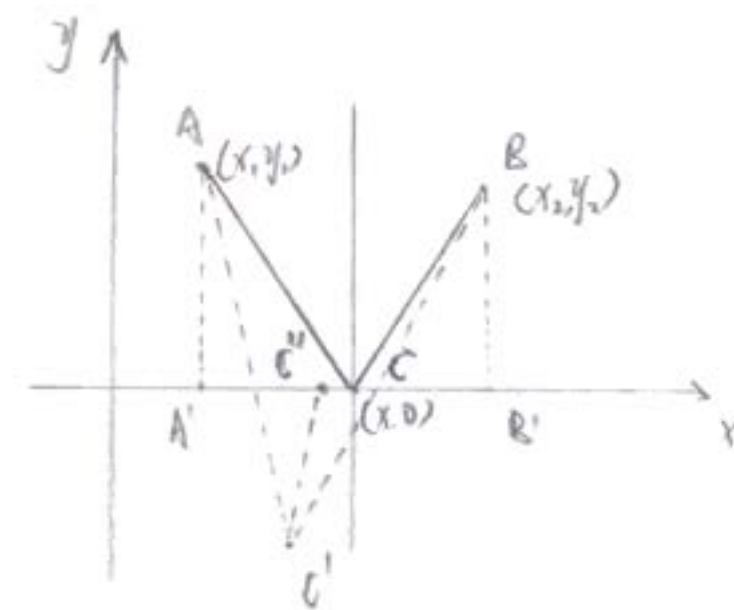


1. 证：设两个均匀介质的分界面是平面，它们的折射率为  $n_1$  和  $n_2$ 。光线通过第一介质中指定的 A 点后到达同一介质中指定的 B 点。为了确定实际光线的路径，通过 A,B 两点作平面垂直于界面， $\overline{OO'}$  是他们的交线，则实际 光线在界面上的反射点 C 就可由费马原理来确



定（如右图）。

(1) 反正法：如果有一点  $C'$  位于线外，则对应于  $C'$ ，必可在  $OO'$  线上找到它的垂足  $C''$ 。由于  $\overline{AC'} > \overline{AC''}$ ,  $\overline{C'B} > \overline{C''B}$ , 故光谱  $AC'B$  总是大于光程  $AC''B$  而非极小值，这就违背了费马原理，故入射面和反射面在同一平面内得证。

(2) 在图中建立坐  $oxy$  标系，则指定点 A,B 的坐标分别为  $(x_1, y_1)$  和  $(x_2, y_2)$ ，未知点 C 的坐标为  $(x, 0)$ 。C 点在  $A', B'$  之间是，光程必小于 C 点在  $\overline{A'B'}$

以外的相应光程，即  $x_1 < x < x_2$ ，于是光程 ACB 为：

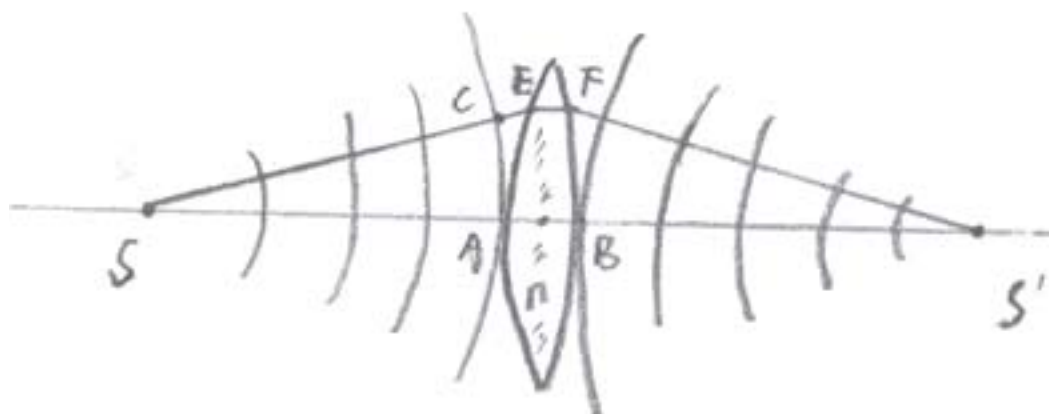
$$n_1 \overline{ACB} = n_1 \overline{AC} + n_1 \overline{CB} = n_1 \sqrt{(x - x_1)^2 + y_1^2} + n_1 \sqrt{(x_2 - x)^2 + y_2^2}$$

根据费马原理，它应取极小值，即：

$$\frac{d}{dx}(n_1 \overline{ACB}) = \frac{n_1(x - x_1)}{\sqrt{(x - x_1)^2 + y_1^2}} - \frac{n_1(x_2 - x)}{\sqrt{(x_2 - x)^2 + y_2^2}} = n_1 \left( \frac{\overline{A'C}}{\overline{AC}} - \frac{\overline{CB}}{\overline{CB}} \right)$$

$$\therefore i'_1 = i_1, \therefore \frac{d}{dx}(n_1 \overline{ACB}) = 0$$

取的是极值，符合费马原理。故问题得证。



2.(1)证：如图所示，有位于主光轴上的一个物点  $S$  发出的光束

经薄透镜折射后成一个明亮的实象点  $S'$ 。由于球面  $AC$  是由  $S$  点

发出的光波的一个波面，而球面  $DB$  是会聚于  $S'$  的球面波的一个

波面，因而  $SC = SB$ ， $S'D = S'B$ 。又  $\because$  光程  $CEFD = CE + nEF + FD$ ,

而光程  $AB = nAB$ 。根据费马原理，它们都应该取极值或恒定值，

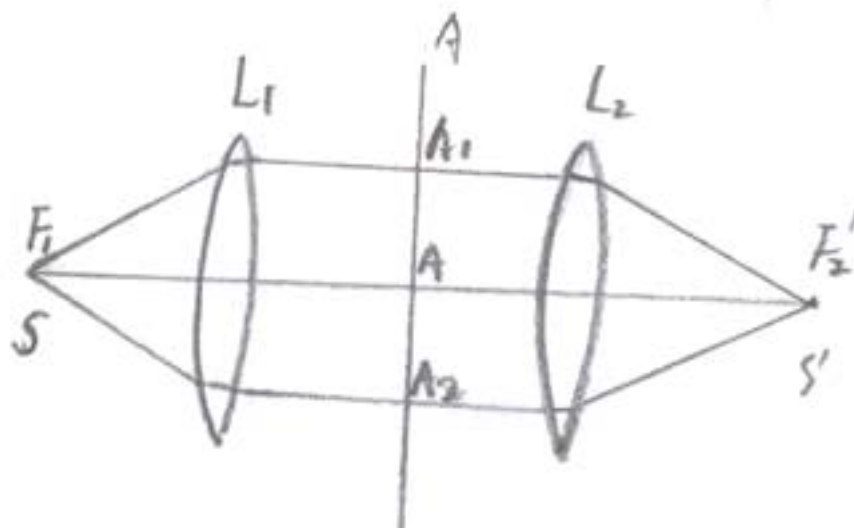
这些连续分布的实际光线，在近轴条件下其光程都取极大值或

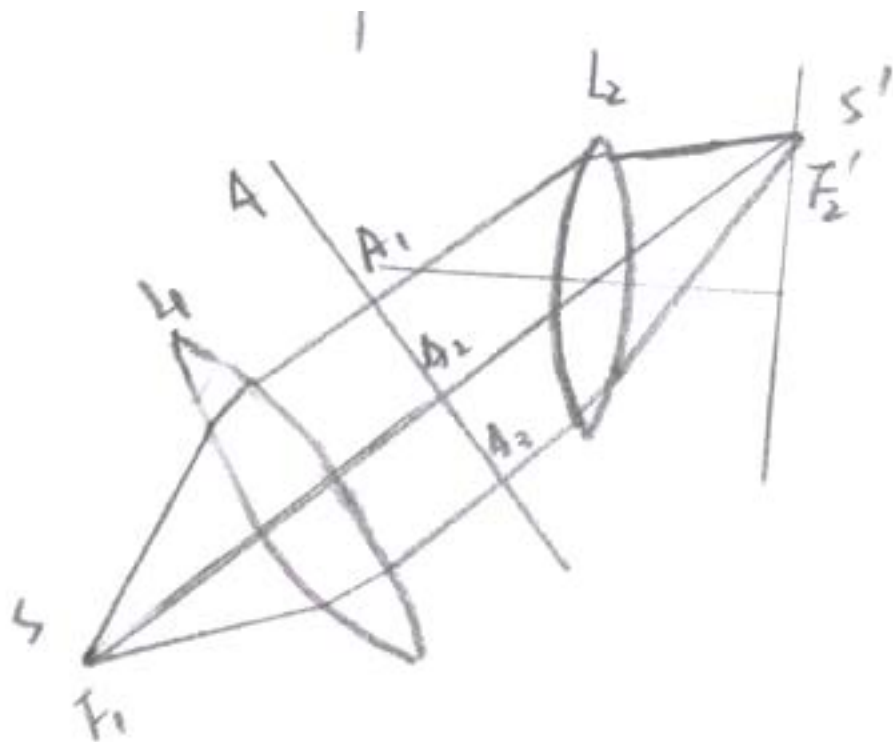
极小值是不可能的，唯一的可能性是取恒定值，即它们的光程却相等。

由于实际的光线有许多条。我们是从中去两条来讨论，故从物点发出

并会聚到像点的所有光线的光程都相等得证。

除此之外，另有两图如此，并与今后常用到：

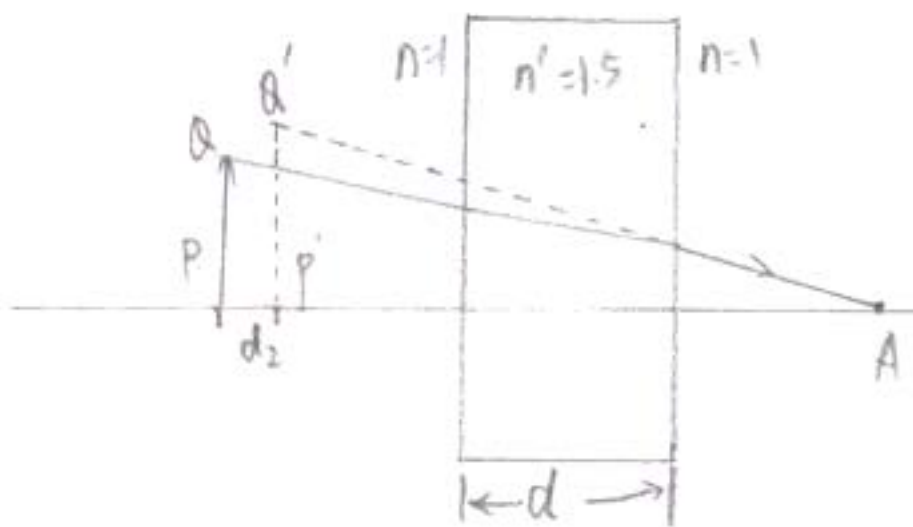


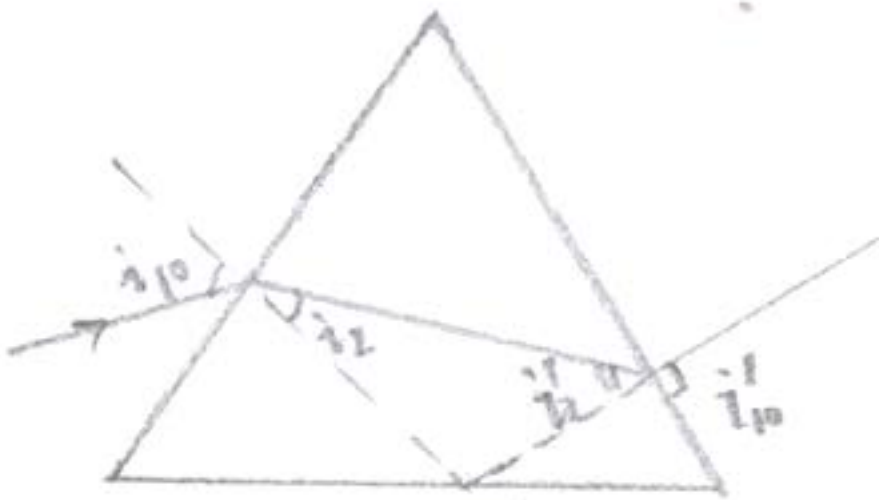


3.解：由  $P_{164} L 3-1$  的结果

$$\overline{PP'} = h \left(1 - \frac{1}{n}\right) \text{ 得:}$$

$$\begin{aligned} d_2 &= d \left(1 - \frac{1}{n}\right) \\ &= 30 \times \left(1 - \frac{1}{1.5}\right) \\ &= 10 \text{ (cm)} \end{aligned}$$





