Math, Inner Product Space#2

OSM Lab instructor,Zachary Boyd

OSM Lab student, CHEN Anhua

Due Wednesday, July 5 at 8:00am

1. (4.2)

   D should take the form of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

   $p_D(z) = det(zI - D) = z^3$. If $p_D(z) = 0$, it indicates that eigenvalue of D is 0 with the algebric multiplicity of 3. Also $\mathcal{N}(0I - D) = \mathcal{N}(-D) = span\{[x, 0, 0]^T\}$. The geometric multiplicity is 1, since $dim(\mathcal{N}(-D)) = 1$.

2. (4.4)

   Using 4.3, we could show if $(tr(A)^2 - 4det(A))$ is non-negative, then the matrix only got real eigenvalues, otherwise, it only gets imaginary eigenvalues.

   i. If the matrix is Hermitian, the $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$, then $(tr(A)^2 - 4det(A)) = (a-d)^2 + 4b^2 \geq = 0$

   ii. If the matrix is skew-Hermitian, then $\begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix}$. Therefore, $(tr(A)^2 - 4det(A)) = -4b^2 < 0$ for $b \neq 0$

3. (4.6)

   Let $A_{n\times n}$ be an upper triangular matrix. $det(\lambda I - A) = 0 \implies \prod_{i=1}^{n}(\lambda_i - a_{ii}) = 0$. Therefore, the eigenvalues of matrix A are its diagonal elements.

4. (4.8)

   i. To prove $\{sin(x), cos(x), sin(2x), cos(2x)\}$ is the basis for V, we need to prove they are linearly independent. This is equivalent to prove that for $\forall x \in \mathbb{R}, asin(x) + bcos(x) + csin(2x) + dcos(2x) = 0 only when a = b = c = d = 0$. When $x = 0, b + d = 0$. When $x = \pi, -b + d = 0$. Therefore, $b = d = 0$. Also, when $x = \frac{\pi}{2}, a - d = 0 \implies a = 0$. Then $\forall x \in \mathbb{R}, dcos(2x) = 0 \implies d = 0$. This completes the proof that S is a basis for V.

   ii. $(asin(x) + bcos(x) + csin(2x) + dcos(2x))' = (-b)sin(x) + acos(x) + (-2d)sin(2x) + 2ccos(x)$. So the matrix representation of D is $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

   iii.