C_{n+1}\hat{x}_{n|n})
\end{aligned}
$$

where:

$$
\begin{aligned}
\overline{y_{n+1}} &= y_{n+1} - \mathrm{Proj}_{y_1,\ldots,y_n} y_{n+1} = y_{n+1} - \mathrm{Proj}_{y_1,\ldots,y_n}(C_{n+1}x_{n+1} + w_{n+1}) \\
&= y_{n+1} - \mathrm{Proj}_{y_1,\ldots,y_n}(C_{n+1}x_{n+1}) = y_{n+1} - C_{n+1}\hat{x}_{n|n} \\
\hat{x}_{n+1|n} &= \mathrm{Proj}_{y_1,\ldots,y_n} x_{n+1} = \mathrm{Proj}_{y_1,\ldots,y_n}(A_n x_n + v_n) = A_n \hat{x}_{n+1|n} \\
K_{n+1} &= E(\hat{x}_{n+1} y_{n+1}^T)[E(y_{n+1} y_{n+1}^T)]^{-1}
\end{aligned}
$$

Kalman Filter Algorithm Steps

Input: Initial states $\hat{x}_{0|0}$, $\hat{z}_0$, matrices A_0, B, C, noise covariances V_0, W_0, control inputs u_0, step sizes $\{t_k\}$.

For each time step $k = 1, 2, \ldots$, with new data Cx_k, control input u_{k-1}, and matrix A_{k-1}:

- Predict next state and error covariance:

  $$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1|k-1} + Bu_{k-1}$$

  $$P_{k|k-1} = A_{k-1}P_{k-1|k-1}A_{k-1}^{\top} + V_{k-1}$$

Continuing for each time step k:

- Compute Kalman gain and update state estimate:

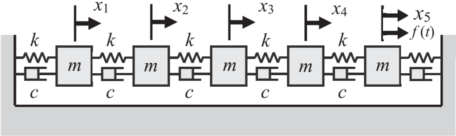
  $$K_k = P_{k|k-1}C_k^{\top}[C_kP_{k|k-1}C_k^{\top} + W_k]^{-1}$$

  $$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - C_k\hat{x}_{k|k-1})$$

- Update error covariance:

  $$P_{k|k} = (I - K_kC_k)P_{k|k-1}$$

Numerical Example: MDOF systems



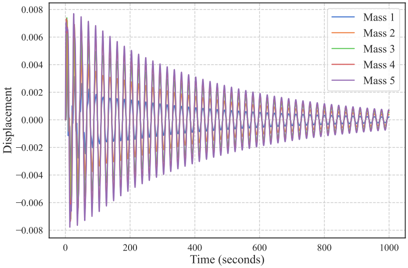
- The damping matrix, C, is proportionally related to the stiffness matrix, defined as $C = (c/k)K$, with c being the damping coefficient.
- For the purposes of numerical example generation, the parameters are set to $m = 1$, $c = 0.06$, and $k = 1$.

The stiffness matrix K is given by:

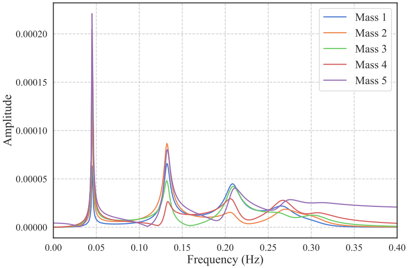
$$K = \begin{bmatrix} 2k & -k & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 \\ 0 & -k & 2k & -k & 0 \\ 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & -k & 2k \end{bmatrix}$$

Numerical Example: MDOF systems

- An initial excitation is applied to the fifth node.
- The Newmark-beta numerical method is implemented.
- The process continues with a time step of 0.005 seconds, covering an extensive duration of 1000 seconds.



