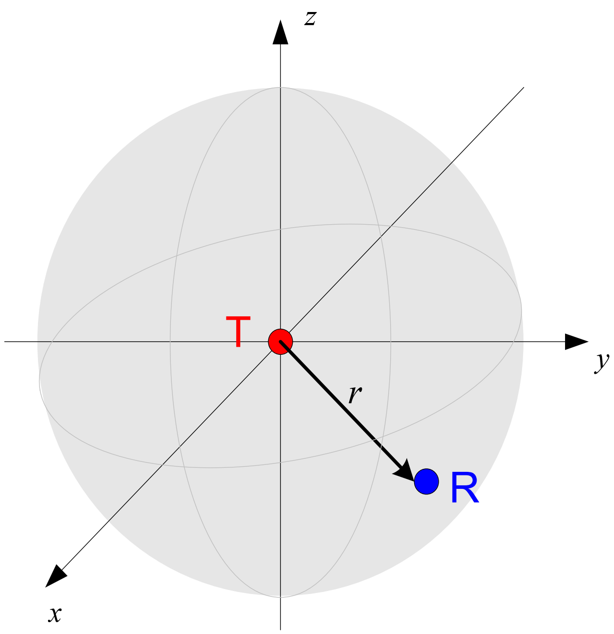
Lecture 2. Fading Channel

- Characteristics of Fading Channels
- Modeling of Fading Channels
- Discrete-time Input/Output Model

Radio Propagation in Free Space

- Speed: c = 299,792,458 m/s
- Isotropic
  - Received power at a particular location decays with distance: $P \sim r^{-2}$



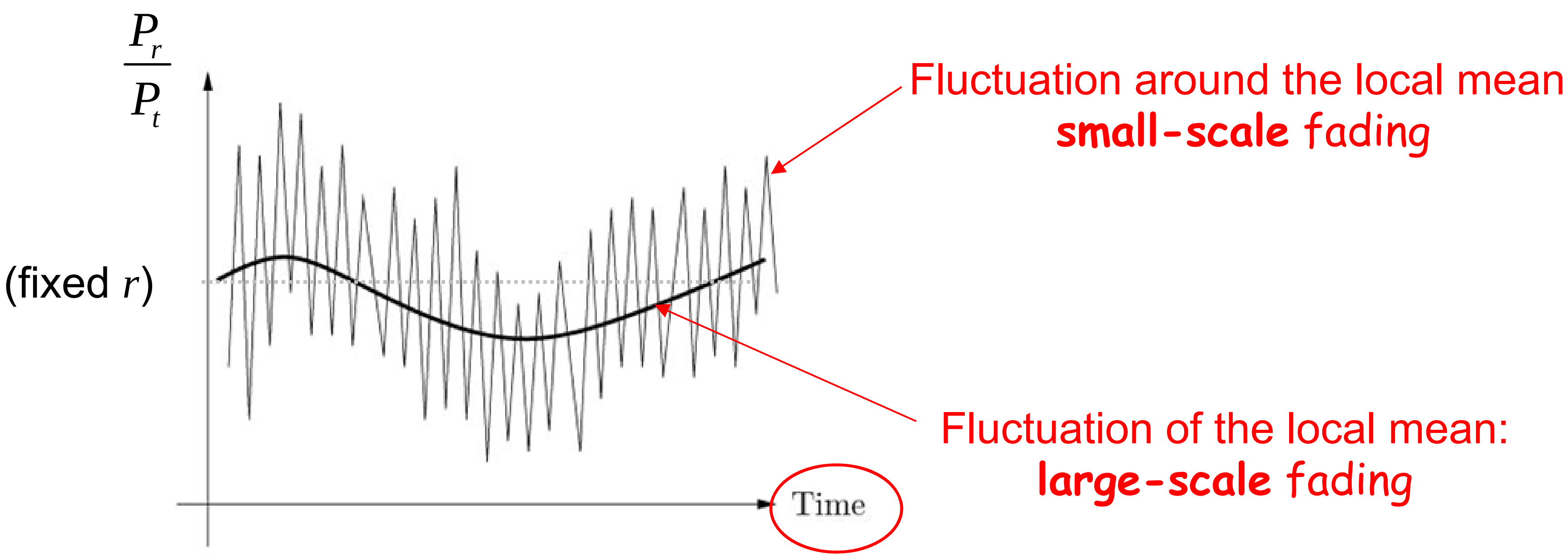
✓ Transmitted signal: $s(t) = \cos 2\pi ft$