4.解：由  $P_{170}$  结果知：

(1)  $\because$

$$n = \frac{\sin \frac{\theta_0 + A}{2}}{\sin \frac{A}{2}}, \quad n \sin \frac{A}{2} = \sin \frac{\theta_0 + A}{2}$$

$$\begin{aligned} \therefore \theta_0 &= 2 \sin^{-1} \left[ n \sin \frac{A}{2} \right] - A \\ &= 2 \sin^{-1} \left[ 1.6 \times \sin \frac{60^\circ}{2} \right] - 60^\circ \\ &= 2 \sin^{-1} [0.8] - 60^\circ \\ &= 2 \times 53.13^\circ - 60^\circ \\ &= 46.26^\circ \end{aligned}$$

$$\approx 46^\circ 16'$$

$$(2) \quad i' = \frac{\theta_0 + A}{2} = \frac{46^\circ 16' + 60^\circ}{2} = 53^\circ 08'$$

$$(3) \quad \because n = \frac{\sin i_2}{\sin i_{10}}$$

$$\therefore \sin i'_2 = \frac{\sin i'_{10}}{n} = \frac{\sin 90^\circ}{1.6} = \frac{1}{1.6}$$

$$i'_2 = \sin^{-1} \frac{1}{1.6} = 38.68^\circ = 38^\circ 41'$$

$$\text{而 } i_2 = A - i'_2 = 60^\circ - 38^\circ 41' = 21^\circ 19'$$

$$\frac{\sin i_2}{\sin i_{10}} = \frac{d}{n}$$

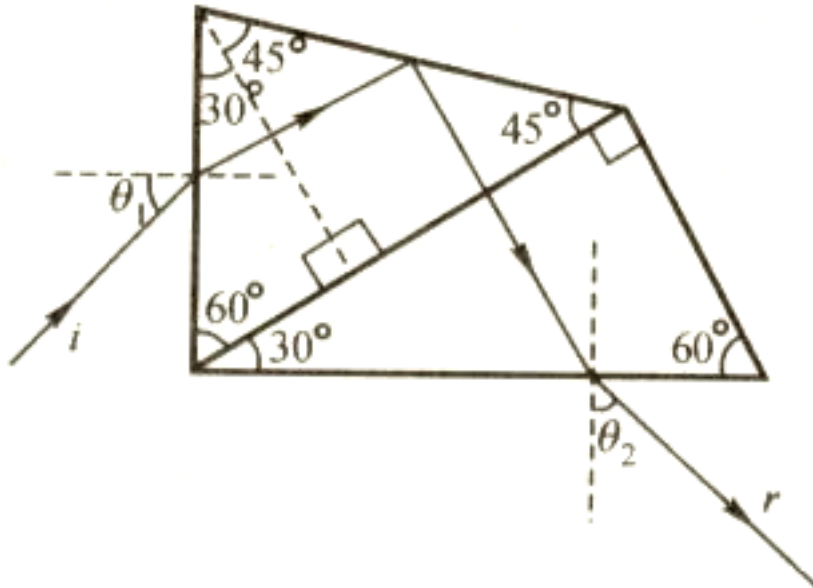
$$\text{又 } \because \sin i_{10} = n \sin i_2$$

$$\therefore i_{10} = \sin^{-1}(1 \sin 21^\circ 19')$$

$$= 35.57^\circ \approx 35^\circ 34'$$

$$\text{故: } i_{\min} = i_{10} = 35^\circ 34'$$

5.证:



$$\therefore \sin \theta_1 = n \sin i_2$$

$$\text{若 } \sin \theta_1 = \frac{n}{2}$$

$$\text{则 } \sin i_2 = \frac{1}{2} \quad i_2 = 30^\circ$$

$$\text{即: } i'_2 = i_2 = 30^\circ$$

$$\text{而 } \sin \theta_2 n \sin i'_2 = n \sin 30^\circ = \frac{1}{2}$$

$$\therefore \theta_1 = \theta_2 \quad \text{得证。}$$

$$\text{又 } \because \theta_1 + \alpha_1 = 90^\circ \quad \text{而 } \theta_1 = \theta_2,$$

$$\therefore \theta_2 + \alpha_1 = 90^\circ \quad \text{即 } \gamma \perp i \text{ 得证。}$$

$$\text{or: 又 } \because \theta_2 + \alpha_2 = 90^\circ, \therefore \alpha_1 = \alpha_2,$$

$$\text{故: } \theta_1 + \theta_2 = 90^\circ \quad \text{即 } \gamma \perp i \text{ 得证。}$$

$$\text{讨论: 1. 由此可推论 } \theta_1 = \theta_2 = 45^\circ$$

$$2. n = \sin \theta_1 = 2 \sin 45^\circ = 2 \times \frac{\sqrt{2}}{2} = \sqrt{2} = 1.414$$

6.解:

$$\therefore \frac{1}{s'} + \frac{1}{s} = \frac{1}{f'}$$

$$\therefore \frac{1}{s'} = \frac{1}{f'} - \frac{1}{s}$$

$$\text{即: } \frac{1}{s'} + \frac{1}{-10} - \frac{1}{-12} = -\frac{1}{60}$$

$$\therefore s' = -60 \text{ (cm)}$$

$$\text{又} \therefore \frac{-y'}{y} = \frac{-s'}{-s}$$

$$\begin{aligned} \therefore y' &= -\frac{s'}{s} y \\ &= -\frac{-60}{-12} \times 5 \\ &= -25 \text{ (cm)} \end{aligned}$$

7.解: (1)

$$\therefore \beta = \frac{y'}{y} = -\frac{s'}{s}$$

$$\therefore s' = -\frac{y'}{y} s = -\frac{1}{5} \times (-10) = 2 \text{ (cm)}$$

$$\text{又} \therefore \frac{1}{s'} + \frac{1}{s} = \frac{2}{r}$$

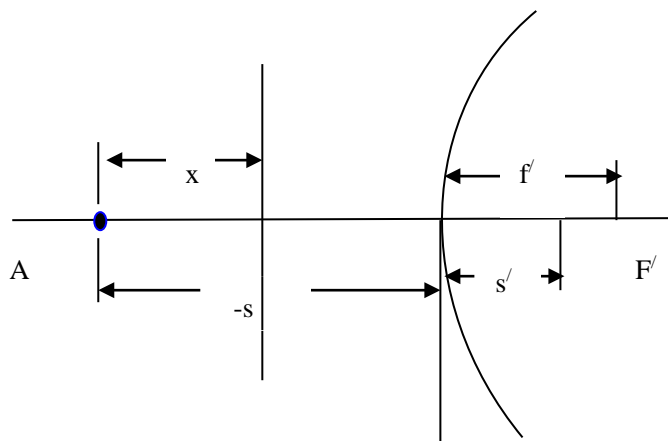
$$\text{即} \frac{2}{r} = \frac{1}{2} - \frac{1}{10} = \frac{2}{5}$$

$$\therefore r = 5 \text{ (cm)}$$

(2)  $\therefore r = 5 \text{ cm} > 0$   
是凸透镜.

8.解:

$$\therefore x = \frac{s' + (-s)}{2} = \frac{8 + 40}{2} = 24(cm)$$


$$\therefore y' = \frac{n_2}{n_1} y$$

## 第一次折射:

$$n_2 = n, \quad n_1 = 1$$

$$y = \overline{D P'_1}$$

$$\therefore y' = n \overline{D P'_1}$$

### 第二次折射:

$$n_2 = 1, \quad n_1 = \mathfrak{n}$$

$$y = y'_1 - d, y'_2 = \frac{1}{n}(y'_1 - d) = \overline{EP'_1}$$

$$\therefore \overline{PP'} = (-\overline{EP'}) - (-\overline{EP'}) = \overline{EP'} - \overline{EP'}$$

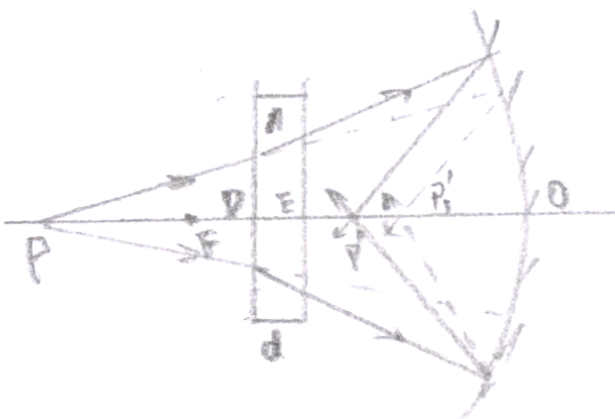
$$= \frac{1}{n}(y'_1 - d) - (\overline{D p'_1} - d)$$

$$= \frac{1}{n}(n\overline{D p'_1} - d) - (\overline{D p'_1} - d)$$

$$= \overline{D p'_1} - \frac{1}{n}d - \overline{D p'_1} + d = d(1 - \frac{1}{n}) = d \frac{n-1}{n}$$

$$d^{\frac{n-1}{2}}$$

由图可知，若使凹透镜向物体移动  $n$  的距离亦可得到同样的结果。





10.解:

$$\therefore \frac{n}{s'} - \frac{n}{s} = \frac{n' - n}{\gamma}$$



而:  $s = \infty \quad s' = 2\gamma \quad n = 1$

$$\therefore \frac{n'}{2\gamma} - \frac{n' - 1}{\gamma}, \quad \frac{n'}{2} = n' - 1 \quad n' - \frac{n'}{2} = 1$$

故  $n' = 2$

P'

11.解:

(1) 由  $P_{208} L3-7$  经导知:

$$f' = \frac{nR}{2(n-1)} = \frac{1.5 \times 4}{2(1.5-1)} = 6 \text{ (cm)}$$

按题意, 物离物方主点 H 的距离为  $-(6+4)$ ,  
于是由

$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f'} \quad \text{得}$$

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f'} = \frac{1}{6} + \frac{1}{-10} = \frac{5-3}{30} = \frac{1}{15}$$

$$\therefore s' = 15 \text{ (cm)}$$

(2)

$$\beta = \frac{s'}{s} = \frac{15}{6+4} = 1.5$$

12.解:

$$\therefore \frac{n}{s'} - \frac{n}{s} = \frac{n' - n}{r}$$

$$\therefore \frac{n}{s} = \frac{n'}{s'} - \frac{n' - n}{r}$$

(1)

$$\therefore s' = r$$

$$\therefore \frac{n}{s_1} = \frac{n'}{r} - \frac{n' - n}{r} = \frac{n}{r}$$

$$\text{即 } s_1 = r$$

仍在原处 (球心), 物像重合

(2)

$$\therefore s'_1 = \frac{r}{2}$$

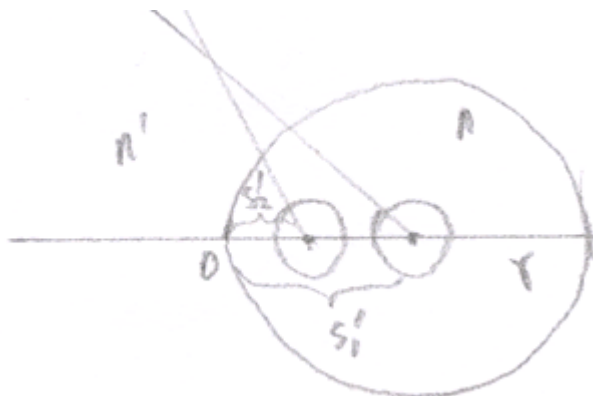
$$\therefore \frac{n}{s_2} = \frac{n'}{\frac{r}{2}} - \frac{n' - n}{r} = \frac{2n'}{r} - \frac{n' - n}{r} = \frac{n' + n}{r}$$

$$s_2 = \frac{nr}{n' + n} = \frac{nD}{2(n' + n)}$$

$$= \frac{1.57 \times 20}{2 \times (1.53 + 1)} \approx 6.05 \text{ (cm)}$$

13.解:

(1)



$$\therefore \frac{n'}{s'} - \frac{n}{s} = \frac{n' - n}{r}$$

$$\therefore \frac{n}{s_2} = \frac{n' - n}{r} + \frac{n}{s}$$

$$\text{又} \because s = r \quad \frac{n'}{s'} = \frac{n' - n}{r} + \frac{n}{r} = \frac{n'}{r}$$

$\therefore s' = r = 15\text{cm}$  即鱼在原处

(2)

$$\therefore \beta = \frac{y'}{y} = \frac{s'}{s} \cdot \frac{n}{n'} = \frac{15}{15} \times \frac{1.33}{1} = 1.33$$

14 解:

(1)

$$\therefore f = \frac{n'}{n' - n} r = -\frac{1.33}{1.50 - 1.33} \times 2 = -15.647\text{cm}$$

$$f' = \frac{n'}{n' - n} r = \frac{1.50}{1.50 - 1.33} \times 2 = 17.647\text{cm}$$



$$\text{而} \frac{f'}{s'} + \frac{f}{s} = 1 \quad \text{即} \frac{f'}{s'} = 1 - \frac{f}{s} = \frac{s - f}{s}$$

$$\therefore s' = \frac{sf'}{s - f} = \frac{-8 \times 17.647}{-8 - (-15.647)} = -\frac{141.176}{7.647} \approx -18.46 \approx -18.5\text{cm}$$

(2)

$$\beta = \frac{y'}{y} = \frac{s'}{s} \cdot \frac{n}{n'} = \frac{-18.5}{-8} \times \frac{1.33}{1.50} \approx 2.046 \approx 2$$

(3)

光路图如右:

15 解:

(1)

$$\because f = \frac{-n_1}{\left(\frac{n-n'}{r_1} - \frac{n_2-n}{r_2}\right)}, n_1 = n_2 = n, r_1 = r_2 = r, n = n'$$

$$\therefore f_1 = \frac{-n}{\frac{n-n'}{r} + \frac{-n+n'}{-r}} = \frac{1.33 \times 10}{2 \times (1.5 - 1.33)} \approx -39.12 = -f'_1$$

$$\text{又} \because \frac{f'}{s'} + \frac{f}{s} = 1 \quad f'_1 = -f_1$$

$$\therefore \frac{-f'}{s'} + \frac{f}{s} = 1$$

$$\frac{1}{s'} = \frac{1}{s} - \frac{1}{f} = \frac{1}{-20} - \frac{1}{-39.12} = -0.0244$$

$$\therefore s'_{\text{凸}} = s'_1 = -40.92 \text{ cm}$$

(2)

$$\because f = \frac{n_1}{\left(\frac{n-n'}{r_1} + \frac{n_2-n}{r_2}\right)}, n_1 = n_2 = n, r_1 = r_2 = r, n = n'$$

$$\therefore f'_2 = \frac{n}{\frac{-n+n'}{-r} + \frac{n-n'}{r}} = -\frac{nr}{2(n'-n)} = -\frac{1.33 \times 10}{2 \times (1.5 - 1.33)} \approx -39.12 = -f_2$$

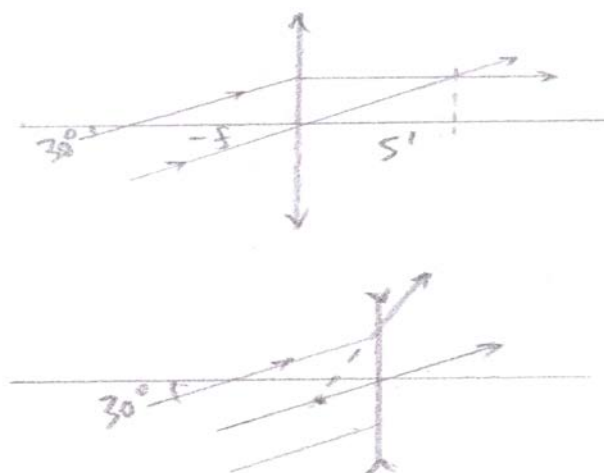
$$\text{又} \because \frac{f'}{s'} + \frac{f}{s} = 1 \quad f'_2 = -f_2$$

$$\therefore \frac{f'_2}{s'_2} + \frac{-f_2}{s} = 1$$

$$\frac{1}{s'_2} = \frac{1}{s} + \frac{1}{f'_2} = \frac{1}{-20} + \frac{1}{-39.12} = -0.0756$$

$$\therefore s'_{\text{凹}} = s'_2 = -13.23 \text{ cm}$$

(3)



16.解：（1）透镜在空气中和在水中的焦距分别为：

$$\begin{aligned} \frac{1}{f'_1} &= (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) & \frac{1}{f'_2} &= \frac{n-n'}{n'}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \\ \therefore \frac{f'_2}{f'_1} &= \frac{n'(n-1)}{n-n'} & n'(n-1) &= (n-n')\frac{f'_2}{f'_1} \\ n'n - n' &= n\frac{f'_2}{f'_1} - n'\frac{f'_2}{f'_1} & n(n' - \frac{f'_2}{f'_1}) &= n'(1 - \frac{f'_2}{f'_1}) \\ \therefore n &= \frac{n'(1 - \frac{f'_2}{f'_1})}{n' - \frac{f'_2}{f'_1}} = \frac{1.33 \times (1 - \frac{136.8}{40})}{1.33 - \frac{136.8}{40}} = \frac{-3.22}{-2.09} \approx 1.54 \\ \frac{1}{r_1} - \frac{1}{r_2} &= \frac{1}{f'_1(n-1)} = \frac{1}{40 \times (1.54-1)} = \frac{1}{21.6} \end{aligned}$$

