

# Signals and Systems MATLAB HW3

**Deadline: 2024/04/30 23:59**

## Digital Filters

The objective of this section is to learn several MATLAB commands about the digital filter design and its applications.

### 1. Background

A causal, non-ideal lowpass filter is designed with frequency response  $H(e^{jw})$ , and its difference equation is specified as

$$a_1 y[n] = \sum_{k=1}^{N+1} b_k x[n-k+1] - \sum_{k=2}^{M+1} a_k y[n-k+1],$$

where  $x[n]$  and  $y[n]$  denote the input signal and the output signal, respectively.

- To obtain the frequency response of the filter, we may use the MATLAB command **freqz**:

$$[H, w] = \text{freqz}(b, a, K);$$

This returns the  $K$ -point complex frequency response vector  $H$  and the  $K$ -point frequency vector  $w$  in radians/sample of the filter:

$$H(e^{jw}) = \frac{b_1 + b_2 e^{-jw} + \dots + b_{N+1} e^{-jNw}}{a_1 + a_2 e^{-jw} + \dots + a_{M+1} e^{-jMw}},$$

given the numerator and denominator coefficients in vectors  $b$  and  $a$ , where

$$a = [a_1, a_2, \dots, a_{M+1}]$$

$$b = [b_1, b_2, \dots, b_{N+1}].$$

- The MATLAB command **butter** designs an IIR filter with a Butterworth response of order  $L$ :

$$[b, a] = \text{butter}(L, f_c);$$

This returns the transfer function with numerator  $b$  and denominator  $a$  coefficient vectors (of length  $L+1$ ) of an  $L^{\text{th}}$ -order lowpass digital Butterworth filter with normalized cutoff frequency  $f_c$ . The cutoff frequency  $f_c$  of the filter is normalized so that it lies in the interval  $[0, 1]$ , with 1 corresponding to  $w = \pi$ . Moreover, you can design a lowpass, highpass, bandpass, or bandstop Butterworth

filter by specifying the value of `fc`:

$$[b, a] = \text{butter}(L, f_c, \text{ftype});$$

- Given an input signal vector  $x$ , and filter it by a lowpass filter with numerator  $b$  and denominator  $a$ . To obtain the output signal  $y$ , we may use the MATLAB command **filter**:

$$y = \text{filter}(b, a, x);$$

## 2. Questions

### Part I

A discrete-time signal is written as

$$x[n] = \cos(2\pi(n-1)T_s), \quad n = 1, 2, \dots, 100.$$

$T_s$  denotes the sampling interval, and the sampling frequency is  $f_s = 1/T_s = 20$  Hz.

Please write a MATLAB script (saved as **mybutter1.m**) to implement the following problems.

- (a) (5%) Use the MATLAB function **plot** to plot  $x[n]$  vs  $n$ .
- (b) (15%) Obtain a Butterworth lowpass digital filter  $H(e^{j\omega})$  by using the MATLAB function **butter** with the following specifications:

Filter order:  $L = 3$

Cutoff frequency:  $f_c = 0.05$

Please write down the transfer function  $H(e^{j\omega})$  of the filter in your report, and use the MATLAB function **plot** to plot the magnitude response (in dB) vs  $\omega$  (in interval  $[0, \pi]$ ) and the phase response (in degree) vs  $\omega$  (in interval  $[0, \pi]$ ) of this filter  $H(e^{j\omega})$ . In addition, use the MATLAB function **plot** to plot the output signal  $y[n]$  vs  $n$  when inputting  $x[n]$  into the filter  $H(e^{j\omega})$ . There will be 3 figures in total in this problem.

- (c) (15%) Please repeat Problem (b) with  $L = 7$ ,  $f_c = 0.05$  and  $f_s = 20$  Hz.
- (d) (15%) Please repeat Problem (b) with  $L = 3$ ,  $f_c = 0.5$  and  $f_s = 20$  Hz.
- (e) (10%) What is the effect of increasing  $L$ ? What about increasing  $f_c$ ? Please give some explanation in your report.

**Note:** It would be better to show the 9 figures from Part I (b)(c)(d) in 9 sub-figures, which are integrated into one figure. For example, you can use the MATLAB function **subplot** in your **mybutter1.m** file.

## Part II

An input signal is written as

$$x[n] = \cos(2\pi(n-1)T_s) + 2\cos(2\pi f_1(n-1)T_s), \quad n = 1, 2, \dots, M,$$

where  $T_s = 0.002$ ,  $f_1 = 100$  and  $M = 1000$ .

Please write a MATLAB script (saved as **mybutter2.m**) to implement the following problems.

- (a) (10%) Use the MATLAB function **plot** to plot  $x[n]$  vs  $n$ .
- (b) (15%) Obtain a 16-order Butterworth lowpass digital filter by using the MATLAB function **butter** such that the output

$$y[n] \approx \cos(2\pi(n-1)T_s), \quad n = 1, 2, \dots, M$$

when inputting  $x[n]$  into the filter.

Please write down the transfer function  $H(e^{j\omega})$  of this filter and the cutoff frequency in your report, and use the MATLAB function **plot** to plot the output signal  $y[n]$  vs  $n$ .

- (c) (15%) Obtain a 16-order Butterworth bandpass digital filter by using the MATLAB function **butter** such that the output

$$y[n] \approx 2\cos(2\pi f_1(n-1)T_s), \quad n = 1, 2, \dots, M$$

when inputting  $x[n]$  into the filter.

Please write down the transfer function  $H(e^{j\omega})$  of this filter and the bandpass frequency in your report, and use the MATLAB function **plot** to plot the output signal  $y[n]$  vs  $n$ .

**Note:** It would be better to show the 3 figures from Part II (a)(b)(c) in 3 sub-figures, which are integrated into one figure. For example, you can use the MATLAB function **subplot** in your **mybutter2.m** file.

## 3. NTU COOL Submission

- Please add "clear **all**; clc;" at the beginning of the MATLAB program and use "%%" to separate each sub-question.
- Please upload three files (.zip), including your **MATLAB files** (saved as **ID\_mybutter1.m** and **ID\_mybutter2.m**) and a **pdf file** (saved as **ID\_report.pdf**). Please show the figures mentioned above in the pdf file (ID\_report.pdf) and answer the questions.