✓ Received signal: $y(t) = \dfrac{\beta \cos 2\pi f(t - r/c)}{r}$

Radio Propagation in Terrestrial Environment

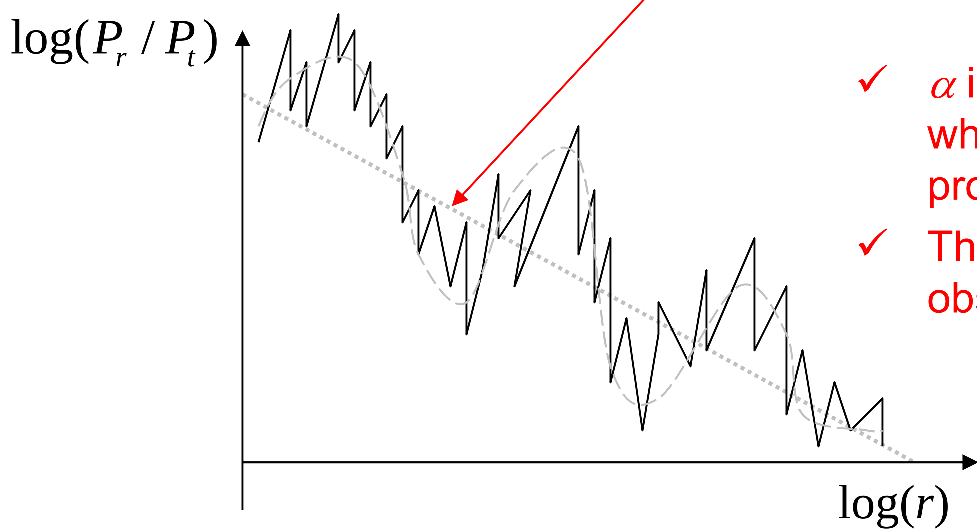
Transmission Power: P_t

Received Power: P_r



Large-scale Fading

- Large-scale fading -- *Log-normal shadowing*
  - Attributed to the random variation of propagation environment.
  - Empirically modeled as a log-normal random variable with mean μ and variance σ^2.
  - Mean μ is determined by the path loss: $\mu \sim r^{-\alpha}$, $\alpha > 2$.



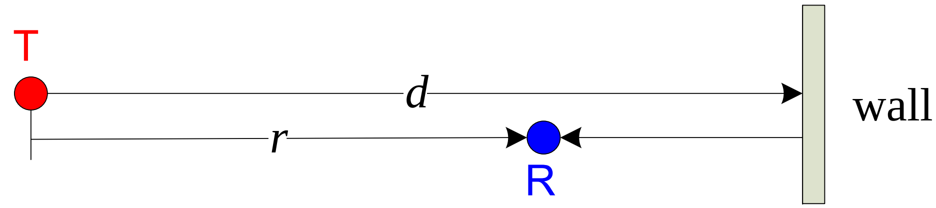
- ✓ α is called path-loss factor, which depends on the propagation environment.
- ✓ The higher the density of obstacles, the larger α.

Small-scale Fading

- Small-scale fading
  - Attributed to 1) multiple arriving paths; and 2) movements of the transmitter and/or receiver.