(a) Time response of displacements

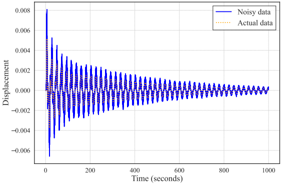


(b) Frequency response of accelerations

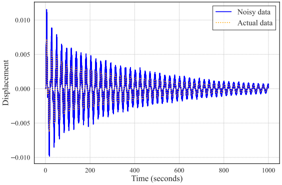
Figure: Time response of Five degrees of freedom system

Numerical Example: MDOF systems

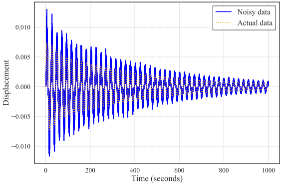
The analysis incorporates white noise proportional to signal amplitude, with each data point increased by noise amounting to 10% of its amplitude.



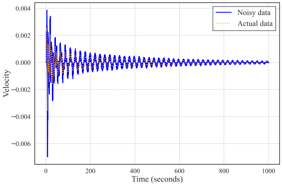
(a) $x_1(t)$ Displacement



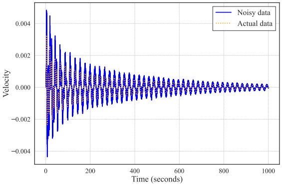
(b) $x_2(t)$ Displacement



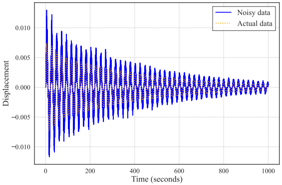
(c) $x_3(t)$ Displacement



(d) $x_1(t)$ Velocity



(e) $x_2(t)$ Velocity



(f) $x_3(t)$ Velocity

Normalized Mean Squared Error

The Normalized Mean Squared Error (NMSE) is a metric used to evaluate the accuracy of a predictive model:

$$\text{NMSE} = \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2}$$

where y_i is the i-th true value, $\hat{y}_i$ is the i-th predicted value, $\bar{y}$ is the mean of the true values calculated as $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$, and N is the number of samples.

- **NMSE ≈ 0**: Indicates nearly perfect model predictions with minimal error. The predicted values are very close to the true values.
- **0 < NMSE < 0.1:** Indicates very accurate model predictions with small errors. The predicted values are very close to the true values. Such models are considered highly accurate.
- **NMSE ≥ 1:** Indicates very high prediction errors. The predicted values may be worse than using the mean value as the prediction. Such models are generally considered invalid and require major improvements.

The form of the Equations of Motion

Differential and Integral Forms:

- The differential form refers to describing the state and changes of the system at every moment using differential equations. This form focuses on the specific behavior of the system at each instant.
- The integral form refers to describing the state and changes of the system over a longer period using integral equations. This form emphasizes the overall behavior of the system over a certain period.

Non-Variational and Variational Forms:

- The non-variational form refers to describing the motion of the system using physical quantities such as force, mass, and acceleration, focusing on the general laws of all actual movements.
- The variational form refers to describing the system's motion through variational principles. The variational principle considers that, among all possible motion paths, the actual motion path is the one that extremizes a certain action.

Hamilton's Principle

Hamilton's variational statement of Dynamics:

$$\int_{t_1}^{t_2} \delta[T(t) - V(t)]\, dt + \int_{t_1}^{t_2} \delta W_{nc}(t)\, dt = 0$$

where $T(t)$: The kinetic energy of the system at time t, $V(t)$: The potential energy of the system at time t, and $W_{nc}(t)$: The work done by nonconservative forces at time t.

- Hamilton's Principle can be interpreted as stating that the actual path taken by the system between times t_1 and t_2 is such that the integral of the variation of the difference between kinetic and potential energies plus the integral of the variation of the work done by nonconservative forces is zero.
- The application of Hamilton's Principle leads directly to the equations of motion for any given system.

