Computer Architecture
Final Project: Matrix Chain Multiplication

Speaker：Billy

Advisor：Prof. An-Yeu Wu

Date：2025/4/28

Overview

- Given a sequence of matrices, implement **assembly code** to compute the matrix chain multiplication.

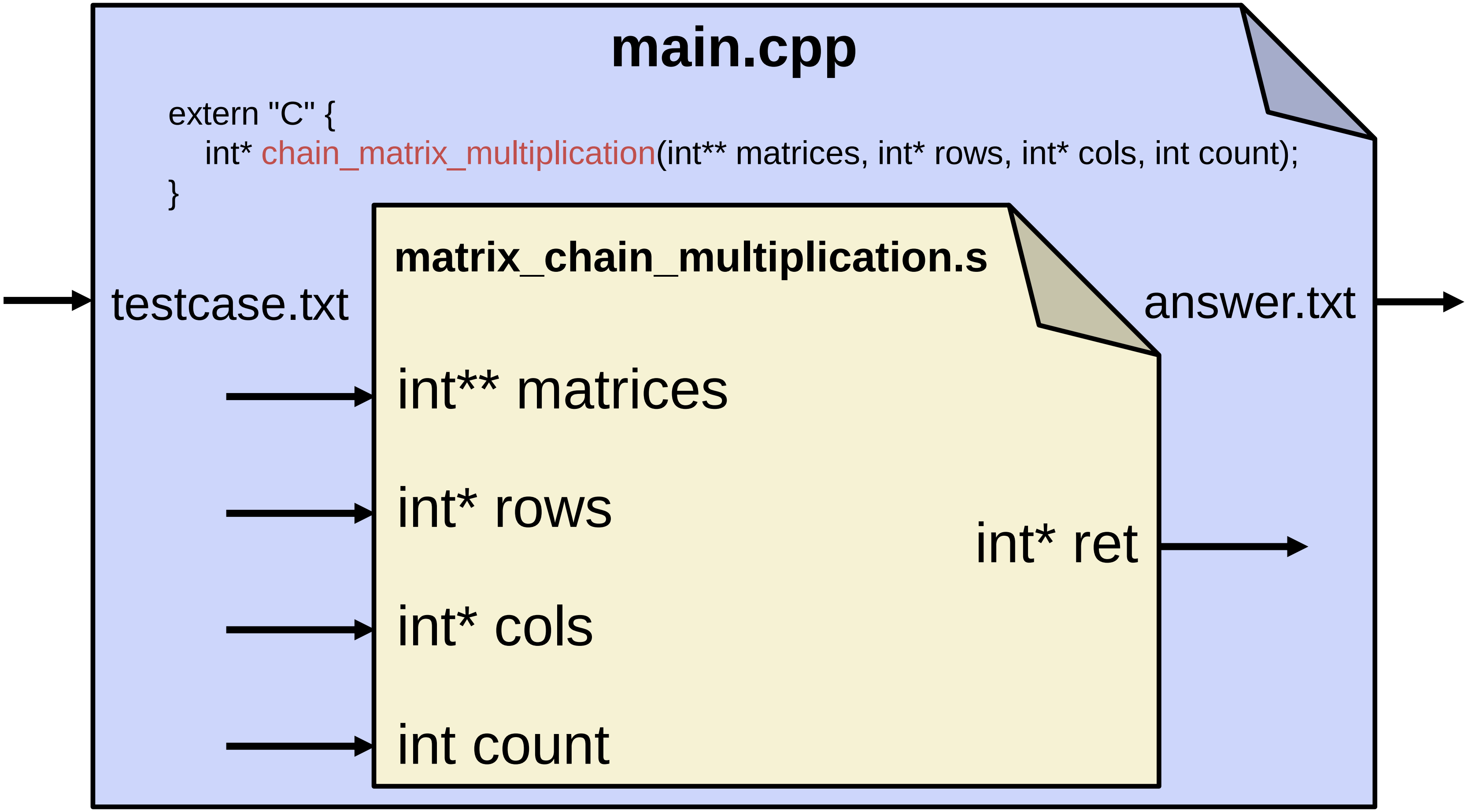
- Your implementation will be scored based on its performance, which is determined by **the number of cycles and cache size**.

