ECON, DSGE#1

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1. (Exercise 1)
   Through substituting K_{t+1} and K_{t+2} by $Ae^{z_t}K_t^\alpha$ and $Ae^{z_{t+1}}K_{t+1}^\alpha$, we can find that $A = \alpha\beta$

2. (Exercise 2)

$$\text{Budget cosntraint:} \quad c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \tag{1}$$

$$\text{Intertemporal Euler:} \quad \frac{1}{c_t} = \beta E_t\{\frac{1}{c_{t+1}}[(r_{t+1} - \delta)(1-\tau) + 1]\} \tag{2}$$

$$\text{Consumption-leisurel Euler:} \quad \frac{\alpha}{1-l_t} = \frac{1}{c_t} w_t(1-\tau) \tag{3}$$

$$\text{Capital FOC:} \quad r_t = \alpha e^{z_t} k_t^{\alpha-1} l_t^{1-\alpha} \tag{4}$$

$$\text{Labor FOC:} \quad w_t = (1-\alpha)e^{z_t} k_t^\alpha l_t^{-\alpha} \tag{5}$$

$$\text{Government budget:} \quad \tau[w_t l_t + (r_t - \delta)k_t] = T_t \tag{6}$$

$$\text{Law of motions:} \quad z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z \tag{7}$$

3. (Exercise 3)

$$\text{Budget cosntraint:} \quad c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \tag{8}$$

$$\text{Intertemporal Euler:} \quad c_t^{-\gamma} = \beta E_t\{c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\} \tag{9}$$

$$\text{Consumption-leisure Euler:} \quad \frac{a}{1-l_t} = c_t^{-\gamma} w_t(1-\tau) \tag{10}$$

$$\text{Capital FOC:} \quad r_t = \alpha e^{z_t} k_t^{\alpha-1} l_t^{1-\alpha} \tag{11}$$

$$\text{Labor FOC:} \quad w_t = (1-\alpha)e^{z_t} k_t^\alpha l_t^{-\alpha} \tag{12}$$

$$\text{Government budget:} \quad \tau[w_t l_t + (r_t - \delta)k_t] = T_t \tag{13}$$

$$\text{Law of motions:} \quad z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z \tag{14}$$

4. (Exercise 4)

$$\text{Budget cosntraint:} \quad c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \tag{15}$$

$$\text{Intertemporal Euler:} \quad c_t^{-\gamma} = \beta E_t\{c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\} \tag{16}$$

$$\text{Consumption-leisure Euler:} \quad a(1-l_t)^{-\xi} = c_t^{-\gamma} w_t (1-\tau) \tag{17}$$

$$\text{Capital FOC:} \quad r_t = \alpha e^{z_t} k_t^{\eta-1}[\alpha k_t^\eta + (1-\alpha) l_t^\eta]^{\frac{1}{\eta}-1} \tag{18}$$

$$\text{Labor FOC:} \quad w_t = (1-\alpha) e^{z_t} l_t^{\eta-1}[\alpha k_t^\eta + (1-\alpha) l_t^\eta]^{\frac{1}{\eta}-1} \tag{19}$$

$$\text{Government budget:} \quad \tau[w_t l_t + (r_t - \delta)k_t] = T_t \tag{20}$$

$$\text{Law of motions:} \quad z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z \tag{21}$$

5. (Exercise 5)

$$\text{Budget cosntraint:} \quad c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \tag{22}$$

$$\text{Intertemporal Euler:} \quad c_t^{-\gamma} = \beta E_t\{c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\} \tag{23}$$

$$\text{Capital FOC:} \quad r_t = \alpha k_t^{\alpha-1}(l_t e^{z_t})^{1-\alpha} \tag{24}$$

$$\text{Labor FOC:} \quad w_t = (1-\alpha) k_t^\alpha l_t^{-\alpha} e^{z_t(1-\alpha)} \tag{25}$$

$$\text{Government budget:} \quad \tau[w_t l_t + (r_t - \delta)k_t] = T_t \tag{26}$$

$$\text{Law of motions:} \quad z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z \tag{27}$$

If $l_t = 1$, then the steady state version of these equations will be:

$$\text{Budget cosntraint:} \quad \bar{c} = (1-\tau)[\bar{w} + (\bar{r} - \delta)\bar{k}] + \bar{T} \tag{28}$$

$$\text{Intertemporal Euler:} \quad \bar{c}^{-\gamma} = \beta E\{\bar{c}^{-\gamma}[(\bar{r} - \delta)(1-\tau) + 1]\} \tag{29}$$

$$\text{Capital FOC:} \quad \bar{r} = \alpha \bar{k}^{\alpha-1} e^{\bar{z}(1-\alpha)} \tag{30}$$

$$\text{Labor FOC:} \quad \bar{w} = (1-\alpha)\bar{k}^\alpha e^{\bar{z}(1-\alpha)} \tag{31}$$

$$\text{Government budget:} \quad \tau[\bar{w} + (\bar{r} - \delta)\bar{k}] = \bar{T} \tag{32}$$

When being solved algebrically, $\bar{k} = \alpha^{\frac{1}{1-\alpha}}(\frac{1}{1-\tau}(\frac{1}{\beta} - 1) + \delta)^{\frac{1}{\alpha-1}} e^{\bar{z}}$. Taking the value given in the question, we get $\bar{k} = 7.28749795069$
The numerical solutions for Problem 5:
Steady state capital: [7.28749795]
Steady state output: [2.21325461]
Steady state investment: [0.7287498]

6. (Exercise 6)

$$\text{Budget cosntraint:} \quad c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \tag{33}$$

$$\text{Intertemporal Euler:} \quad c_t^{-\gamma} = \beta E_t\{c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\} \tag{34}$$

$$\text{Consumption-leisure Euler:} \quad a(1-l_t)^{-\xi} = c_t^{-\gamma} w_t (1-\tau) \tag{35}$$

$$\text{Capital FOC:} \quad r_t = \alpha k_t^{\alpha-1}(l_t e^{z_t})^{1-\alpha} \tag{36}$$

$$\text{Labor FOC:} \quad w_t = (1-\alpha)k_t^{\alpha} l_t^{-\alpha} e^{z_t(1-\alpha)} \tag{37}$$

$$\text{Government budget:} \quad \tau[w_t l_t + (r_t - \delta)k_t] = T_t \tag{38}$$

$$\text{Law of motions:} \quad z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z \tag{39}$$

The numerical solutions for Problem 6:

Steady state capital: 4.22522902519
Steady state labor: 0.579791453145
Steady state output: 1.28322610861
Steady state investment: 0.422522902519