A Simple Two-path Model (1)

- Reflecting wall, fixed antenna



✓ Transmitted signal: $s(t) = \cos 2\pi ft$

✓ Received signal: $y(t) = \dfrac{\beta \cos 2\pi f(t - r/c)}{r} - \dfrac{\beta \cos 2\pi f(t-(2d-r)/c)}{2d-r}$

The phase difference between the two waves:

$$\Delta\phi = \left(\frac{2\pi f(2d-r)}{c} + \pi\right) - \left(\frac{2\pi fr}{c}\right)$$

$$= \frac{4\pi f}{c}(d-r) + \pi$$

What happens if $\Delta\phi$ changes from π to 2π?

$\Delta\phi = 2k\pi$: Constructively add

$\Delta\phi = (2k+1)\pi$: Destructively add

Coherence Bandwidth vs. Delay Spread

- An appreciable change of received signal will be observed if the frequency of transmitted signal f changes by: $\dfrac{c}{4(d-r)}$

Coherence Bandwidth W_c

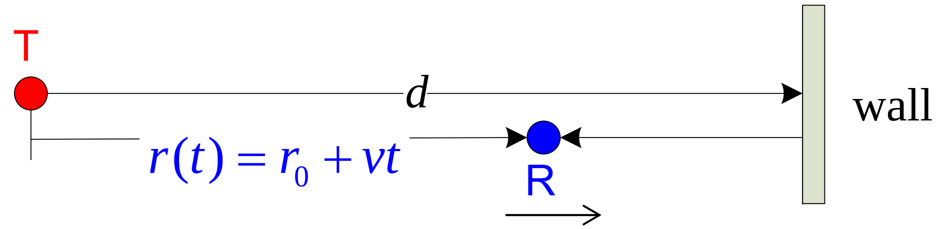
- The difference between the propagation delays along the two signal paths is: $\dfrac{2d-r}{c}-\dfrac{r}{c}=\dfrac{2(d-r)}{c}$

Delay Spread T_d

- $T_d \sim \dfrac{1}{W_c}$

A Simple Two-path Model (2)

- Reflecting wall, moving antenna



  - ✓ Transmitted signal: $s(t) = \cos 2\pi ft$
  - ✓ Received signal:

$$y(t) = \frac{\beta \cos 2\pi f(t - r(t)/c)}{r(t)} - \frac{\beta \cos 2\pi f(t - (2d - r(t))/c)}{2d - r(t)}$$

Doppler Shift: $D_1 = -fv/c$ Doppler Shift: $D_2 = fv/c$

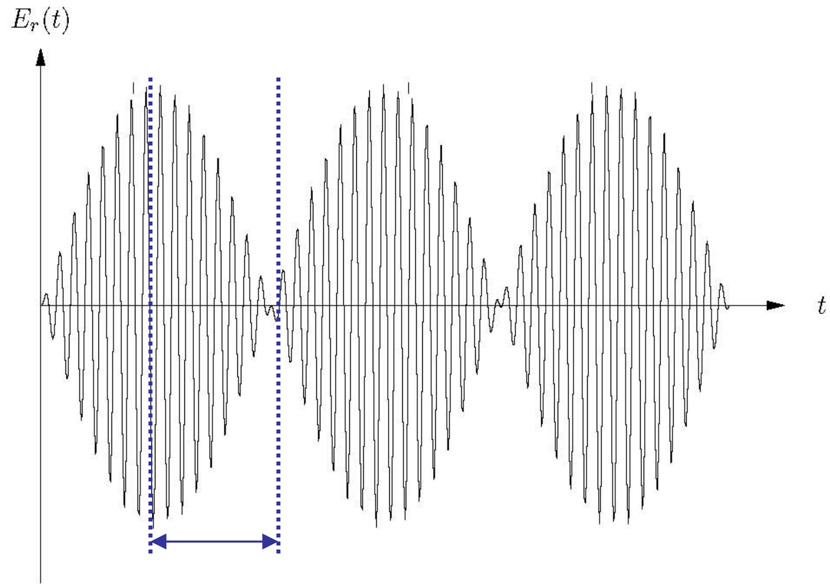
$$= \frac{\beta \cos 2\pi f((1 - v/c)t - r_0/c)}{r_0 + vt} - \frac{\beta \cos 2\pi f((1 + v/c)t + (r_0 - 2d)/c)}{2d - r_0 - vt}$$

$$\approx \frac{2\beta \sin 2\pi f(vt/c + (r_0 - d)/c) \sin 2\pi f(t - d/c)}{r_0 + vt}$$