Project Goal

- Accelerate the function to achieve better performance
  - Increase hit rate of cache
  - Use better algorithm (e.g. Dynamic Programming)

- Performance Formula
  - $Time \times \left(\log_2 Size_{L1\_ICache} + \log_2 Size_{L1\_DCache} + \frac{1}{2}\log_2 Size_{L2\_Cache}\right)$
  - Goal: minimize the performance formula

Block Diagram



Input / Output

Type	Port	Reg	I/O	Description
Int **	matrices	`x10 (a0)`	I	An array storing the addresses of all matrices*.
Int *	rows	`x11 (a1)`	I	An array storing the row size of each matrix.
Int *	cols	`x12 (a2)`	I	An array storing the column size of each matrix.
Int	count	`x13 (a3)`	I	An integer representing the number of matrices.
Int *	ret	`x10 (a0)`	O	An address storing the result matrix* after computation.

*All matrices are stored in row-major order.

- In the RISC-V calling convention
  - Function arguments are passed starting from the `a0` register
  - Return value is stored in the `a0` register too

Input

- Example:

$$\begin{bmatrix} 23 & 7 \\ 16 & 11 \end{bmatrix} \times \begin{bmatrix} 2 & 13 & 5 \\ 8 & 26 & 18 \end{bmatrix} \times \begin{bmatrix} 9 & 15 & 17 \\ 31 & 28 & 4 \\ 14 & 27 & 21 \end{bmatrix}$$

2×2 matrix   2×3 matrix   3×3 matrix

Addr	Value
matrices	0x1000

Addr	Value
0x1000	0x2000
0x1004	0x2100
0x1008	0x2200

Addr	Value
0x2000	23
0x2004	7
0x2008	16
0x200c	11

Input

- Example:

$$\begin{bmatrix} 23 & 7 \\ 16 & 11 \end{bmatrix} \times \begin{bmatrix} 2 & 13 & 5 \\ 8 & 26 & 18 \end{bmatrix} \times \begin{bmatrix} 9 & 15 & 17 \\ 31 & 28 & 4 \\ 14 & 27 & 21 \end{bmatrix}$$

2x2 matrix　　2x3 matrix　　3x3 matrix

Addr	Value
rows	0x3000

Addr	Value
0x3000	2
0x3004	2
0x3008	3

Addr	Value
cols	0x4000

Addr	Value
0x4000	2
0x4004	3
0x4008	3

Addr	Value
count	3

Output

- Example:

$$\begin{bmatrix}23 & 7\\16 & 11\end{bmatrix} \times \begin{bmatrix}2 & 13 & 5\\8 & 26 & 18\end{bmatrix} \times \begin{bmatrix}9 & 15 & 17\\31 & 28 & 4\\14 & 27 & 21\end{bmatrix} = \underbrace{\begin{bmatrix}19203 & 21505 & 8719\\20286 & 23138 & 9854\end{bmatrix}}_{\text{result}}$$

Addr	Value
ret	0x9000

Addr	Value
0x9000	19203
0x9004	21505
0x9008	8719
0x900c	20286
0x9010	23138
0x9014	9854

You need to store this value in `a0` before return

RISC-V Calling Convention

- Input / output start from `a0`
- Return address is store in `ra`
- `s0` needs to be saved and restored across function call

Register	ABI Name	Description	Saver
x0	zero	Hard-wired zero	—
x1	ra	Return address	Caller
x2	sp	Stack pointer	Callee
x3	gp	Global pointer	—
x4	tp	Thread pointer	—
x5–7	t0–2	Temporaries	Caller
x8	s0/fp	Saved register/frame pointer	Callee
x9	s1	Saved register	Callee
x10–11	a0–1	Function arguments/return values	Caller
x12–17	a2–7	Function arguments	Caller
x18–27	s2–11	Saved registers	Callee
x28–31	t3–6	Temporaries	Caller

Memory Allocation

- Allocation of memory for storing matrices is needed in this project
  - At least for the result matrix of matrix chain multiplication

