Department of Electrical Engineering
National Taiwan University
Algorithms, Spring 2024

Quiz #4

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I pledge to follow the honor code of NTU and do not cheat in the exam.

Signature: _____________________

Problem 1. (30 pts)

Consider two positively weighted graphs $G = (V, E, w)$ and $G' = (V, E, w')$ with the same vertices V and edges E such that, for any edge $e \in E$, we have $w'(e) = \left(w(e)\right)^2$. Prove or disprove the following statement:

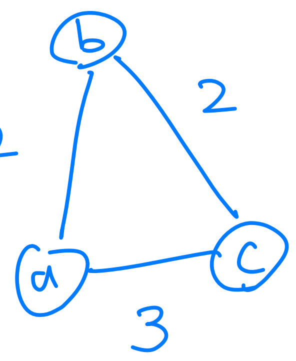
For any two vertices $u, v \in V$, any shortest path between u and v in G' is also a shortest path in G.

Giving a counterexample is sufficient for disproving this statement.

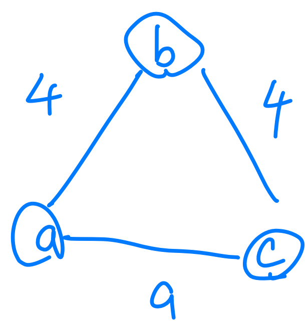
False. $(a + b < c \not\Rightarrow a^2 + b^2 < c^2)$

Counter example:

G:



G':



$a \to b \to c$

$a \to c$

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Problem 2. (30 pts)

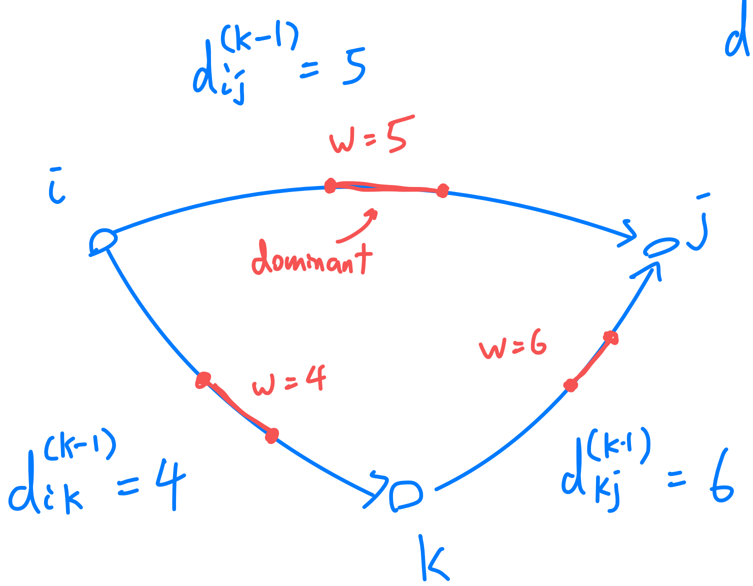
Let $G = (V, E)$ be a weighted directed graph. The dominant of a path is defined as the **maximum** edge weight among all the edges on the path. Suppose that we want to find a **minimum** dominant path between each pair of vertices. Show how to modify Floyd-Warshall's all-pairs shortest-path algorithm shown below to solve this problem in $O(V^3)$ time. **(You only need to give your modifications to save your time.)**

```
Floyd-Warshall(W)
1. n = W.rows
2. D^(0) = W
3. for k = 1 to n
4.     let D^(k) = (d_ij^(k)) be a new n × n matrix
5.     for i = 1 to n
6.         for j = 1 to n
7.             d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1) + d_kj^(k-1))
8. return D^(n)
```

Line 7: $d_{ij}^{(k)} = \min\left(d_{ij}^{(k-1)}, \max\left(d_{ik}^{(k-1)}, d_{kj}^{(k-1)}\right)\right)$

e.g.

Denote the min. dominant of any path from i to j with $\{1, \dots, k\}$ as intermediate nodes as $d_{ij}^{(k)}$.



$$d_{ij}^{(k)} = \min\left(d_{ij}^{(k-1)}, \max\left(d_{ik}^{(k-1)}, d_{kj}^{(k-1)}\right)\right)$$
$$= \min(5, 6)$$
$$= 5$$

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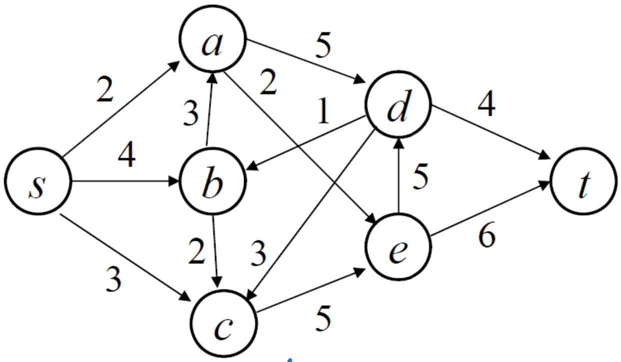
Problem 3. (40 pts)

In the flow network shown below, the number beside an edge denotes its corresponding capacity. Apply the **Edmonds-Karp** algorithm to find a maximum flow from s to t in the network.

Show **every augmentation path** (but you **do NOT** need to show the whole network to save time) and explain why the flow you found is maximum.



Edmonds-Karp: Choose shortest augmenting path at each iteration.

$s \to a \to d \to t$ flow = 2

$s \to c \to e \to t$ flow = 2+3

$s \to b \to c \to e \to t$ flow = 2+3+2

$s \to b \to a \to d \to t$ flow = 2+3+2+2

$\Rightarrow$ Total flow = 9

Proof: By maxflow mincut theorem, the max flow = min capacity of cut, which is the cut: s) a b c

which has capacity 9.

Therefore the flow is a max flow.