科目 机率

長庚大學期中、期末考試答案用紙

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[1]

(1)

$$f_X(x) = \begin{cases} x=0 & b(0,10,\frac{1}{10}) \doteqdot 0.3487 \\ x=1 & b(1,10,\frac{1}{10}) \doteqdot 0.3874 \\ x=2 & b(2,10,\frac{1}{10}) \doteqdot 0.1937 \\ x=3 & b(3,10,\frac{1}{10}) \doteqdot 0.0574 \\ x=4 & b(4,10,\frac{1}{10}) \doteqdot 0.0112 \\ x=5 & b(5,10,\frac{1}{10}) \doteqdot 0.0015 \\ x=6 & b(6,10,\frac{1}{10}) \doteqdot 0.0001 \\ x=7 & b(7,10,\frac{1}{10}) \doteqdot 0 \\ x=8 & b(8,10,\frac{1}{10}) \doteqdot 0 \\ x=9 & b(9,10,\frac{1}{10}) \doteqdot 0 \\ x=10 & b(10,10,\frac{1}{10}) \doteqdot 0 \\ 0, & x \in R \setminus S \end{cases}$$

(2) $E(X) = n \cdot p = 10 \cdot \frac{1}{10} = 1$

(3) $Std(X) = 0.9487$

(4)

2. (1) $f_W(w) = p(w;100) = \frac{e^{-100}(100)^w}{w!}$

(2) $E[W] = 100$ $Std[W] = \sqrt{100} = 10$

$E[W] + Std[W] = 110$

(3) $P(|W - E[W]| \leq 2 \cdot std[W]) = P(|W-100| \leq 20) = P(80 \leq W \leq 120) = \sum_{w=80}^{120} P(w;100)$

(4) $P(W > 120)$

(5) 拒絕它，偏差值过高

3. $p = 0.05$ $n = 100$

(1) $p(X=x) = \binom{100}{x}(0.05)^x(1-0.05)^{100-x}$

$P(X=10) = \binom{100}{10}(0.05)^{x}(0.95)^{90} = 0.016715$

(2) A buyer would suspect the claim is not correct because assuming a correct claim, probability of having 10 detective item in sample is 1.6715×10^{-2} and event would occur only 1.6715% of time.

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The binomial distribution tends toward the Poisson distribution as : $n \to \infty$, $p \to 0$, $\lambda = np$ stays constant

We need to show $\binom{n}{x} p^x (1-p)^{n-x} \to \frac{\lambda^x e^{-\lambda}}{x!}$

We will need : $e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$

$\binom{n}{x} p^x (1-p)^{n-x} = \binom{n}{x}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x} = \frac{n!}{x!(n-x)!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x} = \frac{\lambda^x}{x!}\frac{n!}{(n-x)!}\frac{1}{n^x}\left(1-\frac{\lambda}{n}\right)^{n-x}$

$\frac{n!}{(n-x)!}\frac{1}{n^x} = \frac{n(n-1)(n-2)\cdots(n-x+1)(n-x)!}{(n-x)!\,n^x} = \frac{n(n-1)(n-2)\cdots(n-x+1)}{n^x} = \frac{n}{n}\cdot\frac{n-1}{n}\cdot\frac{n-2}{n}\cdots\frac{n-x+1}{n} = 1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x+1}{n}\right)$

$\binom{n}{x} p^x (1-p)^{n-x} = \frac{\lambda^x}{x!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x+1}{n}\right)\left(1-\frac{\lambda}{n}\right)^{n-x} = \frac{\lambda^x}{x!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x+1}{n}\right)\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x}$

$\binom{n}{x} p^x (1-p)^{n-x} = \frac{\lambda^x}{n!} \times \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{n+1}{n}\right) \times \left(1-\frac{\lambda}{n}\right)^n \times \left(1-\frac{\lambda}{n}\right)^{-x}$

$\lim_{n\to\infty}\binom{n}{x} p^x (1-p)^{n-x} = \frac{\lambda^x}{x!} \times \lim_{n\to\infty}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x+1}{n}\right) \times \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n \times \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-x}$

$\lim_{n\to\infty}\binom{n}{x} p^x (1-p)^{n-x} = \frac{\lambda^x}{x!} \times 1 \times e^{-\lambda} \times 1 = \frac{\lambda^x e^{-\lambda}}{x!}$