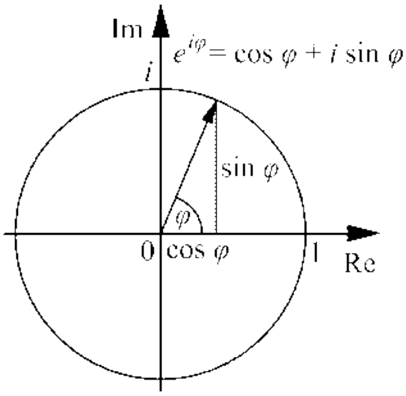
常用数学公式（第三章版本）

2017年2月26日 8:38

复数：



- [ ] $x + iy = Ae^{i\phi}$,
  $A = \sqrt{x^2 + y^2}$ *and* $\tan\phi = y/x$
  其实就是直角坐标到极坐标变换
- [ ] Euler公式 $e^{i\phi} = \cos\phi + i\sin\phi$
  可以看作 A = 1 时的特例
- [ ] Fourier分解（变换）

$$F(x) = \sum_{n=-\infty}^{\infty} F_n e^{inx}$$

可以理解为一个矢量在一个无穷维坐标系下的表示，e^{inx}是坐标轴单位矢量，F_n是坐标轴上的值（或者F(x)在坐标轴e^{inx}上的投影）；也可以理解为由一系列方程相互独立的解 e^{inx} 够早的通解（见第一章讲义）

Fourier 变换

- [ ] $\psi(\boldsymbol{r}) = \dfrac{1}{(2\pi\hbar)^{3/2}} \int \varphi(\boldsymbol{p})\, e^{i\boldsymbol{p}\cdot\boldsymbol{r}/\hbar} d^3p$
- [ ] $\varphi(\boldsymbol{p}) = \dfrac{1}{(2\pi\hbar)^{3/2}} \int \psi(\boldsymbol{r})\, e^{-i\boldsymbol{p}\cdot\boldsymbol{r}/\hbar} d^3x$

微分方程：

- [ ] 微分方程 $\frac{\partial^2 y}{\partial x^2} + k^2 y = 0$ 的通解为 $y = Ae^{ik(x+\phi)}$, 或者实数解为 $y = A\sin k(x + \phi)$

微积分：

- [ ] $\int x^n e^{ax} dx = \dfrac{1}{a} x^n e^{ax} - \dfrac{n}{a} \int x^{n-1} e^{ax} dx \quad (n > 0)$
- [ ] $\int e^{ax} \cos bx\, dx = \dfrac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$
- [ ] $\int e^{ax} \sin bx\, dx = \dfrac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx)$
- [ ] $\int x\sin ax\, dx = \dfrac{1}{a^2}\sin ax - \dfrac{x}{a}\cos ax$
- [ ] $\int x^2 \sin ax\, dx = \dfrac{2x}{a^2}\sin ax + \left(\dfrac{2}{a^3} - \dfrac{x^2}{a}\right)\cos ax$
- [ ] $\int x\cos ax\, dx = \dfrac{1}{a^2}\cos ax + \dfrac{x}{a}\sin ax$
- [ ] $\int x^2 \cos ax\, dx = \dfrac{2x}{a^2}\cos ax + \left(\dfrac{x^2}{a} - \dfrac{2}{a^3}\right)\cos ax$
- [ ] $\int \sqrt{ax^2 + c} dx = \begin{cases} \dfrac{x}{2}\sqrt{ax^2 + c} + \dfrac{c}{2\sqrt{a}} \ln\left(\sqrt{a}x + \sqrt{ax^2 + c}\right) & (a > 0) \\ \dfrac{x}{2}\sqrt{ax^2 + c} + \dfrac{c}{\sqrt{-a}} \arcsin\sqrt{-\dfrac{a}{c}}x & (a < 0) \end{cases}$

- [ ] $$\left.\begin{array}{l}\int_0^{\frac{\pi}{2}}\sin^n x dx\\ \int_0^{\frac{\pi}{2}}\cos^n x dx\end{array}\right\} = \begin{cases}\frac{(n-1)!!}{n!!}\frac{\pi}{2} & (n\text{为正偶数})\\ \frac{(n-1)!!}{n!!} & (n\text{为正奇数})\end{cases}$$
- [ ] $$\int_0^{\infty}\frac{\sin ax}{x}dx = \begin{cases}\frac{\pi}{2} & (a>0)\\ -\frac{\pi}{2} & (a<0)\end{cases}$$
- [ ] $$\int_0^{\infty} e^{-ax}x^n dx = \frac{n!}{a^{n+1}} \quad (n\text{为正整数}, a>0)$$
- [ ] $$\int_0^{\infty} e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$
- [ ] $$\int_0^{\infty} x^{2n}e^{-ax^2}dx = \frac{(2n-1)!!}{2^{n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$
- [ ] $$\int_0^{\infty} x^{2n+1}e^{-ax^2}dx = \frac{n!}{2a^{n+1}}$$
- [ ] $$\int_0^{\infty}\frac{\sin^2 ax}{x^2}dx = \frac{\pi a}{2}$$
- [ ] $$\int_0^{\infty} xe^{-ax}\sin bx\, dx = \frac{2ab}{(a^2+b^2)^2} \quad (a>0)$$
- [ ] $$\int_0^{\infty} xe^{-ax}\cos bx\, dx = \frac{a^2-b^2}{(a^2+b^2)^2} \quad (a>0)$$

级数展开：

- [ ] $$e^x = 1 + x + \frac{x^2}{2!} + \cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$
- [ ] $$\sum_{k=0}^{n-1}(a+kd)q^k = \frac{a-[a+(n-1)d]q^n}{1-q} + \frac{dq(1-q^{n-1})}{(1-q)^2} \quad (n \geqslant 1)$$
$$\sum_{k=0}^{\infty}(a+kd)q^k = \frac{a}{1-q} + \frac{dq}{(1-q)^2} \quad (|q|<1)$$