（2）透镜置于水<sup>CS<sub>2</sub></sup>中的焦距为：

$$\begin{aligned} \frac{1}{f'_3} &= \frac{n-n''}{n''}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \\ &= \frac{1.54-1.62}{1.62} \times \frac{1}{21.6} = \frac{-0.08}{34.992} \\ \therefore f'_3 &= -\frac{34.992}{-0.08} = -437.4 \text{ cm} \end{aligned}$$

17.解：

$$\begin{aligned}
\because f' &= \frac{n_2}{\frac{n-n_1}{r_1} + \frac{n_2-n}{r_2}} \quad n_1 = n_2 = n' \\
\therefore f' &= \frac{n'(n-1)}{n-n'} \bigg/ \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \\
&= \frac{1.33}{1-1.33} \bigg/ \left( \frac{1}{20} - \frac{1}{-25} \right) \\
&= \frac{1.33}{-0.33 \times 0.09} \\
&\approx -44.78 \text{ cm}
\end{aligned}$$

18.解:

(1)

$$\because \frac{1}{s'} + \frac{1}{s} = \frac{1}{f'}$$

$$s = \infty$$

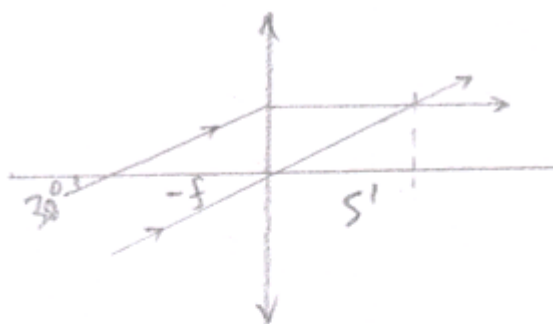
$$\therefore s'_x = f' = 10 \text{ cm}$$

$$s'_y = s'_x \tan 30^\circ = 10 \times 0.577 \approx 5.77 \text{ cm}$$

考虑也可能去负值，而平行光从光面射下射

$\therefore$  像点的坐标(10,5.77).

同理，对于发散透镜其像点的坐标 (10,5.77) 。



(2)

$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

$$s = -f'$$

$$\therefore \frac{1}{s'} = \frac{1}{s} + \frac{1}{f'} = \frac{1}{f'} + \frac{1}{-f'} = 0$$

$s' = \infty$  即，发射光束仍为平行光无像点

$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'} \quad s = -f \quad s' = -f$$

$$\therefore \frac{1}{s'} = \frac{1}{s} + \frac{1}{f'} = \frac{1}{-f} + \frac{1}{-f} = -\frac{2}{f}$$

$$\therefore s' = -\frac{f}{2} = -\frac{10}{2} = -5 \text{ cm}$$

$$\text{又} \because \beta = \frac{y'}{y} = \frac{s'}{s}$$

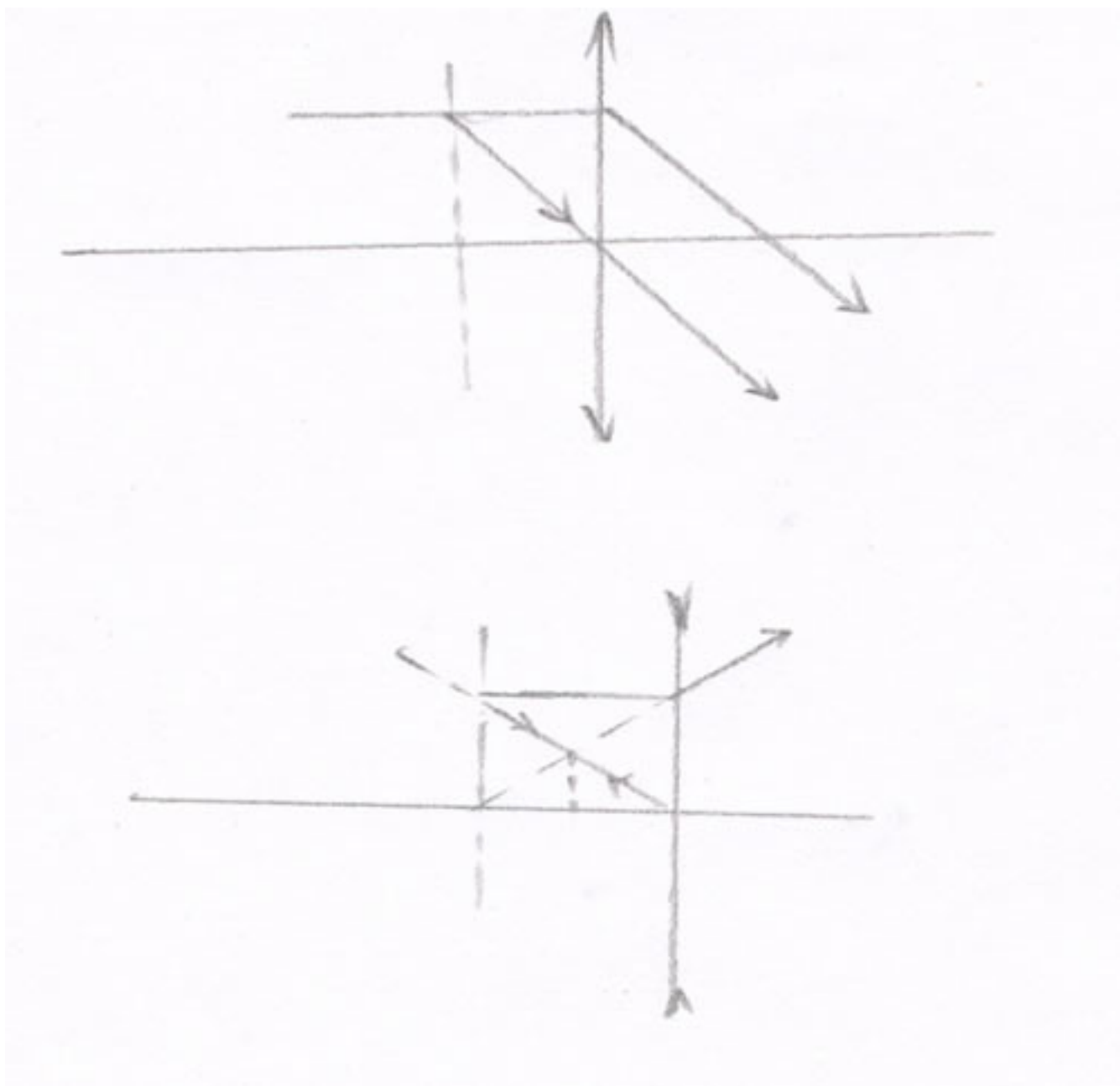
$$\therefore y' = \frac{s'}{s} y = \frac{-5}{-10} \times 1 = 0.5 \text{ cm}$$

再考虑到像点另一种放置，

故像点的坐标为  $(-5, |0.5|)$  cm

19.解：透镜中心和透镜焦点的位置如图所示：





20.解:

$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

$$\therefore \frac{1}{s'} = \frac{1}{s} + \frac{1}{f'}$$

$$= \frac{1}{50} + \frac{1}{-300}$$

$$= \frac{1}{60}$$

$$\therefore s' = 60(\text{cm})$$



$$\text{又} \because \tan \theta = \frac{\theta/2}{-s} = \frac{d/2}{s' - s} \quad \text{即: } d/2 = \frac{s' - s}{2} \cdot \frac{t}{2}$$

$$\therefore d = \frac{s' - s}{s} t = \frac{300 + 60}{300} \times 0.1 = 0.12 \text{ cm}$$

$p_1, p_2$  这两个象点，构成了相干光源，故由双缝干涉公式知，干涉条纹的间距为

$$\Delta y = \frac{r_0}{d} \lambda = \frac{l - s'}{d} \lambda = \frac{450 - 60}{0.12} \times 6328 \times 10^{-8} \approx 0.206 \text{ cm} = 2.06 \text{ mm}$$

21.解:

$\therefore$  该透镜是由 A, B 两部分胶合而成的 (如图所示), 这两部分的主轴都

不在光源的中心轴线上, A 部分的主轴  $O_A P_A$  在系统中心线下方 0.5cm 处, B 部分

的主轴  $O_B F'_B$  则在系统中心线上方 0.5cm 处。由于点光源经凹透镜 B 的成像位置

$P_B$  即可 (为便于讨论, 图 (a) (b) (c) 是逐渐放大图像)

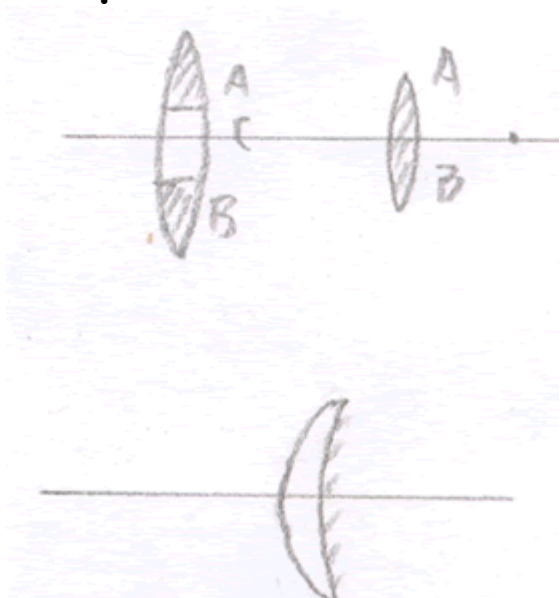
$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

$$\therefore \frac{1}{s'} = \frac{1}{s} + \frac{1}{f'} = \frac{1}{10} + \frac{1}{-5} = -\frac{1}{10}$$

$$s' = -10$$

$$\therefore \beta = \frac{y'}{y} = \frac{s'}{s}$$

$$\therefore y' = \frac{s'}{s} y = \frac{-10}{-5} \times 0.5 = 1 \text{ cm}$$



式中  $y'$  和  $y's'$  分别为点光源 P 及其像点  $P'_B$  离开透镜 B 主轴的距离，

虚线  $P'_B$  在透镜 B 的主轴下方 1cm 处，也就是在题中光学系统对称轴下方 0.5 的地方

同理，点光源 P 通过透镜 A 所成的像  $P'_A$ ，在光学系统对称轴上方 0.5 的处，距离透镜 A

的光心为 10cm，其光路图 S 画法同上。值得注意的是  $P'_A$  和  $P'_B$  构成了相干光源

22.证：经第一界面折射成像：

$$\therefore \frac{n'}{s'} - \frac{n}{s} = \frac{n' - n}{r}$$

$$n' = 1.5, \quad n = 1, \quad r = r_1 = 5\text{cm}, \quad s' = s'_1$$

$$\therefore \frac{n'}{s'_1} = \frac{n' - n}{r_1} + \frac{n}{s} \quad \text{即: } \frac{1.5}{s'_1} = \frac{1.5 - 1}{5} + \frac{1}{s}$$

$$\therefore \frac{1}{s'_1} = \frac{1}{15} + \frac{1}{1.5s}$$

经第二界面（涂银面）反射成像：

$$\therefore \frac{1}{s'} + \frac{1}{s} = \frac{2}{r}$$

$$s' = s'_2 \quad s' = s'_1 \quad r = r_1 = 15\text{cm}$$

$$\therefore \frac{1}{s'} = \frac{2}{r_2} - \frac{1}{s} = \frac{2}{15} - \left( \frac{1}{15} + \frac{1}{1.5s} \right) = \frac{1}{15} - \frac{1}{1.5s}$$

再经第一界面折射成像

$$\therefore \frac{n'}{s'} - \frac{n}{s} = \frac{n' - n}{r}$$

$$n = 1.5, \quad n' = 1, \quad r = r_1 = 5\text{cm}, \quad s' = s'_3, \quad s = s'_1$$

$$\therefore \frac{1}{s'_3} = \frac{1 - 0.5}{r_1} + \frac{1.5}{s'_2} = \frac{1 - 1.5}{5} + 1.5 \times \left( \frac{1}{15} + \frac{1}{1.5s} \right)$$

$$\text{即: } \frac{1}{s'_3} = -0.1 + 0.1 - \frac{1}{s} = -\frac{1}{s}$$

$$\therefore s'_3 = -s$$

$$\beta = \frac{s'}{s}$$

而三次的放大率由  $s$  分别得

$$\beta_1 = \frac{s'_1}{s_1} = \frac{s'_1}{-s} \quad \beta_2 = \frac{s'_2}{s'_1} \quad \beta_3 = \frac{s'_3}{s'_2}$$

$$\therefore \beta = \beta_1 \cdot \beta_2 \cdot \beta_3 = \frac{s'_1}{-s} \cdot \frac{s'_2}{s'_1} \cdot \frac{s'_3}{s'_2} = \frac{s'_3}{-s} = \frac{-s}{-s} = 1$$

又 $\because$ 对于平面镜成像来说有：

$$s' = -s, \quad \beta = 1$$

可见，当光从凸表面如射时，该透镜的成像和平面镜成像的结果一致，故该透镜作用相当于一个平面镜  
证毕。

23.解：

依题意所给数据均标于图中

由于直角棱镜的折射率  $n=1.5$ ，其临界角

$$i_c = \sin^{-1} \frac{n_2}{n_1} = \sin^{-1} \frac{1}{1.5} = 42^\circ < 45^\circ,$$

故，物体再斜面上将发生全反射，并将再棱镜左侧的透镜轴上成虚像。

有考虑到像似深度，此时可将直角棱镜等价于厚度为  $h=6\text{cm}$  的平行平板，

由于  $P_{164-166} L3-1$  的结果可得棱镜所成像的位置为：

$$\Delta h = h(1 - \frac{1}{n}) = 6 \times (1 - \frac{1}{1.5}) = 2 \text{ cm}$$

故等效物距为：

$$s_1 = -[6 + (6 - 2) + 10] = -20 \text{ cm}$$

对凹透镜来说：

$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

$$\text{即：} \frac{1}{s'_1} = -\frac{2}{f'_2} + \frac{1}{s_1} = \frac{1}{20} + \frac{1}{-20} = 0$$