- You can allocate memory by `call malloc` as example below:
  - Allocate 10 bytes memory space in this example

```
li a0, 10          # Store required memory size in a0 (function argument)
call malloc        # Function call to allocate memory

sw t0, 0(a0)       # Allocated memory address will be stored in a0
                   # you can than store data in the allocated memory
```

  - Value in register `a0`, `a1`, `a2`, `a3` will be overwritten after this function call

- Release memory space by `call free`

Matrix Multiplication Assembly Example

- Basic idea of matrix multiplication: Three loops

```
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        for (int k = 0; k < n; k++) {
            C[i][j] += A[i][k] * B[k][j]
        }
    }
}
```

Matrix Multiplication Assembly Example

```
    .data
A:  .word 1, 2, 3, 4                # Matrix A
B:  .word 5, 6, 7, 8                # Matrix B
C:  .space 16                       # Space for Matrix C (2 x 2 = 4 integers = 16 bytes)

    .text
    .globl main
main:
    # load base addresses
    la s0, A                        # s0 = base address of A
    la s1, B                        # s1 = base address of B
    la s2, C                        # s2 = base address of C

    # i initialization
    li t0, 0                        # i = 0 (row index)

outer_loop_i:
    # j initialization

outer_loop_j:
    # k initialization
```

Data definition

Address of matrix data

Loop initializatoin

Matrix Multiplication Assembly Example

```
inner_loop_k:
    li t3, 2                        # matrix size = 2

    # Compute address of A[i][k]
    mul t4, t0, t3                  # t4 = i * matrix size
    add t4, t4, t2                  # t4 = i * matrix size + k
    slli t4, t4, 2                  # offset = (i * matrix size + k) * 4
    add t5, s0, t4                  # (base address of A) + offset
    lw t6, 0(t5)                    # load A[i][k] from memory

    # Compute address of B[k][j]
    # Similar to load A

    # multiplication and accumulation
    mul s7, t6, s6                  # A[i][k] * B[k][j]
    add s3, s3, s7                  # sum += A[i][k] * B[k][j]

    # Update k = k + 1
    addi t2, t2, 1                  # k++
    li s8, 2                        # matrix size = 2
    blt t2, s8, inner_loop_k        # if k < 2, continue inner loop

    # Compute address of C[i][j]
    # Similar to load A and B

    # Update j = j + 1

    # Update i = i + 1

    # End of program
    jr ra
```