Coherence Time vs. Doppler Spread

- An appreciable change of received signal will be observed if time t changes by: $\frac{c}{4fv}$

Coherence Time T_c



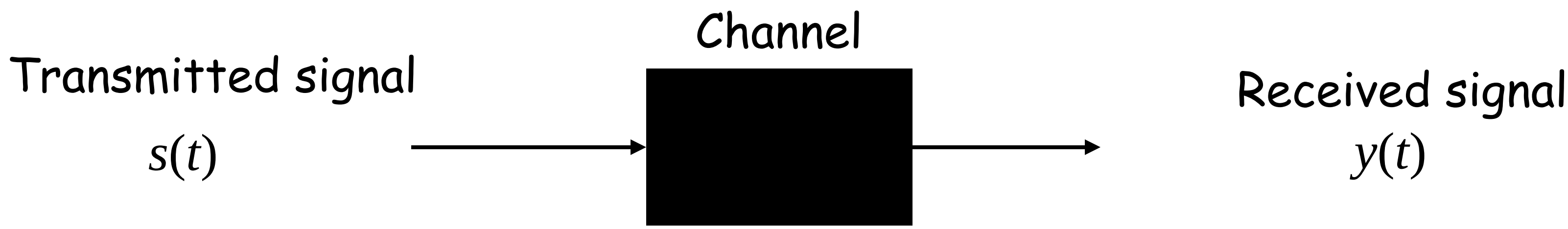
- The difference between the Dopper shifts of the two paths is:

$$D_s = D_2 - D_1 = 2fv/c$$

Doppler Spread D_s

- $T_c \sim \frac{1}{D_s}$

Modeling of Fading Channels



- A fading channel can be modeled as a Linear Time-Varying system.

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t) s(t-\tau) d\tau$$

  - Suppose that h(τ,t) is a deterministic function of time t and delay τ.

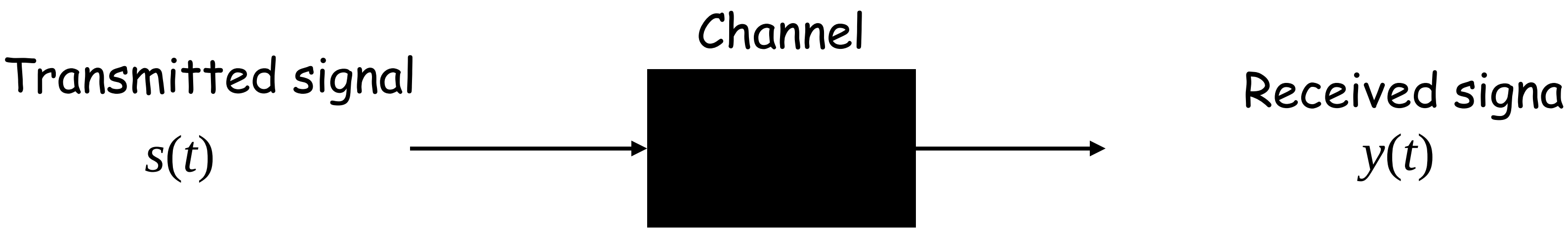
Time-variant transfer function: $H(f,t) = \mathcal{F}_\tau(h(\tau,t))$

Delay Doppler function: $S(\tau,\upsilon) = \mathcal{F}_t(h(\tau,t))$

Doppler-variant transfer function: $D(f,\upsilon) = \mathcal{F}_\tau(S(\tau,\upsilon)) = \mathcal{F}_t(H(f,t))$

$\mathcal{F}(\cdot)$ denotes Fourier transform.