$\therefore s'_1 = \infty$ , 即将成像无限远处。

对凸透镜而言，

$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

$$\text{即：} \frac{1}{s'_2} = \frac{1}{f'_2} - \frac{1}{s'_1} = \frac{1}{-10} - 0,$$

$$\therefore s'_2 = -10 \text{ cm}$$

即在凹透镜左侧 10cm 形成倒立的虚像，其大小为

$$\therefore \beta = \frac{y'}{y} = \frac{s'}{s} \quad \beta_1 = \frac{s'_1}{s_1} \quad \beta_2 = \frac{s'_2}{s_2} = \frac{s'_2}{s'_1}$$

$$\therefore \beta = \beta_1 \cdot \beta_2 = \frac{s'_1}{s'_1} \cdot \frac{s'_2}{s_1} = \frac{s'_2}{s_1} = \frac{-10}{-20} = \frac{1}{2}$$

$$\text{故: } y' = \beta y = \frac{1}{2} \times 1 = 0.5 \text{ (cm)}$$

$$\text{or} \because s_1 = -f'_1 = -20 \text{ cm}$$

$$s'_2 = f'_2 = -10 \text{ cm}$$

$$\text{即: } \beta = \frac{s'_2}{s_1} = \frac{f'_2}{f'_1}$$

$$\therefore y' = \beta y = \left| \frac{f'_2}{f'_1} \right| y = 0.5 \text{ (cm)}$$

24.解:

$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'} , \quad s_2 = d - s'_1$$

$$\therefore \frac{1}{s'_2} = \frac{1}{f'_2} + \frac{1}{s_2} = \frac{1}{f'_2} + \frac{1}{d - s'_1},$$

$$\text{即: } \frac{1}{25} = \frac{1}{3} + \frac{1}{-(20 - s'_1)}$$

$$\frac{1}{20 - s'_1} = \frac{1}{3} - \frac{1}{25} = \frac{22}{75}$$

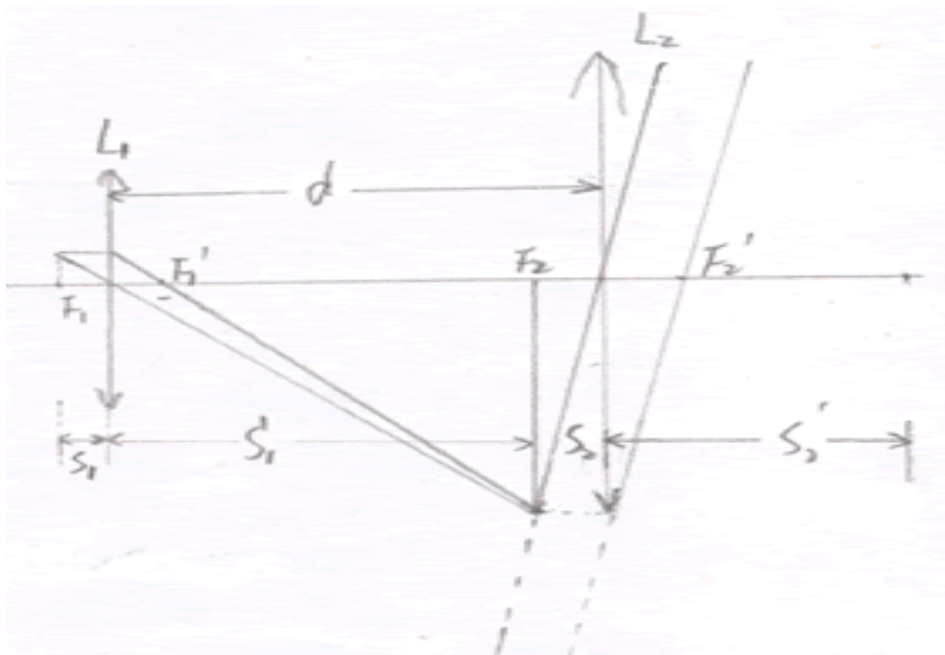
$$20 - s'_1 = \frac{75}{22} \approx -3.4 \quad (cm)$$

$$\therefore s'_1 = 20 - 3.4 = 16.6 cm$$

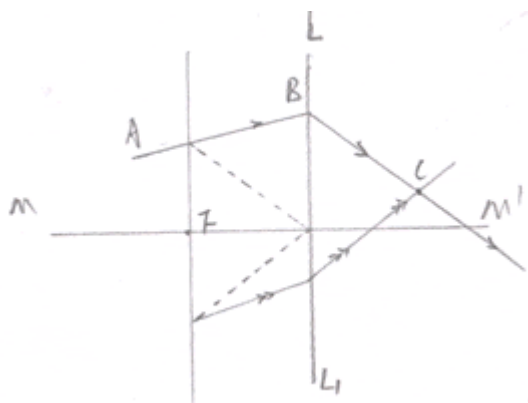
$$\text{又} \therefore \frac{1}{s_1} = \frac{1}{s'_1} - \frac{1}{f'_1} = \frac{1}{16.6} - \frac{1}{1} = -\frac{15.6}{16.6} \quad (\approx -0.96)$$

$$\therefore s = s_1 = -\frac{16.6}{15.6} \approx -1.06 \quad (cm) \quad (\approx -1.0638)$$

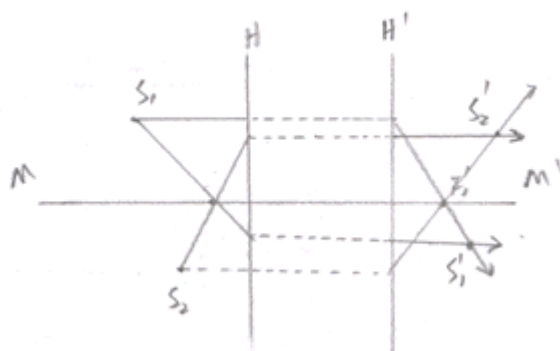
其光路图如下：



25.解：



26.解:



27.解:

经第一界面折射成像:

$$\therefore \frac{n'}{s'} - \frac{n}{s} = \frac{n' - n}{r}$$

$$n' = 1.5, \quad n = 1, \quad r_1 = 10\text{cm}, \quad s_1 = -20\text{cm}$$

$$\therefore \frac{n'}{s'_1} = \frac{n' - n}{r_1} + \frac{n}{s_1}$$

$$\text{即: } \frac{1.5}{s'_1} = \frac{1.5 - 1}{10} + \frac{1}{-20} = \frac{0.5}{10} + \frac{1}{-20} = 0$$

$\therefore s'_1 \rightarrow \infty$ , 即折射光束为平行光束。

经第二界面(涂银面)反射成像:

$$\therefore \frac{1}{s'} + \frac{1}{s} = \frac{2}{r}$$

$$s_2 = s'_1 \rightarrow \infty \quad r_2 = -15\text{cm}$$

$$\therefore \frac{1}{s'_2} = \frac{r_2}{2} = \frac{-15}{2} = -7.5 \quad (\text{cm})$$

再经第一界面折射成像



$$\therefore \frac{n'}{s'} - \frac{n}{s} = \frac{n' - n}{r}$$

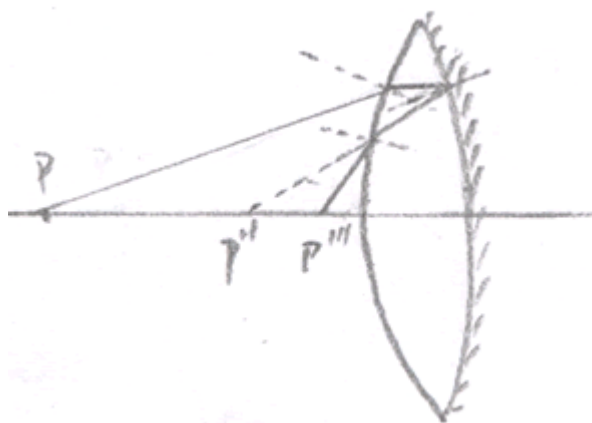
$$n = 1.5, \quad n' = 1, \quad r_1 = 10\text{cm}, \quad s'_2 = s_3 = -7.5\text{cm}$$

$$\therefore \frac{n'}{s'_3} = \frac{n' - n}{r_1} + \frac{n}{s_3}$$

$$\text{即: } \frac{1}{s'_3} = \frac{1 - 0.5}{10} + \frac{1.5}{-7.5} = -\frac{0.5}{10} - \frac{1.5}{7.5} = -0.25$$

$$\therefore s'_3 = -4 \quad (\text{cm})$$

即最后成像于第一界面左方 4cm 处



28.解:

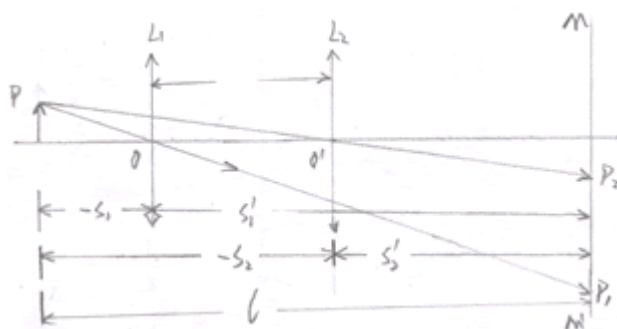
依题意作草图如下:

$$\text{令 } s'_2 = x,$$

$$\text{则 } s_1 = l - (d + x)$$

$$s_2 = l - x$$

第一次成像:



$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'},$$

$$\therefore \frac{1}{s'_1} - \frac{1}{s_1} = \frac{1}{f'}$$

$$\text{即: } \frac{1}{d+x} - \frac{1}{[l-(d+x)]} = \frac{1}{f'}$$

$$\frac{[l-(d+x)] + (d+x)}{(d+x)[l-(d+x)]} = \frac{1}{f'}$$

$$\therefore f' = \frac{(d+x)[l-(d+x)]}{l} \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

第二次成像:

$$\therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'},$$

$$\therefore \frac{1}{s'_2} - \frac{1}{s_2} = \frac{1}{f'}$$

$$\text{即: } \frac{1}{x} - \frac{1}{(l-x)} = \frac{1}{f'}, \quad \frac{x+(l-x)}{x(l-x)} = \frac{1}{f'}$$

$$f' = \frac{(l-x)x}{l} \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

由 (1) (2) 得:

$$(d+x)[l-(d+x)] = x(l-x)$$

$$dl + xl - d^2 - xd - dx - x^2 = xl - x^2$$

$$l - d - 2x = 0$$

$$\therefore x = \frac{l-d}{2} \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

$$\text{即: } s_1 = l - (d + x) = (l - d) - \frac{l - d}{2} = \frac{l - d}{2} \quad \dots$$

$$s'_1 = d + x = d + \frac{l - d}{2} = \frac{l + d}{2} \quad \dots$$

$$s_2 = l - x = l - \frac{l - d}{2} = \frac{l + d}{2} \quad \dots$$

$$s'_2 = lx = \frac{l - d}{2} \quad \dots(4)$$

1) 求两次象的大小之比:

$$\therefore \beta = \frac{y'}{y} = \frac{s'}{s} \quad \text{即 } \beta_1 = \frac{y'_1}{y_1} = \frac{\frac{l + d}{2}}{\frac{l - d}{2}} = \frac{l + d}{l - d}$$

$$\beta_2 = \frac{y'_2}{y_2} = \frac{s'_2}{s_1} = \frac{\frac{l - d}{2}}{\frac{l + d}{2}} = \frac{l - d}{l + d}$$

$$\therefore \frac{\beta_2}{\beta_1} = \frac{\frac{l - d}{l + d}}{\frac{l + d}{l - d}} = \left(\frac{l - d}{l + d}\right)^2$$

$$\text{又 } \therefore \frac{y'_2}{y'_1} = \frac{y_2}{y_1} \cdot \frac{\beta_2}{\beta_1} \quad \text{而 } y_2 = y_1 = y$$

故两次像的大小之比为:

$$\frac{y'_2}{y'_1} = \frac{\beta_2}{\beta_1} = \left(\frac{l - d}{l + d}\right)^2 \quad \dots(5)$$

2)

$$\text{证 } f' = \frac{(l^2 - d^2)}{4l}$$

将 (3) 代入 (4) :

$$f' = \frac{(\frac{d}{2} + \frac{l-d}{2})[(1-d) - \frac{l-d}{2}]}{l} = \frac{\frac{l-d}{2} \cdot \frac{l+d}{2}}{l} = \frac{l^2 - d^2}{4l}$$

或将 (3) 代入 (2) :

$$f' = \frac{(\frac{l-d}{2})(1 - \frac{l-d}{2})}{l} = \frac{\frac{l-d}{2} \cdot \frac{l+d}{2}}{l} = \frac{l^2 - d^2}{4l}$$

$$\text{故有 } f' = \frac{l^2 - d^2}{4l} \quad \text{得证}$$

3)

$$\text{证 } l > 4f'$$

$$\text{由 (6) 得: } l^2 - d^2 = 4lf' \quad d^2 = l^2 - 4lf' = l(l - 4f')$$

$$\therefore d = \sqrt{l(l - 4f')} \quad \dots \dots (7)$$

可见: 若  $l < 4f'$ , 则  $d$  无解, 即得不到对实物能成实像的透镜位置

若  $l = 4f'$ , 则  $d=0$ , 即透镜在 E 中央, 只有一个成像位置,

$$\beta = -1$$

若  $l > 4f'$ , 则可有两个成像位置。

故, 欲使透镜成像, 物和屏的距离  $l$  不能小于透镜焦距的 4 倍  
但要满足题中成两次清晰的像, 则必须有

$$l > 4f' \quad \text{证毕。}$$

注 : 当  $l = 4f'$  时, 有  $d = 0$ , 则

$$x = \frac{d}{2} \quad s_1 = \frac{l}{2} \quad s_2 = \frac{l}{2} \quad s'_1 = \frac{l}{2} \quad s'_2 = \frac{l}{2}。$$

