Math, Intro to optimization#4

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1. (6.1)
   The standard form of the problem is as following:
   minimize $-e^{-w^T x}$
   subject to

$$\begin{aligned} w^T A w - w^T(x + Ay) &\leq -a \\ y^T w - w^T x &= b \end{aligned} \tag{1}$$

2. (6.5)
   Let $x_1, x2$ be the amounts of milk bottle and knob to produce respectively and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Let $w = \begin{bmatrix} 0.7 \\ 0.5 \end{bmatrix}$ and $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 240,000 \\ 6,000 \end{bmatrix}$The optimization problem is as following:

$$\begin{aligned} \text{minimize } & - w^T x \\ \text{subject to } & Ax \leq b \end{aligned} \tag{2}$$

3. (6.6)
   $Df(x, y) = \begin{bmatrix} 6xy + 4y^2 + y \\ 3x^2 + 8xy + x \end{bmatrix} = \mathbf{0} \implies \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -\frac{1}{4} \end{bmatrix}, \begin{bmatrix} -\frac{1}{3} \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{9} \\ -\frac{1}{12} \end{bmatrix}$
   We also have $D^2 f(x, y) = \begin{bmatrix} 6y & 6x + 8y + 1 \\ 6x + 8y + 1 & 8x \end{bmatrix}$. Since $x, y \in \mathbb{R}$, we examine the $D^2 f(x, y)$ by plugging in the critical points and according to **Remark 6.2.10**, we could find that $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -\frac{1}{4} \end{bmatrix}$ *and* $\begin{bmatrix} -\frac{1}{3} \\ 0 \end{bmatrix}$ are saddle points while $\begin{bmatrix} -\frac{1}{9} \\ -\frac{1}{12} \end{bmatrix}$ is the local maximum.

4. (6.11)
   By using the Newton method, $q(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2 = ax_0^2 + bx_0 + c + (2ax_0 + b)(x - x_0) + a(x - x_0)^2 = zx^2 + bx + c$. Therefore, the minimizer of $q(x)$ coincides with that of the orignial funciton and the optimization problem will be done in one iteration.