Modeling of Fading Channels



- A fading channel can be modeled as a Linear Time-Varying system.

$$y(t)=\int_{-\infty}^{\infty} h(\tau,t)s(t-\tau)d\tau$$

- Suppose that h(τ,t) is a WSS process with uncorrelated scatterers:
  The autocorrelation function $R_h(\tau_1,t_1,\tau_2,t_2)=\mathrm{E}[h(\tau_1,t_1)h^*(\tau_2,t_2)]=R_h(\tau,\Delta t)$

  Time-variant transfer function: $R_H(\Delta f,\Delta t)=\mathcal{F}_\tau(R_h(\tau,\Delta t))$

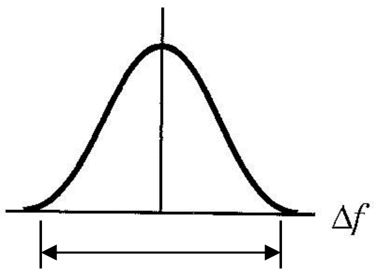
  Delay Doppler function: $R_S(\tau,\upsilon)=\mathcal{F}_{\Delta t}(R_h(\tau,\Delta t))$

  Doppler-variant transfer function: $R_D(\Delta f,\upsilon)=\mathcal{F}_\tau(R_S(\tau,\upsilon))=\mathcal{F}_{\Delta t}(R_H(\Delta f,\Delta t))$

Coherence Bandwidth vs. Delay Spread

- Let $\Delta t = 0$:

$R_H(\Delta f, 0)$

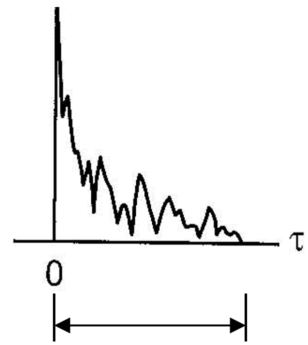


Coherence Bandwidth
W_c

$$R_H(\Delta f, 0) \xrightarrow{\mathcal{F}^{-1}_{\Delta f}(\cdot)} R_h(\tau, 0) \qquad R_h(\tau, 0) \xrightarrow{\mathcal{F}_{\tau}(\cdot)} R_H(\Delta f, 0)$$

$$\boxed{T_d \sim \frac{1}{W_c}}$$

$R_h(\tau, 0)$



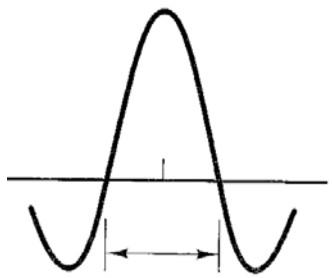
Delay Spread
T_d

- Flat fading: Signal Bandwidth W << Coherence Bandwidth W_c
  Symbol Time T >> Delay Spread T_d
- Frequency-selective fading: Signal Bandwidth W >> Coherence Bandwidth W_c
  Symbol Time T << Delay Spread T_d

Coherence Time vs. Doppler Spread

- Let $\tau = 0$:

$R_h(0, \Delta t)$



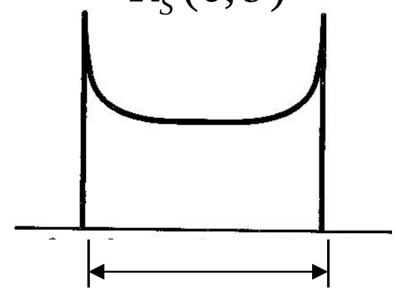
Coherence Time
T_c

$$R_h(0,\Delta t) \xrightarrow{\mathcal{F}_{\Delta t}(\cdot)} R_S(0,\upsilon)$$

$$R_h(0,\Delta t) \xleftarrow[\mathcal{F}_{\upsilon}^{-1}(\cdot)]{} R_S(0,\upsilon)$$

$$\boxed{T_c \sim \frac{1}{D_s}}$$

$R_S(0, \upsilon)$



Doppler Spread
D_s

- Slow fading: Symbol Time T << Coherence Time T_c
  Signal Bandwidth W >> Doppler Spread D_s