即只有能成一个像的位置。

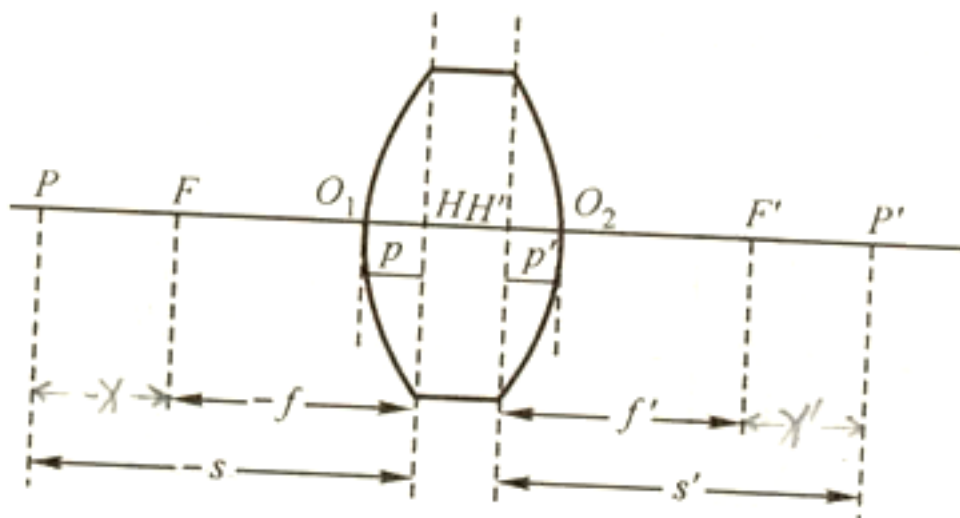
29.解:

$$\therefore xx' = ff'$$

由 (6) 得(作草图如下)

$$f = -60\text{cm}$$

$$f' = 60\text{cm}$$



$\therefore$  (1)当  $x_1 = -20\text{mm}$  时, 有

$$x'_1 = \frac{ff'}{x_1} = \frac{60 \times (-60)}{-20} = 180\text{mm}$$

$$s'_1 = f' + x' = 60 + 180 = 240\text{mm} \quad (p', \text{实像})$$

(2)当  $x_2 = 20\text{mm}$  时, 有

$$x'_2 = \frac{ff'}{x_2} = \frac{60 \times (-60)}{20} = -180\text{mm}$$

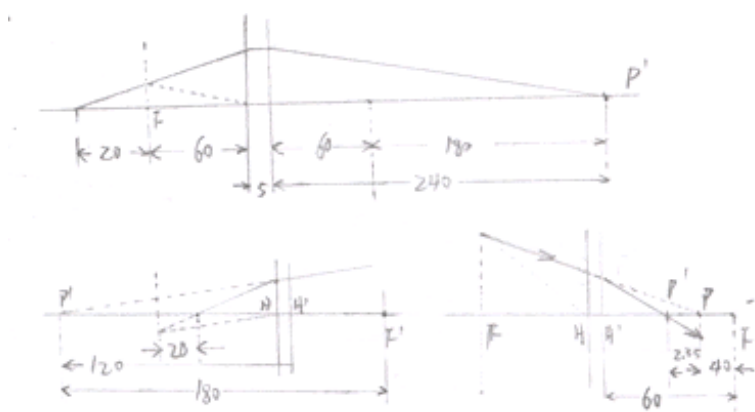
$$s'_2 = f' + x' = 60 + (-180) = -120\text{mm} \quad (p', \text{虚像})$$

(3)当  $x_3 = 60 + 5 + 20 = 85\text{mm}$  时, 有

$$x'_3 = \frac{ff'}{x_3} = \frac{60 \times (-60)}{85} \approx -42.35\text{mm}$$

$$s'_3 = f' + x' = 60 + (-42.35) = 17.65\text{mm} \quad (p', \text{实像})$$

其光路图分别如下:



30.解:

$$\because \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}, \quad f = f'$$

$\therefore$  复合光学的焦距为:

$$\frac{1}{f'} = \frac{1}{s'} - \frac{1}{s} = \frac{1}{60} - \frac{1}{80} = \frac{7}{240}$$

$$\text{即 } f' = \frac{240}{7} \approx 34.29 \text{ (cm)}$$

$$\text{又 } \because \frac{1}{f'} = \frac{1}{f'_1} - \frac{1}{f'_2} - \frac{d}{f'_1 f'_2} \quad \text{及 } d = 0$$

$$\text{即: } \frac{1}{f'} = \frac{1}{f'_1} + \frac{1}{f'_2}$$

$$\therefore \frac{1}{f'} = \frac{1}{f'_1} + \frac{1}{f'_2} = \frac{7}{240} - \frac{1}{10} = -\frac{17}{240}$$

$$\text{故: } f'_2 = -\frac{240}{17} \approx -14.12$$

31.解:

$$\begin{aligned}
\therefore \frac{1}{f'} &= (n-1) \left[ \frac{1}{r_1} - \frac{1}{r_2} + \frac{t(n-1)}{n r_1 r_2} \right] \\
&= (1.5-1) \left[ \frac{1}{100} - \frac{1}{-200} + \frac{10 \times (1.5-1)}{1.5 \times 100 \times (-200)} \right] \\
&= 0.5 \times [0.01 + 0.005 - 0.00017] \\
&= 0.5 \times 0.01483 = 0.007415
\end{aligned}$$

$$\therefore f' \approx 134.86 \text{ (mm)}$$

$$f = -f' = -134.86 \text{ mm}$$

$$\text{又} \because \frac{1}{f'_1} = \frac{n-1}{r_1} = \frac{1.5-1}{100} = \frac{0.5}{100} = \frac{1}{200}, \text{ 即 } f'_1 = 200(\text{mm})$$

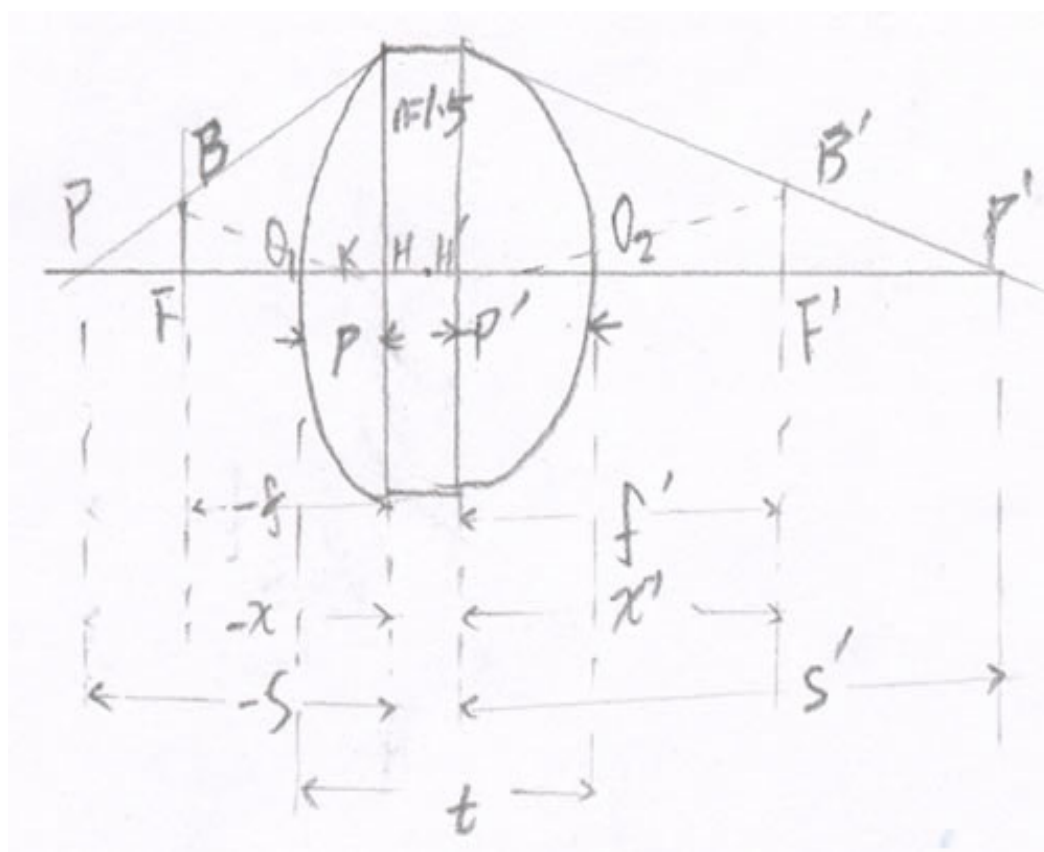
$$\frac{1}{f'_2} = \frac{n-1}{r_2} = \frac{1.5-1}{-200} = -\frac{1}{400}, \text{ 即 } f'_2 = -400(\text{mm})$$

$$\therefore p = \frac{t f'}{n f_2} = \frac{10 \times 134.86}{1.5 \times (-400)} = 2.2477(\text{mm})$$

$$p' = -\frac{t f'}{n f'_1} = -\frac{10 \times 134.86}{1.5 \times 200} = -4.495(\text{mm})$$

$$x = f' = 134.86 \text{ mm} \quad x' = f = -134.86 \text{ mm}$$

其草图绘制如下



32.解:  
(1)



$$\therefore \frac{1}{f'} = \frac{1}{f'_1} + \frac{1}{f'_2} - \frac{d}{f'_1 f'_2}$$

$$f'_1 = f'_2 = 2\text{cm}$$

$$d = \frac{4}{3}$$

$$\text{即: } \frac{1}{f'} = \frac{1}{2} + \frac{1}{2} - \frac{\frac{4}{3}}{2 \times 2} = 1 - \frac{1}{3} = \frac{2}{3}$$

$\therefore$

$$f' = \frac{3}{2} = 1.5(\text{cm})$$

$$\therefore p = \frac{f'd}{f_2} = \frac{f'd}{f'_2} = \frac{1.5 \times \frac{4}{3}}{2} = 1(\text{cm})$$

$$p' = -\frac{f'd}{f'_1} = -\frac{1.5 \times \frac{4}{3}}{2} = -1(\text{mm})$$

$$x = \overline{FK} = f' = 1.5\text{mm} \quad x' = f = \overline{F'K'} = -1.5\text{mm}$$

(2)

$$\therefore f'_1 = 6(\text{cm}), \quad f'_2 = 2(\text{cm}), \quad d = 4(\text{cm})$$

$$\therefore \frac{1}{f'} = \frac{1}{6} + \frac{1}{2} - \frac{4}{6 \times 2} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$\text{即} \quad f' = 3(\text{cm}) \quad f = -f' = -3(\text{cm})$$

$$p = \frac{3 \times 4}{2} = 6(\text{cm}),$$

$$p' = -\frac{3 \times 4}{6} = -2(\text{cm})$$

$$x = f' = 3\text{cm} \quad x' = f = -3\text{cm}$$

33.解:

$$\because f'_1 = 20\text{cm} \quad f'_2 = -20\text{cm} \quad d = 6\text{cm}$$

$$(1) \quad \frac{1}{f'} = \frac{1}{f'_1} + \frac{1}{f'_2} - \frac{d}{f'_1 f'_2} = \frac{1}{20} + \frac{2}{-20} - \frac{6}{20 \times (-20)} \\ = \frac{3}{200}$$

$\therefore$

$$f' = \frac{200}{3}(\text{CM}) = \frac{2}{3}(\text{m}) \quad f = -f' = -\frac{2}{3}$$

$$\therefore p = \frac{f'd}{f_2} = \frac{f'd}{f'_2} = \frac{\frac{2}{3} \times 6}{-20} = -0.2(\text{m})$$

$$p' = -\frac{f'd}{f'_1} = \frac{\frac{2}{3} \times 6}{-20} = -0.2(\text{m})$$

$$(2) \text{ 又 } \because s = \bar{s} - p = -0.30 - (-0.20) = -0.10(\text{m})$$

$$\text{而 } \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

$$\text{即: } \frac{1}{s'} = \frac{1}{s} + \frac{1}{f'} = \frac{1}{\frac{2}{3}} + \frac{1}{-0.10} = 1.5 - 10 = -8.5$$

$$\therefore s' \approx -0.117647 \approx -0.118(\text{cm})$$

$$\beta = \frac{s'}{s} = \frac{-0.118}{-0.10} = 1.18$$

注：该体也可用光焦度（ $\Phi$ ）发计算，  
也可用逐次像发.....

34.解:

$$\because \frac{1}{s'} + \frac{1}{s} = 1$$

$$f' = 6cm$$

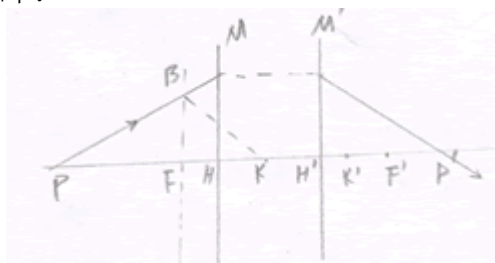
$$f = -5cm$$

$$s = -20cm$$

$$\text{即: } \frac{6}{s'} + \frac{-5}{-20} = 1 \quad \frac{6}{s'} = 1 - \frac{5}{20} = \frac{3}{4}$$

$$\therefore s' = \frac{6 \times 4}{3} = 8cm$$

其光路图如下:



35.解: (1) 由折射定律:

$$n \sin \alpha = \sin \beta$$

$$\text{所以 } \alpha = \sin^{-1}(\sin \phi / n)$$

$$\text{又 临界角 } \alpha_c = \sin^{-1}(1/n)$$

即  $\alpha < \alpha_c$  故是部分反射。

$$(2) \text{ 由图知: } \alpha = (\phi - \alpha) + \theta, \text{ 即 } \theta = 2\alpha - \phi,$$

$$\text{而 } \delta = \pi - 2\theta, \text{ 所以 } \delta = \pi - 4\alpha + 2\phi.$$

$$(3) \text{ 因为 } d\delta/d\phi = -4d\alpha/d\phi + 2 = 0, \text{ 即: } d\alpha/d\phi = 1/2,$$

$$\text{而: } \alpha = \sin^{-1}(\sin \phi / n), d\sin^{-1}x/dx = 1/(1-x^2)^{1/2}.$$

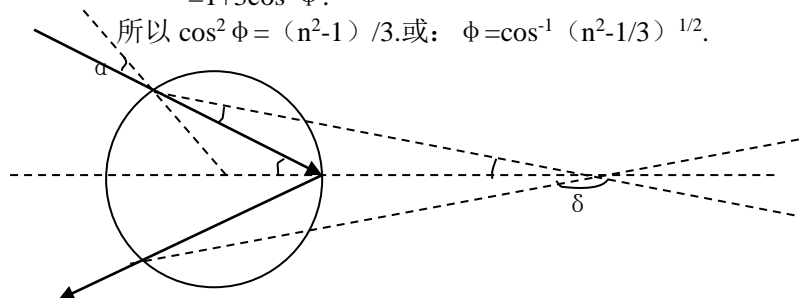
$$\text{即: } d\alpha/d\phi = \cos \phi / n(1 - \sin^2 \phi / n^2)^{1/2} = 1/2,$$

$$1 - \sin^2 \phi / n^2 = 4\cos^2 \phi / n^2$$

$$1 = \sin^2 \phi / n^2 + \cos^2 \phi / n^2 + / n^2$$

$$= 1 + 3\cos^2 \phi.$$

$$\text{所以 } \cos^2 \phi = (n^2 - 1) / 3. \text{ 或: } \phi = \cos^{-1} (n^2 - 1 / 3)^{1/2}.$$



36. 因为  $n'/s' - n/s = (n' - n)/r$ .

