Econ, Dynamic programming #2, Proof

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Due Wednesday, July 5 at 8:00am

Arguments:

1. It's easy to show that $Uw(y)$ is in $\mathbb{R}_+$ since u, β and w all belong to $\mathbb{R}_+$

2. Define $\rho(g, f) = \sup_{y \geq 0} |g(y) - f(y)|$. It's easy to prove that $(\mathbb{R}_+, \rho)$ is a complete space.

3. Now we will prove that U is a contraction map on $(\mathcal{C}, \rho)$.
   $|Uw(y) - Uw^{'}(y)| = |\beta \int w(f(y - \sigma(y))z)\phi(dz) - \beta \int w^{'}(f(y - \sigma(y))z)\phi(dz)|$
   $= \beta| \int (w(y) - w^{'}(y))\phi(dz)|$
   $\leq \beta \int |(w(y) - w^{'}(y))|\phi(dz)$
   $\leq \beta \int \sup_y |w(y) - w^{'}(y)|z\phi(dz)$
   $= \beta \sup_y |w(y) - w^{'}(y)|$
   $= \beta\rho(w, w^{'})$
   Therefore, we find U is a contraction map on $(\mathcal{C}, \rho)$.