Get $A[i][k]$ address and load

Get $B[k][j]$ address and load

Compute $C[i][j]$, iterate k

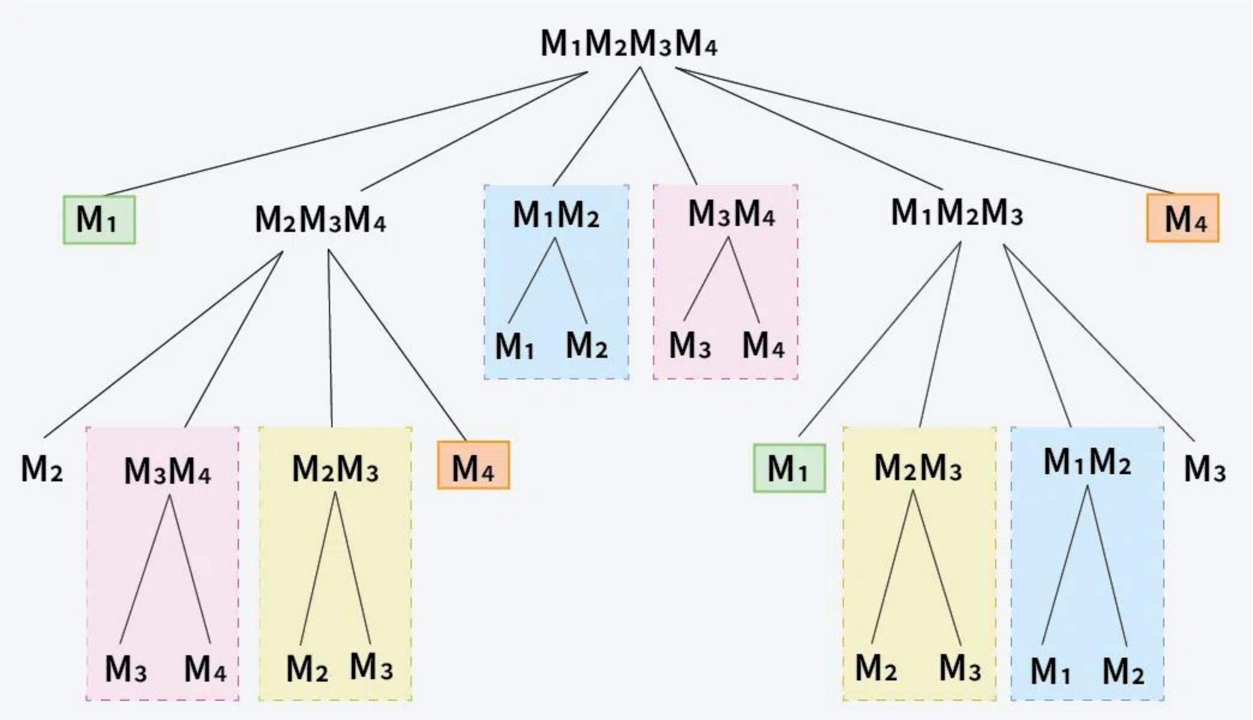
Get $C[i][j]$ address and store

Iterate i, j

End of program

Dynamic Programming

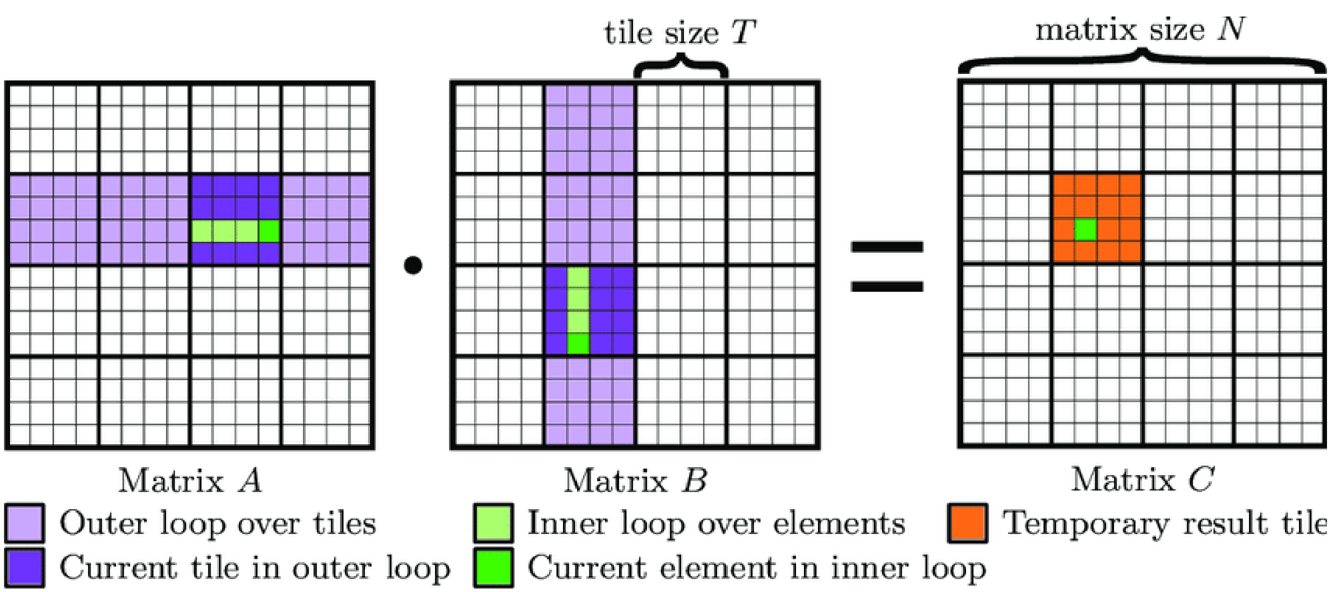
- When multiplying a $k \times m$ matrix with an $m \times n$ matrix
  - $k \times m \times n$ multiply-add operations are required
  - Dynamic programming to find the optimal multiplication order



source: www.geeksforgeeks.org

Tiled Matrix Multiplication

- Tiling matrix into small block that can fit in DCache
  - Outer loop: iterate through each block of matrices
  - Inner loop: iterate through elements inside the block
- Co-optimization of algorithm and cache size