(1) 1 因为  $n' = 1.5, n = 1, s_1 = r_1 = 4(cm)$

$$\text{所以 } 1.5/s'_1 - 1/4 = (1.5 - 1)/4, 1.5/s'_1 = 1/4 + 0.5/4 = 3/8.$$

所以  $s_1' = 8 \times 1.5/3 = 4(\text{cm})$ . 即在球心处。

2 因为  $n' = 1, n = 1, s_2 = s' + (9-8)/2 = 4.5\text{cm}$ .

所以  $1/s_2' - 1/s_1' = 0, s_2' = s_2 = 4.5\text{cm}$ . 即像仍在球心处。

(3) 1 因为  $n' = 1.33, 1.5, r = 1.5\text{mm}, s = 1\text{mm}$ .

所以  $1.33/s_1' - 1.5/1 = (1.33-1.5)/1.5$ .

1.  $33/s_1' = 1.5 + 1.33/1.5 - 1 = 1.39$ .

所以  $s_1' = 1.33/1.39 = 0.96(\text{mm})$

又  $s_2 = 50 - (1.5 - 0.96) = 49.46(\text{mm})$ .

故  $1/s_2' - 1.33/49.46 = 1 - 1.33/50 \quad s_2 = 0.0203 \quad s_2' = 49.26 (\text{mm})$

所以  $d(\text{内}) = 2r(\text{内}) = 2 \times (50 - 49.26) = 1.48 \approx 1.5 (\text{mm})$

2 由  $n' = 1 \quad n = 1.33 \quad r = 50\text{mm} \quad s = 48.5 (\text{mm})$

所以  $1/s_1' - 1.33/48.5 = 1.33/50 \quad 1/s_1' = 1.33/48.5 + 1/50 - 1.33/50 = 0.0208$

所以  $s' \approx 48.1 (\text{mm}) \quad d(\text{外}) = 2r'(\text{外}) = 2 \times (50 - 48.1) \approx 4 (\text{mm})$



(2) 1  $\because n' = 1.5 \quad n = 1.0 \quad r_1 = 4\text{cm}$

$s_1 = 4 - 0.15 = 3.85\text{cm}$

$\therefore 1.5/s_1' - 1/3.85 = (1.5-1.0)/4$

$1.5/s_1' = 1/3.85 + 0.5/4 \approx 0.385$

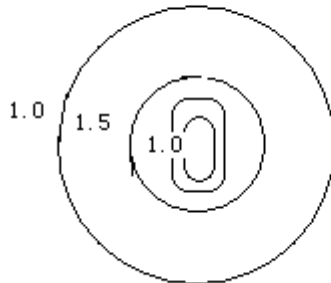
$\therefore s_1' = 1.5/0.385 \approx 3.896(\text{cm})$

2 又  $\because n' = 1.0 \quad n = 1.5\text{cm} \quad s_2 = 3.896 + 0.5 = 4.396(\text{cm})$

$\therefore 1/s_2' - 1.5/4.396 = (1-1.5)/4.5 \quad 1/s_2' = 1.5/4.396 - 0.5/4.5 \approx 0.23$

$\therefore s_2' \approx 4.348(\text{cm})$

$d = 2 \times (4.5 - 4.348) \approx 0.304(\text{cm}) \approx 3\text{mm}$



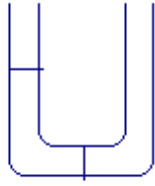
37. (1) 证:  $\because$  物像具有等光程性,

即:  $sl_1ps_1 = \Delta so_1o_2s_2s_1$

$\Delta sl_2s_2 = \Delta so_1o_2s_2$

$\Delta sl_1p = \Delta sl_1ps_1 - \Delta ps_1 = \Delta sl_1ps_1 - ps_1$





38.  $\because d \ll a \quad d \ll b,$

该玻璃板可视为薄透镜，且是近轴光线。

圆板中心处的折射率为  $n(0)$ ,

半径为  $r$  处的折射率为  $n(r)$ ,

则由物像之间的等光程性知：

$$n_1 L + n_2 L' = n_1 a + n(0)d + n_2 b,$$

$$\text{而： } n_1 = n_2 = 1 \quad L = (a^2 + r^2)^{1/2} \quad L' = (b^2 + r^2)^{1/2}$$

$$\text{即： } (a^2 + r^2)^{1/2} + n(r)d + (b^2 + r^2)^{1/2} = a + b + n(0)d$$

$$\therefore n(r)d = n(0)d + a + b - (a^2 + r^2)^{1/2} - (b^2 + r^2)^{1/2}$$

$$\text{故 } n(r) = n(0) + \{a + b - (a^2 + r^2)^{1/2} - (b^2 + r^2)^{1/2}\} / d$$

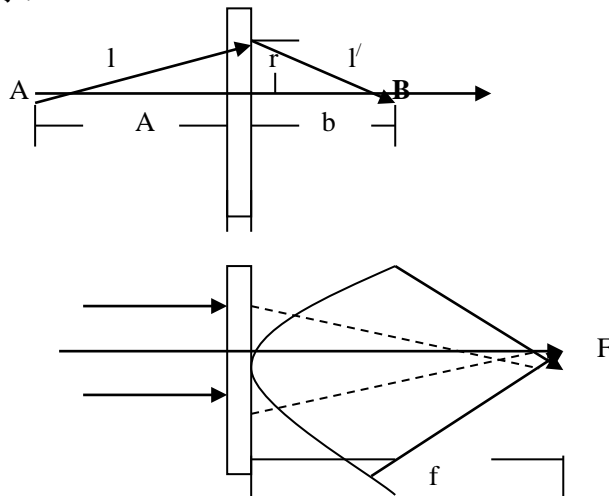
讨论：若为平行光照射时，且折射后会聚于焦点  $F$ ,

$$\text{则有 } n(r)d + (f^2 + r^2)^{1/2} = n(0)d + f$$

$$\text{即： } n(r) = n(0) + \{f - (f^2 + r^2)^{1/2}\} / d.$$

$$\text{当 } d \ll f \text{ 时，有： } n(r) = n(0) - r^2 / 2df.$$

图示：



$$39. (1) \because n'/s_1' - n/s_1 = (n' - n)/r_1$$

$$n' = 1.5, \quad n = 1.0, \quad s_1 = -40\text{cm}, \quad r_1 = -20\text{cm}$$

$$\therefore 1.5/s_1' = 1/(-40) + (1.5 - 1.0)/(-20) = -1/20,$$

$$s_1' = -20 \times 1.5 = -30(\text{cm}).$$

$$(2) \because 1/s_2' + 1/s_2 = 2/r_2, \quad s_2 = s_1' = -30\text{cm}, \quad r_2 = -15\text{cm}$$

$$\therefore 1/s_2' = 2/r_2 - 1/s_2 = 2/(-15) - 1/(-30) = -1/10, \quad s_2' = -10(\text{cm}).$$

$$(3) \because n'/s_3' - n/s_3 = (n' - n)/r_1 \quad s_3 = s_2' = -10\text{cm} \quad r_1 = -20\text{cm}$$

$$n' = 1.0 \quad n = 1.5.$$

$$\therefore 1/s_3' = (1.0 - 1.5)/(-20) + 1.5/(-10) = -1/8$$

$$(4) \because \beta = \beta_1 \beta_2 \beta_3, \quad \beta = y'/y = ns/n's.$$

$$\beta_1 = ns_1'/n's_1 = 1/2$$

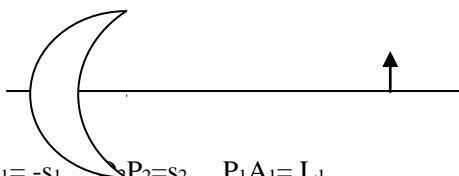
$$\beta_2 = ns_2'/(-n's_2) = -s_2'/s_2 = -1/3$$

$$\beta_3 = ns_3/n's_3 = 6/5.$$

$$\therefore \beta = 1/2 \times (-1/3) \times 6/5 = -1/5 = -0.2.$$

故最后像在透镜左方 8cm 处，为一大小是原物的 0.2 倍倒立缩小实像。

图示：



40. 证：  $\because O_1P_1 = -s_1, \quad P_2 = s_2, \quad P_1A_1 = L_1,$   
 $A_2P_2 = L_2, \quad A_1M = A_2N = h, \quad O_1O_2 = d.,$   
 $L_1 = \{[(-s_1) + O_1M]^2 + h^2\}^{1/2},$   
 $L_2 = \{[s_2 + O_2N]^2 + h^2\}^{1/2}.$

在近轴条件下， $O_1M \ll R_1, \quad O_2N \ll -R_2$  即：  $O_1M \approx h^2/2R_1$

$$O_2N \approx h^2/2(-R_2).$$

$$\therefore \Delta P_1A_1A_2P_2 = n_1L_1 + n[d - O_1M - O_2N] + n_2L_2$$

$$= n_1\{[(-s_1) + O_1M]^2 + h^2\}^{1/2} + n[d - O_1M - O_2N] + n_2\{[s_2 + O_2N]^2 + h^2\}^{1/2}$$

$$= n_1\{[-s_1 + h^2/2R_1]^2 + h^2\}^{1/2} + n[d - h^2/2R_1 - h^2/2(-R_2)] + n_2\{[s_2 + h^2/2(-R_2)]^2 + h^2\}^{1/2}$$

当  $A_1$  点在透镜上移动时， $R_1$  和  $R_2$  是常量， $h$  是常量，根据费马原理，

对  $h$  求导，并令其等于 0，即  $d\Delta P_1A_1A_2P_2/dh = 0$ ，得：

$$n_1\{[-s_1 + h^2/2R_1]h/R_1 + h\}/L_1 - nh/R_1 - nh/(-R_2) + n_2\{[s_2 + h^2/2(-R_2)]h/(-R_2) + h\}/L_2 = 0.$$

$\therefore$  在近轴条件下， $h \ll R_1, \quad h \ll (-R_2), \quad L_1 \approx -s_1, \quad L_2 \approx s_2$ ，并略去  $h^2$  项，得：

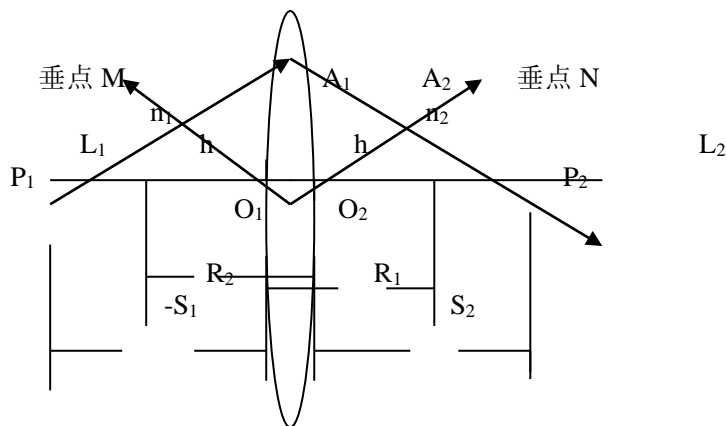
$$h[n_2/s_2 - n_1/s_1 - (n - n_1)/R_1 + (n_2 - n)/R_2] = 0,$$

$$\text{即： } n_2/s_2 - n_1/s_1 = (n - n_1)/R_1 + (n_2 - n)/R_2 = \phi.$$

$$\text{又 } \because f_1 = \lim_{s_2 \rightarrow \infty} s_2 = -n_1/\phi, \quad f_2 = \lim_{s_1 \rightarrow -\infty} s_1 = n_2/\phi$$

$$\therefore f_1/s_1 + f_2/s_2 = 1$$

得证。



1.解：  $f = -\frac{n}{n' - n} \cdot r = -\frac{1}{\frac{4}{3} - 1} \times 5.55$

$$= -3 \times 5.55 \div -16.65 \text{ (mm)} \div -1.67 \text{ (cm)}$$

$$f' = \frac{n'}{n' - n} \cdot r = \frac{\frac{4}{3}}{\frac{4}{3} - 1} \times 5.55$$

$$= 4 \times 5.55 = 22.20 \text{ (mm)} = 2.22 \text{ (cm)}$$

$$\therefore y' = d \cdot \theta' \quad . \quad n' \theta' = n \theta \quad (\text{折射定理}), \quad d = f'$$

$$\therefore y' = f' \cdot \frac{n}{n'} \theta = 2.22 \times \frac{1}{\frac{4}{3}} \times \frac{\pi}{180} \div 0.029 \text{ (cm)}$$

$$2. \text{解: (1).} \quad \therefore \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

$$\therefore \frac{1}{f'_{\text{远}}} = \frac{1}{2} - \frac{1}{-300} = \frac{151}{300}$$

$$f'_{\text{远}} = \frac{300}{151} \div 1.987 \text{ (cm)}$$

$$\frac{1}{f'_{\text{近}}} = \frac{1}{2} - \frac{1}{-100} = \frac{151}{100}$$

$$f'_{\text{近}} = \frac{100}{51} \div 1.961 \text{ (cm)}$$

(2) 此人看不清 1m 以内的物体, 表明其近点在角膜前 1m 出, 是远视眼, 应戴正光焦度的远视镜。要看清 25cm 处的物体, 即要将近点矫正到角膜前 0.25m (即 25cm)

处, 应按  $s' = -1.0m$  (即 -100cm) 和  $s = -0.25m$  (即 -25cm) 去选择光焦度。

$$\therefore \Phi = \frac{1}{f'} = \frac{1}{s'} - \frac{1}{s}$$

$$= \frac{1}{-1.0} - \frac{1}{-0.25}$$

$$= -1 + \frac{100}{25}$$

$$= +3.0 \text{ (D)} = +300 \text{ 度}$$



即眼镜的光焦度  $\Phi$  为+3.0 (D) (屈光度), 在医学上认为这副眼镜为 300 度的远视眼镜 ( $3.0 \times 100$ )。

另: 要看清远处的物体, 则:

$$\Phi' = \frac{1}{f'} = \frac{1}{s'} - \frac{1}{s} = \frac{1}{-3.0} - \frac{1}{\infty} = -0.33 D \quad \text{即33度的凹透镜。}$$

3.解:  $\because \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$

当看远物时有  $s_1 \rightarrow \infty, f'_{\max} = s'_1$

当看近物时, 有

$$\begin{aligned} \frac{1}{s'_2} - \frac{1}{s_2} &= \frac{1}{f'} \\ \frac{1}{s_2} &= \frac{1}{s'_2} - \frac{1}{f'} \leq \frac{1}{s'_2} - \frac{1}{s'_1} \\ &= \frac{1}{20} - \frac{1}{18} = -\frac{1}{180} \end{aligned}$$

$$\therefore s_2 \geq -180 (cm)$$

即 目的物在镜前最近不得小于180cm.