Hamilton's Principle

When applied to statics problems, the kinetic-energy term T vanishes, and the remaining terms in the integrands are invariant with time. Thus, the equation reduces to:

$$\delta(V - W_{nc}) = 0$$

which is the well-known **principle of minimum potential energy**. The equations of motion for an N-Degrees of Freedom system can be derived directly from Hamilton's Principle by simply assuming:

$$\begin{aligned} T &= T(q_1, q_2, \cdots, q_N, \dot{q}_1, \dot{q}_2, \cdots, \dot{q}_N) \\ V &= V(q_1, q_2, \cdots, q_N) \\ \delta W_{nc} &= Q_1 \delta q_1 + Q_2 \delta q_2 + \cdots + Q_N \delta q_N \end{aligned}$$

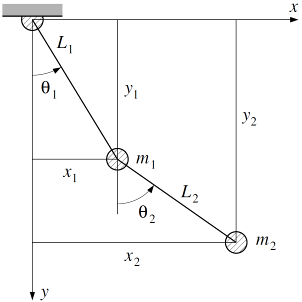
where $q_1, q_2, \cdots, q_N$ represent a set of generalized coordinates, and the coefficients $Q_1, Q_2, \cdots, Q_N$ are the generalized forcing functions corresponding to the coordinates $q_1, q_2, \cdots, q_N$, respectively.

Lagrange's Equations

By substituting the aforementioned expressions into Hamilton's variational statement of dynamics, Lagrange's equations of motion are given by:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

Consider the **double pendulum under free vibration conditions**[1]. The equations of motion can be derived using Lagrange's Equations as follows:



Lagrange's Equations

The kinetic energy and potential energy of the double pendulum can be expressed in terms of the set of generalized coordinates $q_1 \equiv \theta_1$ and $q_2 \equiv \theta_2$ as follows:

$$
\begin{aligned}
T &= \frac{1}{2} m_1 L_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 \left[L_1^2 \dot{q}_1^2 + L_2^2 \dot{q}_2^2 + 2 L_1 L_2 \dot{q}_1 \dot{q}_2 \cos(q_2 - q_1) \right] \\
V &= (m_1 + m_2) g L_1 (1 - \cos q_1) + m_2 g L_2 (1 - \cos q_2)
\end{aligned}
$$

Since there are no nonconservative forces acting on this system, substituting into Lagrange's Equation yields:

$$
\begin{aligned}
(m_1 + m_2) L_1^2 \ddot{q}_1 + m_2 L_1 L_2 \ddot{q}_2 \cos(q_2 - q_1) &- m_2 L_1 L_2 \dot{q}_2^2 \sin(q_2 - q_1) \\
&+ (m_1 + m_2) g L_1 \sin q_1 = 0 \\
m_2 L_2^2 \ddot{q}_2 + m_2 L_1 L_2 \ddot{q}_1 \cos(q_2 - q_1) &+ m_2 L_1 L_2 \dot{q}_1^2 \sin(q_2 - q_1) \\
&+ m_2 g L_2 \sin q_2 = 0
\end{aligned}
$$

The General Equations of Motion for Linear Systems

As long as the energy and work terms can be expressed in terms of the generalized coordinates, and of their time derivatives and variations, Lagrange's equations are valid, regardless of whether the system is linear or nonlinear.

The kinetic and potential energies of linear engineering systems subjected to small-amplitude oscillations can be expressed in the quadratic forms:

$$T = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{N} m_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}}$$

$$V = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{N} k_{ij}q_iq_j = \frac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q}$$

where N is the number of degrees of freedom in the system.

The General Equations of Motion for Linear Systems

For such systems, the second term of Lagrange's equations, namely, $\partial T/\partial q_i$ ($i = 1, 2, \cdots, N$), equals zero, which reduces Lagrange's equations to the form:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) + \frac{\partial V}{\partial q_i} = Q_i \quad i = 1, 2, \cdots, N$$

Lagrange's equations of motion, when placed in matrix form, become:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q}$$

All nonconservative forces, including damping forces, are contained here in the generalized forcing functions $Q_1, Q_2, \cdots, Q_N$. It is evident that

$$Q_i = p_i - \sum_{j=1}^{N} c_{ij}\dot{q}_j$$

Substituting into Lagrange's equations gives the governing equations of motion in matrix form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{p}$$

Outline

Conclusion

- Summarize your main points.
- Provide conclusions and future work.

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