Ch6 例 6-4

$E(X_i)=\mu$, $V(X)=\sigma^2=E(X_i^2)-\mu^2$

$E(\bar{X})=\mu$, $V(\bar{X})=\frac{\sigma^2}{n}=E(\bar{X}^2)-\mu^2$

$$E(\hat{\theta}_1)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n}(n\sigma^2+\cancel{n\mu^2}-\sigma^2-\cancel{n\mu^2})=\frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n-1}(n\sigma^2+\cancel{n\mu^2}-\sigma^2-\cancel{n\mu^2})=\sigma^2$$

由上推導可知 $\hat{\theta}_2=\sum_{i=1}^{n}(X_i-\bar{X})^2/n-1$ 為母体 $V(X)$ 之不偏估計量，而 $\hat{\theta}_1=\sum_{i=1}^{n}(X_i-\bar{X})^2/n$ 為母体 $V(X)$ 之偏誤估計量