- Fast fading: Symbol Time T >> Coherence Time T_c
  Signal Bandwidth W << Doppler Spread D_s

Discrete-time Input/Output Model

- $y(t)=\int_{-\infty}^{\infty} h(\tau,t)s(t-\tau)d\tau = \sum_n s[n]\sum_i a_i(t)\Psi_{T_0}(t-\tau_i(t)-nT_0)$

  $h(\tau,t)=\sum_i a_i(t)\delta(\tau-\tau_i(t))$

  $s(t)=\sum_n s[n]\Psi_{T_0}(t-nT_0)$

  - $a_i(t)$: attenuation at time t of path i
  - $\tau_i(t)$: propagation delay at time t of path i
  - $\Psi_{T_0}(t)$: modulation pulse

- Sampled output at $t=mT_0$:

$$y[m]=\sum_n s[n]\sum_i a_i(mT_0)\Psi_{T_0}((m-n)T_0-\tau_i(mT_0))$$

Let $l=m\text{-}n$.

$$y[m]=\sum_l s[m-l]\sum_i a_i(mT_0)\Psi_{T_0}(lT_0-\tau_i(mT_0)) \quad h_l[m]$$

- Discrete-time Input/Output Model: $y[m]=\sum_l h_l[m]s[m-l]+z[m]$

Discrete-time Input/Output Model

$$y[m] = \sum_{l=0}^{L\text{-}1} h_l[m]s[m-l] + z[m]$$

- How many channel filter taps can be obtained (how to determine the value of L)?

  Delay spread: $T_d = \max_{i,j} |\tau_i - \tau_j|$ $\Longrightarrow$ $\boxed{L = \lceil WT_d \rceil}$

  Sampling rate: W

  - Flat fading: L=1
  - Frequency-selective fading: L>>1

- How fast does the channel filter tap gain $h_l[m]$ vary with time in one symbol time T? Depends on the coherence time T_c.

  - Slow fading: $h_l[m]$ can be regarded as constant in T
  - Fast fading: $h_l[m]$ varies with time in T

More about $h_l[m]$

- Without line-of-sight (LOS) paths: $h_l[m]$ can be modeled as a zero-mean complex Gaussian random variable $\mathcal{CN}(0,\sigma_s^2)$.
  - The magnitude follows Rayleigh distribution.

- With one LOS path: $h_l[m]$ can be modeled as

$$h_l[m]=\sqrt{\frac{k}{k+1}}\sigma_s e^{j\theta}+\sqrt{\frac{1}{k+1}}\mathcal{CN}(0,\sigma_s^2)$$

k: the ratio of the power in the LOS path and that in the scattered paths.

  - The magnitude follows Ricean distribution.

More about $h_l[m]$

- Availability of Channel State Information (CSI)
  - CSIR: CSI is available at the receiver side.
    - available through channel measurement
    - required for coherent detection
  - CSIT: CSI is available at the transmitter side.
    - available through channel measurement and feedback
    - optional
- Ergodicity
    - indispensable condition to achieve the Shannon capacity
    - may not hold in delay-limited scenarios

Summary

- Fading Channel
  - Large-scale fading
    - Path loss
    - Shadowing
  - Small-scale fading
    - Delay spread (caused by multipath) → Flat fading, Frequency-selective fading
    - Doppler spread (caused by mobility) → Slow fading, Fast fading
  - Discrete-time input/output model

$$y[m] = \sum_{l=0}^{L-1} h_l[m]s[m-l] + z[m]$$