Signals and Systems HW4

Deadline: 2024/04/05 23:59

You can convert your handwritten paper to a .pdf file by taking photos, file scanning or typing. Please name the file with your student ID (e.g., B11901xxx.pdf), and then upload the .pdf file to NTU COOL.

1. (20%) A linear time invariant system is characterized by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 9\frac{dy(t)}{dt} + 14y(t) = \frac{dx(t)}{dt} + 3x(t)$$

   (a) (10%) Find the frequency response $H(j\omega)$ and unit impulse response $h(t)$ of the system.

   (b) (10%) Assume the input is $x(t) = e^{-t}u(t)$. Find the output $y(t)$.

2. (50%) Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted below.

   (a) (10%) $X(j\omega)$ can be expressed as $A(j\omega)e^{jB(j\omega)}$, where $A(j\omega)$ and $B(j\omega)$ are both real-values. Find $B(j\omega)$

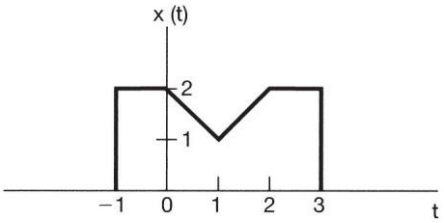
   (b) (10%) Find $X(j0)$

   (c) (10%) Find $\int_{-\infty}^{+\infty} X(j\omega)\ \mathrm{d}\omega$

   (d) (10%) Evaluate $\int_{-\infty}^{+\infty} X(j\omega)\frac{2sin\omega}{\omega}e^{j2\omega}\ \mathrm{d}\omega$

   (e) (10%) Evaluate $\int_{-\infty}^{+\infty} |X(j\omega)|^2\ \mathrm{d}\omega$

Note: You should perform all these calculations without explicitly evaluating $X(j\omega)$



3. (30%)

   (a) (15%) Show that the three LTI systems with impulse responses

$$h_1(t) = u(t)$$
$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t)$$

   and

$$h_3(t) = 2te^{-t}u(t)$$

   All have the same response to $x(t) = \cos(t)$

   (b) (15%) Find the impulse response system with the same response to $\cos(t)$