Signals and Systems HW2

Deadline: 2024/03/15 23:59

You can convert your handwritten paper to a .pdf file by taking photos, file scanning or typing. Please name the file with your student ID (e.g., B11901xxx.pdf), and then upload the .pdf file to NTU COOL.

1. (30%) Let
$$x(t) = u(t-2) - u(t-5) \text{ and } h(t) = e^{-2t}u(t-3)$$
   (a) (15%) Compute $y(t)$ for the convolution integral as
   $$y(t) = x(t) * h(t)$$
   (b) (15%) Compute $g(t)$ for the convolution integral as
   $$g(t) = \left(\frac{dx(t)}{dt}\right) * h(t)$$

2. (10%) For the following statements, determine whether it is true or false with your answer justified. (* denotes the convolution operation.)

   "If $y[n] = x[n] * h[n]$, then $y[n-1] = x[n-1] * h[n-1]$"

3. (40%) Consider a CT system with the input/output relationship given by
$$y(t) = \int_{t-1}^{\infty} (t - \alpha + 3)\, x(1+\alpha) d\alpha.$$
   (a) (20%) Find the unit-impulse response of the system. Justify your answer.
   (b) (20%) Find the output $y(t)$ for $-1 \le t \le 0$ if the input $x(t)$ is given by $u(t) - u(t-1)$, where $u(t)$ is the unit-step function.

4. (20%) A system produces an output signal as below. Is it linear? Is it time-invariant? Verify your answer.
   (a) (10%) $y(t) = x(5t)$ for a given input signal $x(t)$
   (b) (10%) $y(t) = Odd\{x(t)\}$ for a given input signal $x(t)$, where $Odd\{\cdot\}$ is the odd part of a signal.