Source: https://www.researchgate.net/figure/Performance-critical-A-B-part-of-the-GEMM-using-a-tiling-strategy-A-thread-iterates_fig1_320499173

Provided Files

Files	Description
Files you should not modify	
main.cpp	Main program
final_config.py	Gem5 configuration file
testbench.py	Python file for checking the correctness
score.py	Python file for calculate performance score
testcase/public/testcase_xx.txt	Public testing data
golden/public/golden_xx.txt	Public golden data
Makefile	Including make commands used in final
Files you need to modify	
matrix_chain_multiplication.s	Main design file
gem5_args.conf	Gem5 argument file, including cache setting

About final_config.py

- Currently doesn't including config for L2 cache
  - Implementation of L2 cache is part of HW3 (upcoming)
- After HW3 due, final_config.py with L2 cache config will be released
  - Please be aware of the upcoming updates to the final project file
  - Your final result should be simulated by latest version of final_config.py

Design & Simulation Flow (1)

- Start docker

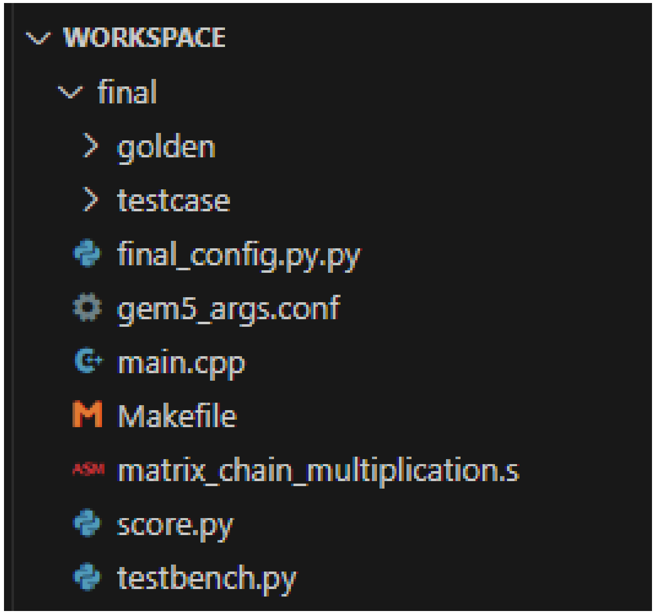
```
docker start -i <docker_name>
```

- Create folder for final

```
mkdir workspace/final
```

```
cd workspace/final
```

- Put all provided file in workspace/final



- Complete the design in matrix_chain_multiplication.s

Design & Simulation Flow (2)

- Modify the cache setting in gem5_args.conf
  - Settings you can change (mark in red):
  - GEM5_ARGS = --l1i_size 4kB --l1i_assoc 2 --l1d_size 4kB --l1d_assoc 2 --l2_size 16kB --l2_assoc 4

```
# Modify the GEM5 arguments for the simulation
GEM5_ARGS = --l1i_size 4kB --l1i_assoc 2 --l1d_size 4kB --l1d_assoc 2 --l2_size 16kB --l2_assoc 4
```

- Compile main.cpp with matrix_chain_multiplication.s

```
make g++_final
```

Design & Simulation Flow (3)

- Run simulation with all testcases or single testcase
  - This step might take a few minutes

```
make gem5_public_all
```

```
make gem5_public ARGS=P0/P1/P2/P3/P4/P5
```

- Checking if generated answer.txt match golden.txt

```
make testbench_public
```

- Compute performance score based on performance formula
  - Read content from m5out/config.json and m5out/out_exec.txt

```
make score_public
```

Test Cases Information

- Constraints:
  - Maximum number of matrices (N): 16
  - Maximum dimension of matrices (D): 64
  - Maximum value of matrices (V): 128