4.解:  $\because U = \frac{-y'}{f'_1}$

$$\begin{aligned} \therefore f'_1 &= \frac{-y'}{U} = \frac{-(-1)}{4'} = \frac{+1}{\frac{4}{60} \times \frac{\pi}{180}} \doteq 859.87(mm) \\ &= 85.987 \quad cm \end{aligned}$$

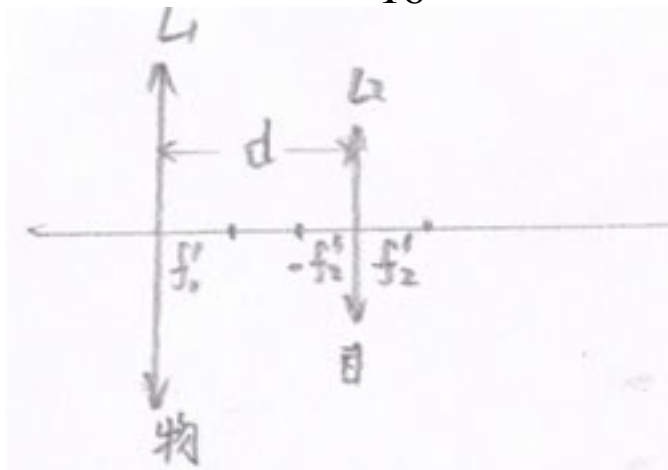
5.解:  $\because M = \beta M' = (-\frac{S'}{f_1})M'$

$$\therefore M_{\max} = \beta_{\max} M'_{\max} = (-\frac{S'}{f_{1\min}})M'_{\max}$$

$$= -\frac{160}{1.9} \times 10 \doteq -842$$

$$M_{\min} = \beta_{\min} M'_{\min} = (-\frac{S'}{f_{1\max}})M'_{\min}$$

$$= -\frac{160}{16} \times 5 = -50$$



6.解:  $\because$  最后观察到的象在无穷远出, 即  $s_2' \rightarrow \infty$ .  
 $\therefore$  经由物镜成象必定在目镜的焦平面上。

$$\text{即: } s_2 = -f'_2 = f_2 \quad (\because \frac{1}{s'_2} - \frac{1}{s_2} = \frac{1}{f'_2}, f'_2 = -f_2)$$

$$\text{故: } s'_1 = d - s_2 = d - f_2 = 22 - 2 = 20 \text{ (cm)}.$$

$$\text{又} \because \frac{1}{s'_1} - \frac{1}{s_1} = \frac{1}{f'_1} \quad \text{即: } \frac{1}{s_1} = \frac{1}{s'_1} - \frac{1}{f'_1}.$$

$$\therefore \frac{1}{s'_1} = \frac{1}{20} - \frac{1}{0.5} = \frac{1}{20} - \frac{40}{20} = \frac{-39}{20}$$

$$s'_1 = -\frac{20}{39} \doteq -0.51 \text{ (cm)}.$$

$$\text{又} \because \beta = \frac{s'_1}{s_1} = -\frac{20}{0.51} = -39.$$

$$M' = \frac{25}{f'} = \frac{25}{2} = 12.5$$

$$\therefore M = \beta M' = -39 \times 12.5 = -487.5$$

$$\text{or: 解: (1) } \because \frac{1}{f'} = \frac{1}{f'_1} + \frac{1}{f'_2} - \frac{d}{f'_1 f'_2} = \frac{1}{0.5} + \frac{1}{2} - \frac{22}{0.5 \times 2} = -19.5$$

$$\text{即: } f' \doteq -0.051 \text{ (cm)}$$

$$\text{而: } \frac{1}{f'} = \frac{1}{s'} - \frac{1}{s}, \quad s' \rightarrow \infty.$$

$$\text{即: } s = -f' = 0.051 \text{ (cm)}$$

$$p = -\frac{f'd}{f_2} = -\frac{-0.051 \times 22}{-2} = -0.561.$$

$$\therefore \bar{s} = s + p = 0.051 \times -0.561 = -0.51 \text{ (cm)}$$

此时是从 0 量起.

$$(2) \quad M = \frac{25}{f'} = 25 \times (-19.5) = -487.5$$

$$\text{或: } M = \frac{-25\Delta}{f'_1 f'_2} = \frac{-25 \times 19.5}{0.5 \times 2} = -487.5$$

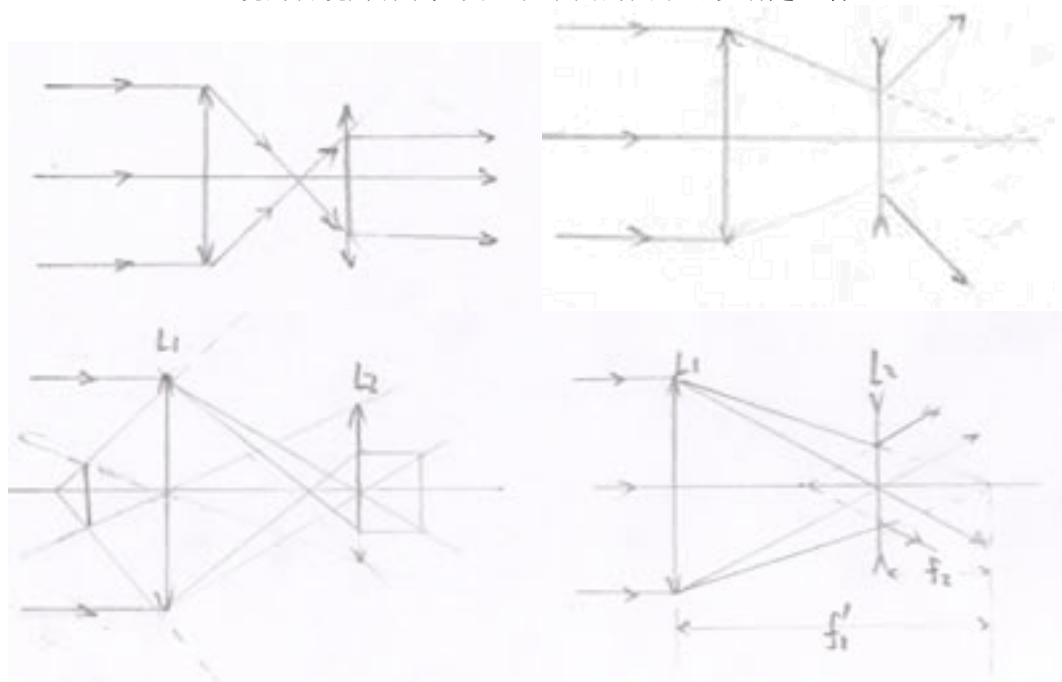
$$\Delta = d - f'_1 + f'_2 = 22 - 0.5 - 2 = 19.5$$

$$\text{或: } M \approx -\frac{\ell}{f'_1} \cdot \frac{25}{f'_2} = -\frac{22}{0.5} \times \frac{25}{2} = -550.$$

7. 证:  $\because$  开氏和伽氏望远镜的物镜都是会聚透镜, 其横向放大率都小于 1, 在物镜和目镜的口径相差不太悬殊的情况下经过物镜边缘的光线, 并不能完全经过目镜, 在整个光具组里, 真正起限制光束作用光圈的是 (会聚透镜)

物镜的边缘。

∴望远镜的物镜为有效光圈（从下面的图中可以清楚地看出。



8. 解：∵有效光阑是在整个光具组的最前面，∴入射光瞳和它重合，其大小就是物镜的口径，位置就是物镜所在处。

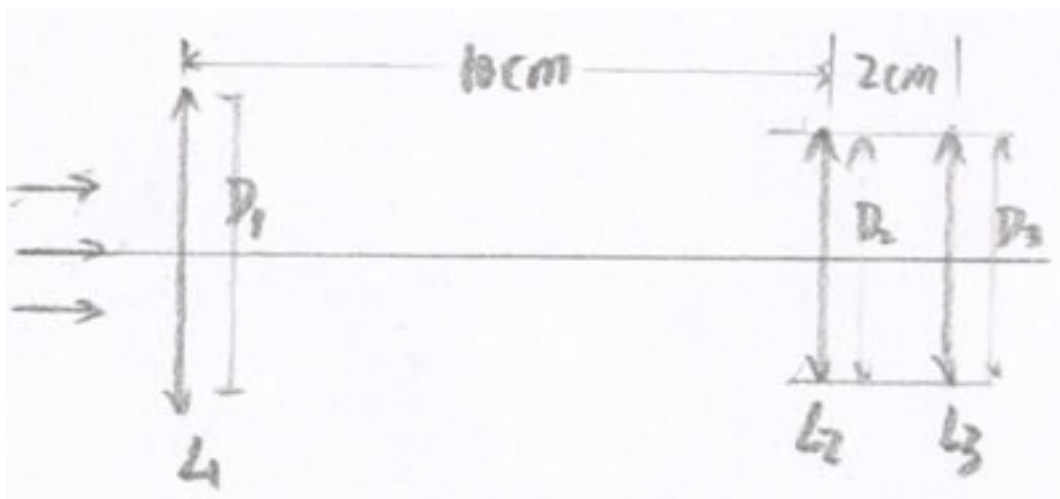
而有效光阑对于后面的光具组所成的象即为出射光瞳

即  $l_1$  对  $l_2$  成的象为出射光瞳。

$$\text{又} \because -s = f_1' + (-f_2), \quad f_1' = -f_2 \quad \text{而} \quad \frac{1}{s'} - \frac{1}{f_2'} + \frac{1}{s} = \frac{1}{-f_2'} - \frac{1}{f' - f}$$

$$\text{即: } s' = \frac{f's}{f' + s} = \frac{(-f_2)(f_2 - f_1')}{(-f_2) + (f_2 - f_1')} = \frac{(f_2 - f_1')}{f'}$$

$$y' = \frac{s'}{s} y = \frac{f_2(f_2 - f_1')/f'}{f_2 - f'} \cdot y = \frac{f_2}{f_1'} y$$



9.解:  $\because L_1$  是该望远镜的有效光阑和入射光瞳, 它被  $L_2$ 、 $L_3$  所成的象为出射光瞳。

$\therefore$  把  $L_1$  对  $L_2$ 、 $L_3$  相继成像, 由物象公式  $\frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$  便可得出出射光瞳的位置。

即:  $\frac{1}{s'_2} - \frac{1}{s_2} = \frac{1}{f'_2}$ ,  $\frac{1}{s'_3} - \frac{1}{s_3} = \frac{1}{f'_3}$

而:  $s_2 = 10 \text{ (cm)}$ ,  $f'_2 = 2 \text{ (cm)}$ ,  $f'_3 = 2 \text{ (cm)}$

$$s_3 = s'_2 - d = s'_2 - 2 \text{ (cm)}$$

$$\frac{1}{s'_2} = \frac{1}{f'_2} + \frac{1}{s_2} = \frac{1}{2} + \frac{1}{-10} = \frac{2}{5}$$

$$s'_2 = \frac{2}{5} = 0.25 \text{ (cm)}$$

$$s'_3 = s'_2 - 2 = 2.5 - 2 = 0.5 \text{ (cm)}$$

$$\frac{1}{s'_3} = \frac{1}{f'_3} + \frac{1}{s_3} = \frac{1}{2} + \frac{1}{0.5} = \frac{5}{2}$$

故  $\therefore s'_3 = \frac{2}{5} = 0.4 \text{ (cm)} = 4 \text{ (mm)}$ .

即 出射光瞳在  $L_3$  的右方  $4\text{mm}$  处。

出射光瞳的大小为:

$$d' = \frac{f'_3}{f'_1} d_1 = \frac{2}{10} \times 4 = 0.8 \text{ (cm)} = 8 \text{ (mm)}$$

or  
将

$$f' = f'_1 f'_2 / (f'_1 - f'_2 - d) = 2\text{cm}$$

$$f = -cc = -2\text{cm}$$

$$p = -fd / f'_2 = 2\text{cm}$$

$$p' = -fd / f'_1 = -2\text{cm}$$

将

$$f' = 2\text{cm}$$

$$f = -2\text{cm}$$

$$s = -12\text{cm}$$

代入

$$f' / s' + f / s = 1$$

得

$$s' = 2.4\text{cm}$$

$$\beta = s' / s = -1/5$$

$$h = 0.8\text{cm}$$

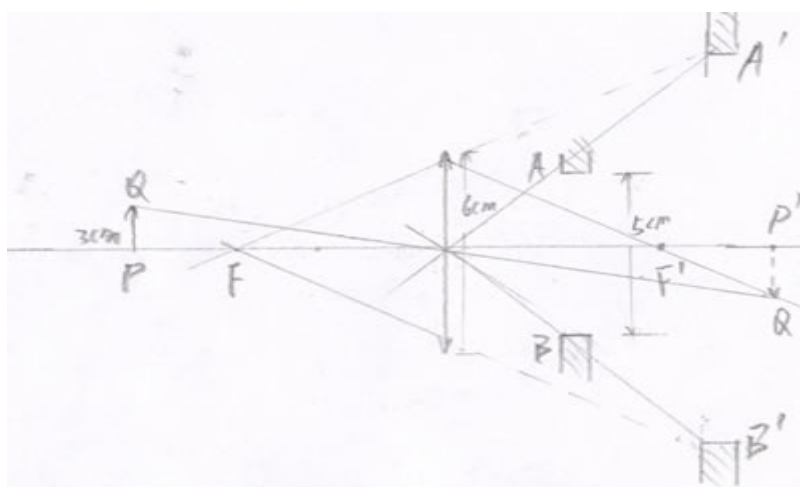
- 10.解：(1) ∵ 光阑放在了透镜后，  
∴ 透镜束就是入射光瞳和出射光瞳，对主轴上 P 点的位置  
均为 12cm，其大小为 6cm.

$$(2) \quad \because \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'} .$$

$$\therefore \frac{1}{s'} = \frac{1}{f'} + \frac{1}{s} = \frac{1}{5} + \frac{1}{-12} = \frac{1}{5} - \frac{1}{12} = \frac{7}{60}$$

$$\text{故： } s' = \frac{60}{7} \doteq 8.57 \text{ (cm)} \doteq 8.6 \text{ (cm)} .$$

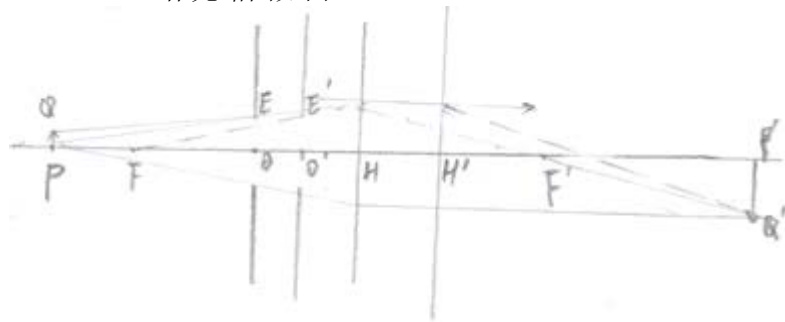
(3): 其光路图如下:



若为凹透镜，则  $s' = -3.53\text{cm}$

11. 解:  $\because \overline{EO} = 2\text{ cm}$   
 $\overline{HP} = 20\text{ cm}$   
 $\overline{HF} = 15\text{ cm}$   
 $\overline{HO} = 5\text{ cm}$   
 $\overline{H'F'} = 15\text{ cm}$   
 $\overline{HH'} = 5\text{ cm}$   
 $\overline{PQ} = 0.5\text{ cm}$

$\therefore$  作光路图如由:



$$(1) \quad \because \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

$$\therefore \frac{1}{s'_1} = \frac{1}{f'} + \frac{1}{s} = \frac{1}{15} + \frac{1}{-20} = \frac{1}{15} + \frac{1}{20} = \frac{1}{60}$$

$$s'_1 = 60 \text{ (cm)}$$

$$(2) \quad \because \beta = \frac{y'}{y} = \frac{s'}{s}$$

$$\therefore y'_1 = \frac{s'_1}{s_1} y_1 = \frac{60}{-20} \times 0.5 = -1.5 \text{ (cm)}$$

$$(3) \quad u = \operatorname{tg}^{-1} \frac{\overline{EO}}{\overline{PO}} = \operatorname{tg}^{-1} \frac{\overline{EO}}{\overline{HP} - \overline{HO}} = \operatorname{tg}^{-1} \frac{2}{15} = \operatorname{tg}^{-1} \frac{2}{15} \\ \doteq 7.595^\circ \doteq 7^\circ 35' 42''$$

$$(4) \quad \because \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'} \quad f' = \overline{H'F'} = 15 \text{ cm}, s_2 = -\overline{HO} = -5 \text{ cm}$$

$$\therefore \frac{1}{s'_2} = \frac{1}{f'} + \frac{1}{s_2} = \frac{1}{15} + \frac{1}{-5} = -\frac{2}{15} \quad .$$

$$\text{故 出射光瞳的位置为: } s'_2 = -\frac{15}{2} = -7.5 \text{ (cm)} \quad .$$

出射光瞳的半径为:

$$R = \overline{E'O'} = y'_2 = \frac{s'_2}{s_2} y_2 = \frac{s'_2}{s_2} \times \overline{EO} = \frac{-7.5}{-5} \times 2 = 3 \text{ (cm)}$$

出射光瞳的孔径角为:

$$u' = \operatorname{tg}^{-1} \frac{\overline{E'O'}}{\overline{P'O'}} = \operatorname{tg}^{-1} \frac{3}{67.5} \doteq 2.545^\circ = 2^\circ 32' 42'' \quad .$$

$$\text{其中 } \overline{P'O'} = s'_1 - s'_2 = 60 - (-7.5) = 67.5 \text{ (cm)}$$

12.解: 设桌的边缘的照度为 E,



$$\text{则: } E = I_0 \frac{\cos \alpha}{\ell^2} = I_0 \frac{x/\ell}{\ell^2} = I_0 \frac{x}{\ell^3}$$

$$= I_0 \cdot \frac{x}{(x^2 + R^2)^{3/2}}$$

$$\frac{dE}{dx} = I_0 \frac{(x^2 + R^2)^{3/2} - x \cdot \frac{3}{2}(x^2 + R^2)^{\frac{3}{2}-1} \cdot 2x}{(x^2 + R^2)^3}$$

$$= I_0 \frac{(x^2 + R^2)^{3/2} - 3x^2(x^2 + R^2)^{\frac{1}{2}}}{(x^2 + R^2)^3}$$

$$= I_0 \frac{(x^2 + R^2) - 3x^2}{(x^2 + R^2)^{5/2}} = 0$$

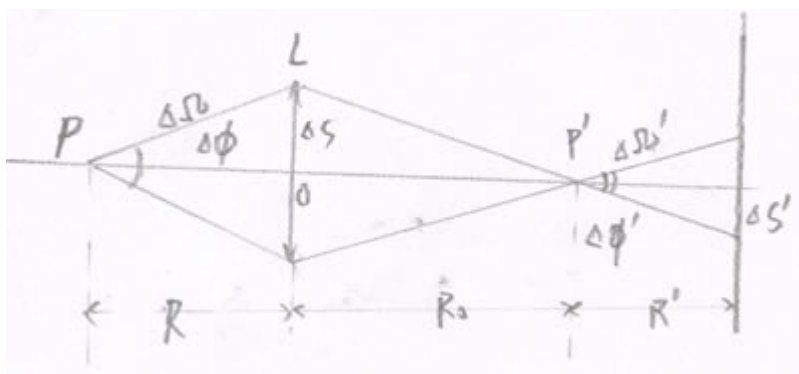
$$\text{即: } x^2 + R^2 - 3x^2 = 0$$

$$R^2 - 2x^2 = 0$$

$$\text{故: } x = \frac{\sqrt{2}}{2} R (h) .$$

即灯应悬在离桌面中心  $\frac{\sqrt{2}}{2} R$  处。

$$13. \text{解: } \because \frac{1}{s'} - \frac{1}{s} = \frac{1}{f'} . \quad f' = 20 \text{ cm} , \quad s = -30 \text{ cm} .$$



or 设 B 出发光强度为  $I_1$ ,  $P$  处发光强的为  $I_2$ , 在立体角  $\Omega_1$  内光源发出的光通过是在

顶点为  $P$  的立体角

$\Omega_2$  内传播, 是顶点在  $P$  和  $P'$  的二圆级在屏后结成来那个个相等的小块, 因此有:

$$\Omega_1 s^2 = \Omega_2 s'^2 \quad \text{光能}$$

$$I_1 \Omega_1 = I_2 \Omega_2 \text{ 联立得: } I_1 / I_2 = 1/4 \text{ 所以 } I_2 = 60(\text{cd})$$

则从  $P'$  发出的光在屏上圆镜的中心的强度为  $E = I_2 \cos \alpha / R^2 = 0.15(\text{ph})$  所以  $\alpha = 0$   $R' = 20$

$$14. \text{ 解: } \because \beta = \frac{y'}{y} = \frac{s'}{s} \cdot s' = \frac{y'}{y} s = \frac{-1}{5} \times (-50) = 10 \text{ cm}$$

$$\frac{1}{f'} = \frac{1}{s'} - \frac{1}{s} \cdot f' = \frac{s's}{s - s'} = \frac{10 \times (-50)}{(-50) - (10)} = \frac{500}{60} \doteq 8.33 \text{ (cm)}$$

又  $\because$  照相机在感光底版上所能分辨的最小距离为:

$$\Delta y' = f' \theta \doteq 1.220 \frac{\lambda}{d/f'}$$

通常定义  $R = \frac{1}{\Delta y}$  为照相物镜的分辨本领, 如果  $\Delta y$  的

单位以  $\text{mm}$  来表示, 则  $R$  就表示  $1\text{mm}$  内所能分辨的最小

$$\text{线对, 即: } R = \frac{1}{1.220\lambda} \left( \frac{d}{f'} \right) \quad (\text{线对}/\text{mm})$$

本题中的  $\Delta y' = 1 \text{ mm}$ , 说明所成的象能分辨, 仍是清晰的。

$$\because \text{tg} u' = \frac{d/2}{f'} = \frac{-y'}{x'}$$

$$\text{即 } \frac{d}{f'} = -\frac{2y'}{x'} = -\frac{2y'}{s' - f'} = -\frac{2 \times (-1)}{100 - 83.3} \doteq 0.12$$

$$\therefore F = \frac{f'}{d} = \frac{1}{0.12} \doteq 8.33$$

$$15. \text{ 解: } \because p = \delta \frac{dn}{d\lambda} = \frac{\lambda}{\Delta \lambda}$$

$$\therefore \delta = \frac{\lambda}{\Delta\lambda} \bigg/ \frac{dn}{d\lambda} = \frac{5893}{(-6)} \bigg/ (-360) = 2.37 \text{ (cm)}$$

$$\delta \geq 2.73 \text{ cm}$$

$$16. \text{解: (1) } \because \rho = \frac{\lambda}{\Delta\lambda}$$

$$\therefore N = \frac{\lambda}{\Delta\lambda} \bigg/ j = \frac{6000}{0.2 \times 2} = 15000 \text{ (条)}$$

$$(2) \because d \sin \theta = j\lambda$$

$$\therefore d = \frac{j\lambda}{\sin \theta} = \frac{2 \times 6000 \times 10^{-7}}{\sin 30^\circ} = 2.4 \times 10^{-3} \text{ (mm)}$$

$$(3) \because \text{第三级缺级, } d = 3b$$

$$\therefore b = \frac{d}{3} = 0.8 \times 10^{-3} \text{ (mm)}$$

$$(4) \quad \delta = Nd = 15000 \times 2.4 \times 10^{-3} = 36 \text{ (mm)}$$

$$(5) \because d \sin \theta = j\lambda \quad \sin \theta = 1$$

$$\therefore j = \frac{d}{\lambda} = \frac{2.4 \times 10^{-3}}{6000 \times 10^{-7}} = 4$$

考虑到缺级 $j = \pm 3$ , 则屏幕上现出的全部亮条纹数为 $2 \times (3 - 1) + 1 = 5$ , 即 $j = 0, \pm 1, \pm 2$ . 这里 $j = \pm 4$ 级是 $\sin \theta = \pm 1$ , 对应的衍射角等于 $\frac{\pi}{2}$ , 故无法观察到。

$$17. \text{解: (1)} \quad \because \Delta y = 0.610 \frac{\lambda}{n \sin u}.$$

$$\begin{aligned} \therefore n \sin u &= 0.610 \times \frac{\lambda}{\Delta y} \\ &= 0.610 \times \frac{5500 \times 10^{-7}}{0.000375} \doteq 0.895 \end{aligned}$$

$$(2) \quad \because U' = 2' = \frac{2}{60} \times \frac{\pi}{180}, \quad U = \frac{\Delta y}{25}$$

$$M = \frac{U'}{U} = \frac{25U'}{\Delta y} = \frac{25 \times \frac{2}{60} \times \frac{\pi}{180}}{0.000375 \times 10^{-1}} = 387.65$$

$$18. \text{解: } \because U \doteq \operatorname{tg} U = \frac{\Delta y}{\ell} = 0.610 \frac{\lambda}{R}$$

$$\begin{aligned} \therefore \ell &= \frac{\Delta y}{0.610 \lambda / R} = \frac{\Delta y \cdot R}{0.610 \lambda} \\ &= \frac{1.5 \times 1.5}{0.610 \times 5500 \times 10^{-7}} = 6.7 \times 10^{-3} \text{ cm} = 6.7 \text{ km} \end{aligned}$$

$$19. \text{解: } \because \operatorname{tg} \theta_1 = 1.220 \frac{\lambda}{d} = \frac{\Delta y}{\ell} \quad \lambda = \frac{\Delta y \cdot d}{1.220 \ell}$$

$$\therefore \lambda_1 = \frac{\Delta y \cdot d_1}{1.220 \ell} = \frac{500 \times 20 \times 10^{-2}}{1.220 \times 60 \times 6370 \times 10^3} = 2140 \text{ \AA}$$

$$\therefore \lambda_2 = \frac{\Delta y \cdot d_2}{1.220 \ell} = \frac{500 \times 160 \times 10^{-2}}{1.220 \times 60 \times 6370 \times 10^3} = 17150 \text{ \AA}$$

可见  $\lambda_1 < 3900 \sim 7600 \text{ \AA}$  达不到可见光范围,  
 $\therefore$  孔径为  $20\text{cm}$  的则不能分辨, 而孔径为  $160\text{cm}$  的则可以分辨。

$$20. \text{解: (1)} \quad \because 2u = 8^\circ \quad \lambda = 1 \text{ \AA} \quad n = 1$$

$$\therefore \Delta y = 0.610 \times \frac{\lambda}{n \sin u} = 0.610 \times \frac{1}{1.0 \times \sin 4^\circ}$$

$$\doteq 8.745 \text{ Å} \approx 8.7 \text{ Å}$$

$$(2) \quad M = \frac{\Delta y'}{\Delta y} = \frac{6.7 \times 10^{-2}}{8.7 \times 10^{-7}} = 7.7 \times 10^{-4} \text{ (倍)}$$

$$21, \text{解: } \because P = \lambda / \Delta \lambda = jN \quad L = Nd \quad D = d \theta / d \lambda = j / d \cos \theta = P / L \cos \theta$$

$$\therefore P = DL \cos \theta = 0.5 \times 10^{-2} \times 4 \times 10^7 \times \cos 60^\circ = 1 \times 10^5.$$

$$22, \text{解: (1)} \because \theta_1 = 0.61 \lambda / R = 1.22 \lambda / R \quad (\text{注意: 中央亮度应为其他的 2 倍, 半角亮度 } \theta_1)$$

$$\therefore D_1 = 2(f' \theta_1) = 2.44 \lambda \quad f'/d = 2.44 \times 632.8 \times 10^{-6} \times 3.76 \times 10^5 / 2 = 290(\text{km}).$$

$$(2) \because d_2 = 10^3 d_1 \quad \therefore D_2 = D_1 / 10^3 \approx 290(\text{m}).$$

$$(3) \because d_5 = 2.5 \times 10^3 d_1, \therefore D_5 = D_1 / 2.5 \times 10^3 \approx 116(\text{m})$$

$$23, \text{解: } \because \theta_1 = 0.61 \lambda / R = 1.22 \lambda / d, \quad \theta' = \Delta y / l.$$

$$\text{而 } \theta' \geq \theta_1, \text{ 即 } \Delta y / l \geq 1.22 \lambda / d.$$

$$d \geq 1.22 \lambda l / \Delta y$$

$$\therefore d_{\min} = 1.22 \lambda l / \Delta y = 1.22 \times 550 \times 10^{-9} \times 200 \times 10^3 / 1 = 0.1342(\text{cm}).$$

$$24, \text{解: } \because \theta_1 = 1.22 \lambda / d, \quad \theta' = \Delta y / l$$

$$\text{而 } \theta' \geq \theta_1, \text{ 即: } \Delta y / l \geq 1.22 \lambda / d.$$

$$\Delta y \geq 1.22 \lambda l / d.$$

$$\therefore \Delta y_{\min} = 1.22 \lambda l / d = 1.22 \times 555 \times 10^{-9} \times 3.8 \times 10^8 / 1.56 \approx 164.93(\text{m})$$

$$\approx 165(\text{m}).$$

$$25, \text{解: } \because P = \Delta y = jN, \quad L = Nd.$$

$$\lambda = 589 + 589.6 / 2 = 589.3(\text{nm})$$

$$\Delta \lambda = 589.6 - 589 = 0.6(\text{nm})$$

$$\therefore d = L / N = jL \Delta \lambda / \lambda = 2 \times 15 \times 0.6 / 589.3 \approx 0.031(\text{cm}) \approx 0.03\text{cm}$$