Deadline: 2024/03/26 23:59

The Convolution Sum

In this section, you will learn how to compute the convolution sum of two signals by using MATLAB.

1. Background

Suppose there are two finite-duration signals $x_1[n]$ and $x_2[n]$; now, there are two integers N_1 and N_2 so that $x_1[n] = 0$ outside the interval $1 \le n \le N_1$, and $x_2[n] = 0$ outside the interval $1 \le n \le N_2$.

The convolution sum of the two signals, denoted by $y[n]$, is written as

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]. \tag{1}$$

To obtain the value using MATLAB, you may directly use the function **conv**:

$$y = \boldsymbol{conv}(x_1, x_2), \tag{2}$$

where $x_1 = [x_1[1], x_1[2], \dots, x_1[N_1]]^T$, $x_2 = [x_2[1], x_2[2], \dots, x_2[N_2]]^T$ and the superscript T denotes the transpose operation.

Besides the direct computation of (1), there are some other methods to compute the convolution sum. For instance, it can be computed by the following matrix form:

$$\begin{bmatrix} y[2] \\ \vdots \\ y[N_1+N_2] \end{bmatrix} = \begin{bmatrix} x_1[1] & 0 & \dots & 0 \\ x_1[2] & x_1[1] & \ddots & \vdots \\ \vdots & x_1[2] & \ddots & 0 \\ x_1[N_1] & \vdots & \ddots & x_1[1] \\ 0 & x_1[N_1] & \ddots & x_1[2] \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & x_1[N_1] \end{bmatrix} \begin{bmatrix} x_2[1] \\ x_2[2] \\ \vdots \\ x_2[N_2] \end{bmatrix}. \tag{3}$$

Note: please pay attention to the dimension of each matrix.

2. Questions:

Given two signals

$$x_1[n] = \begin{cases} n, & 1 \le n \le 20 \\ 40 - n, & 21 \le n \le 39 \\ 0, & elsewhere \end{cases} \qquad x_2[n] = u[n-1] - u[n-11],$$

where u denotes the unit step function.

Please write a MATLAB script (saved as **myconv.m**) to implement problems (a) to (d).

(a) (15%) Use the MATLAB function **stem** to plot $x_1[n]$ vs n and $x_2[n]$ vs n.

(b) (15%) Use the MATLAB function **conv** directly to compute equation (1) and use **stem** to plot the output $y[n]$ vs n.

(c) (20%) Create a MATLAB program by yourself to compute equation (1) by using equation (3) matrix form and use **stem** to plot the output $y[n]$ vs n. You should verify whether the answer is the same as Problem (b).

(You should place the code for this part at the end of the report, using neatly formatted text or multiple screenshots.)

(d) (50%) Repeat Problems (a) to (c) again, but $x_1[n]$ and $x_2[n]$ are changed to the following:

$$x_1[\mathrm{n}] = \begin{cases} 3^n, & 1 \le n \le 3 \\ 0, & elsewhere \end{cases} \qquad x_2[n] = \begin{cases} 2^n, & 1 \le n \le 5 \\ 0, & elsewhere \end{cases}.$$

Note: We expect that executing your **myconv.m** file will output 8 figures in order.
(2 figures for Problem (a), 1 figure for Problem (b), 1 figure for Problem (c), and 4 figures for Problem (d))

Hint: The dimensions of y[n] in (b) and (c) should be $N_1 + N_2 - 1 \times 1$, and the x-axis coordinates 'n' should range from 2 to $N_1 + N_2$.

3. NTU COOL Submission

- Please add "`clear all; clc;`" at the beginning of the MATLAB program and use "`%%`" to separate each sub-question.
- Please upload two files, including your **MATLAB file** (saved as **ID_myconv.m**) and a **pdf file** (saved as **ID_report.pdf**). In the report, please paste the **eight figures** from the assignment **sequentially**, including the code for part (c) at the end of report. There is no need to include the code in part (d) that repeats part (c).