Math, Problem Set #1, Probability Theory

OSM Lab, Karl Schmedders

Due Monday, June 26 at 8:00am

1. **Exercises from chapter.** Do the following exercises in Chapter 3 of Humpherys and Jarvis (forthcoming): 3.6, 3.8, 3.11, 3.12 (watch this movie clip link).

2. Construct examples of events A, B, and C, each of probability strictly between 0 and 1, such that

   (a) $P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$, $P(B \cap C) = P(B)P(C)$, but $P(A \cap B \cap C) \neq P(A)P(B)P(C)$.

   (b) $P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$, $P(A \cap B \cap C) = P(A)P(B)P(C)$, but $P(B \cap C) \neq P(B)P(C)$. (Hint: You can let Ω be a set of eight equally likely points.)

3. Prove that Benford's Law is, in fact, a well-defined discrete probability distribution.

4. A person tosses a fair coin until a tail appears for the first time. If the tail appears on the nth flip, the person wins 2^n dollars. Let the random variable X denote the player's winnings.

   (a) (St. Petersburg paradox) Show that $E[X] = +\infty$.

   (b) Suppose the agent has log utility. Calculate $E[\ln X]$.

5. (Siegel's paradox) Suppose the exchange rate between USD and CHF is 1:1. Both a U.S. investor and a Swiss investor believe that a year from now the exchange rate will be either $1.25 : 1$ or $1 : 1.25$, with each scenario having a probability of 0.5. Both investors want to maximize their wealth in their respective home currency (a year from now) by investing in a risk-free asset; the risk-free interest rates in the U.S. and in Switzerland are the same. Where should the two investors invest?

References

Humpherys, Jeffrey and Tyler J. Jarvis, "Foundations of Applied Mathematics, Volume II: Algorithm Design and Optimization," forthcoming. SIAM, Philadelphia, PA.