- Public Test Cases:
  - 00: N = 3, D = 3, V = 10
  - 01: N = 3, D = 5, V = 10
  - 02: N = 4, D = 6, V = 16
  - 03: N = 5, D = 10, V = 20
  - 04: N = 6, D = 8, V = 12
  - 05: public performance test

Group Formation

- 2 students per group
- Find your groupmate and submit the form before 5/4 (Sun.) 23:59
  - https://forms.gle/rKvznPAk3oMZWjWdA
- You may use NTU COOL 討論區 to find your groupmate
  - https://cool.ntu.edu.tw/courses/45288/discussion_topics/393734
- For those who haven't submit the form, the TAs will randomly assign groupmate for you
- Email to andrew@access.ee.ntu.edu.tw for any private questions related to group formation
  - Subject should start with [113-2 CA 分組]

Submission

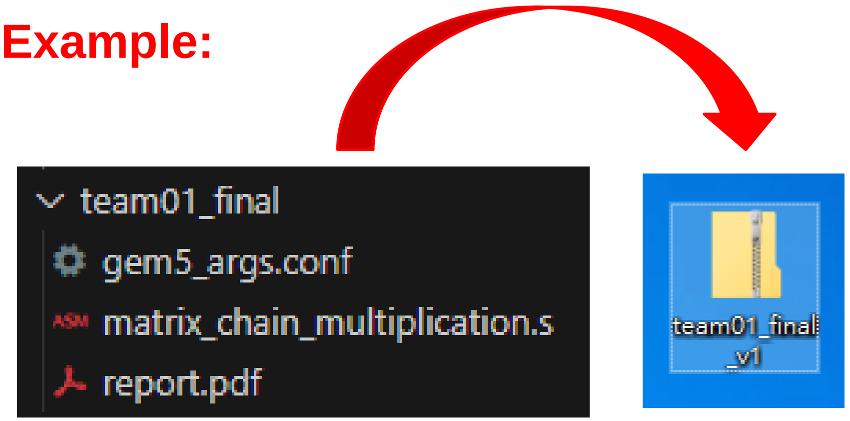
- Deadline: **2025/6/8 23:59:59 (UTC+8)**
- matrix_chain_multiplication.s
  - Your assembly code that implement matrix chain multiplication
- gem5_args.conf
  - Your simulation arguments (cache size & cache associative)
- report.pdf
  - 1-2 pages description of your design spec, special techniques used, team’s division of work, and reflections.
- Submit 組內互評 form
  - Contribution score for you and your teammate

Submission – Group

- Compress teamID_final/ in a zip file
  - named teamID_final_vk.zip (k: version number, e.g., 1,2,...)
- Upload teamID_final_vk.zip to NTUCOOL (ex: team01_final_v1.zip)

- File structure:
- teamID_final_vk.zip
  - teamID_final/
    - matrix_chain_multiplication.s
    - gem5_args.conf
    - report.pdf

Example:



- Incorrect file name or format would get **10% penalty**

Submission – Individual

- Submit 組內互評 form
  - https://forms.gle/x2cKjfPaR9GKfmji8

- Contribution score
  - 0 = no contribution
  - 10 = full contribution
  - Summation of all the group should be 10

- **Everyone** needs to submit this form !

Grading Policy

- Baseline 40% + Performance 40% + Report 20%
  - 0 points for late submission or plagiarism
  - -10% for incorrect naming or format

Item	%	Description
Public Testcases	30	Pass all 6 public testcases (5% each)
Private Testcases	10	Pass all 5 private testcases (2% each)
Public Ranking	20	Assign scores based on the PR value of your performance score of P5 (public).
Private Ranking	20	Assign scores based on the PR value of your performance score of P10 (private).
Report	15	1~2 pages
Contribution	5	Based on 組內互評

Discussion

- NTU COOL Discussion Forum: Final project 討論區
  - TAs will prioritize answering questions on the NTU COOL discussion forum
- Email: r13943002@ntu.edu.tw
  - Title should start with [113-2 CA Final Project]

Reference

- Matrix-chain multiplication
  - Matrix Chain Multiplication | GeeksforGeeks
- Dynamic Programming
  - Dynamic Programming or DP | GeeksforGeeks
- Tiled matrix multiplication
  - penny-xu.github.io/blog/tiled-matrix-multiplication/