# Using SIR Model with Population Flow to Simulate Transmission of COVID-19 between Counties in Taiwan

## 1. Introduction

The virus that made its first appearance in Wuhan City, Hubei Province at the end of 2019 was identified as a new coronavirus after initial research and was tested to be contagious from person to person. On March 11, the World Health Organization officially announced the newly-found coronavirus as a pandemic[1]. The spread of the newly-found coronavirus was both faster and larger in scope than SARS (Severe Acute Respiratory Syndrome) in 2003. Within one month after the spread of COVID-19, the number of active cases had already exceeded 8000 and now the current count is up to millions. Many countries have therefore devised various policies, including tourism restrictions, self-quarantine policies, and commodity control as a means of slowing down the spread of the virus. In addition, employees in various aviation and tourism businesses have suffered heavy losses due to the lack of passengers and tourists resulting from the lock-in policy, which have severely affected the lives of us students. We were forced to stay at home and watch online courses since school was postponed for 2 weeks. In Taiwan, the government has been dealing with the pandemic with a set of strict restrictions, including rules regarding the behaviors of travelers from abroad, that people returning from overseas must stay in their homes for 14 days and must wear masks throughout their journey.

Although the government's measures against the spread of the pandemic were of goodwill, these bans ignited people's dissatisfaction. If the public cannot understand how such measures control the spread of the virus, it may be difficult for people to appreciate the importance of strictly implementing these rules. As long as there are people who refuse to cooperate, the effects of these policies cannot be maximized, and extra time and efforts will have to be further devoted. In order to remind the public of the importance of good isolation policies, we will simulate the effects of various policies on virus control in this research, using the SIR model.

## 2. Research Methods

In our research, only transmission on the Taiwan mainland is considered because transportation to the outer islands proves to be difficult and full of variance. For example, in order to travel to the outer islands from mainland Taiwan, one must go through harbors or airports. The routes one takes to these harbors or airports complicated and, sometimes, unpredictable. We have chosen to use counties and cities as the unit resolution to simulate the interaction between people not only because it’s easier to obtain the population data needed from government websites but also because it will be more realistic, since each county or city is allowed to implement their own policies.

Our model revolves around the introduction of a set amount of coronavirus cases in a certain city and observing the overall trend and extent of disease transmission after various lengths

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[1] World Health Organization. (2020, March 11). *WHO Director-General's opening remarks at the media briefing on COVID-19 - 11 March 2020*. World Health Organization. https://www.who.int/dg/speeches/detail/who-director-general-s-opening-remarks-at-the-media-briefing-on-covid-19---11-march-2020.

of time. We will first be simulating how these cases will spread without any contact prevention policies. Then, we will be simulating how the spread of the pandemic will be influenced in response to reduced contact of individuals within certain regions. This reduced contact can be interpreted as the implementation of new government-introduced prevention policies such as but not limited to, reducing the passengers or frequency of the high-speed rail, distributing pandemic provisions (face masks, hand sanitizers), cities conducting lockdown protocol, etc. Our simulation results on varying population flow can be utilized to deduce the impact of restricting tourism and commuting policies on the spread of the outbreak. [2]

### 2.1. SIR Model

Our research mainly uses the SIR analysis model to calculate and simulate the spread of the virus between Taiwanese counties. The letters S, I, R represent the percentage of people who are "susceptible" to infection, are currently "infected", and have "recovered", respectively. In addition, $\gamma$ is used to represent the recovery rate, the percentage of the infected population that recovers each day, and $\beta$ is used to represent the effective contact rate of the virus, the number of individuals an infected individual comes into contact with per unit time. In order for an individual susceptible to infection (S) to become infected (I), that individual must come into contact with at least one other infected individual and, in the process, be infected by the virus. Consequently, the value of $\beta$ and $\gamma$ will be affected by the frequency at which people come into contact and the type of virus.[3][4][5]

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

In the equations above, $\frac{dS}{dt}$, $\frac{dI}{dt}$,and $\frac{dR}{dt}$ represent the variation of the percentages of susceptible individuals, infected individuals, and recovered individuals in the population over time t.

Because the equations above are only applicable to the application of the SIR model in a single restricted area, they must be modified in order to fit the purpose of this study: the simulation of COVID-19 transmission between multiple counties in Taiwan. To this end, we must consider the influence of counties on nearby counties as well as counties farther away, so we have derived

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[2]Prem, K., Liu, Y., Russell, T. W., Kucharski, A. J., Eggo, R. M., Davies, N., ... & Abbott, S. (2020). The effect of control strategies to reduce social mixing on outcomes of the COVID-19 epidemic in Wuhan, China: a modelling study. *The Lancet Public Health*.

[3]Smith, D., & Moore, L. (2004). The SIR model for spread of disease: the differential equation model. *Loci.(originally Convergence.)* *https://www.maa.org/press/periodicals/loci/joma/the-sir-model-for-spread-of-disease-the-differential-equation-model*.

[4]Nesteruk, I. (2020). Statistics based predictions of coronavirus 2019-nCoV spreading in mainland China. *medRxiv*.

[5]Wangping, J., Ke, H., Yang, S., Wenzhe, C., Shengshu, W., Shanshan, Y., ... & Miao, L. (2020). Extended SIR prediction of the epidemics trend of COVID-19 in Italy and compared with Hunan, China. *Frontiers in medicine*, *7*, 169.

the following equations that describe the application of the SIR model to two geographically distinct areas, represented by the letters A and B:

$$\frac{dS_A}{dt} = -(\beta_{AA}S_AI_A + \beta_{AB}S_AI_B) \qquad \frac{dS_B}{dt} = -(\beta_{BB}S_BI_B + \beta_{BA}S_BI_A)$$

$$\frac{dI_A}{dt} = \beta_{AA}S_AI_A + \beta_{AB}S_AI_B - \gamma I_A \qquad \frac{dI_B}{dt} = \beta_{BB}S_BI_B + \beta_{BA}S_BI_A - \gamma I_B$$

$$\frac{dR_A}{dt} = \gamma I_A \qquad \frac{dR_B}{dt} = \gamma I_B$$

$\beta_{ij}$ is the effective contact rate between individuals in area i and and area j. For instance, $\beta_{AA}$ would represent the effective contact rate between people in area A and other people and area A. Similarly, $\beta_{AB}$ would represent the effective contact rate between people in area A and area B. Because the contact rate between individuals from two distinct areas would most likely be lower than individuals in the same area, $\beta_{AA}$would most likely be greater than $\beta_{AB}$.

In order to generalize these equations to further fit the purpose of our study, we have derived more equations to fit a situation where there are n areas, as opposed to a singular closed area or two distinct areas.

## 2.2. Population Gravity Model

Since we are observing transmission between counties, the S and I input values of each county will change according to the strength of their connections with various counties, defined through a population flow calculation described below. The calculation method of population flow can be derived by referring to the formula of Newton's law of gravitation. The equation that expresses the concept of the population gravity model is as follows: $C = \frac{\theta K P_1 P_2}{r^2}$. [6]

In the equation above, C is a value that represents the population flow between two places, θ (theta) is an input variable that will be later explained in more detail, K is a constant utilized to standardize our data, $P_1$ is the population of Place 1, $P_2$ is the population of Place 2, and r is the distance between Place 1 and Place 2. In this research, the location of each county will be represented by the centroid of each county’s contour, so the distance r will be the distance between the centroids of two cities. According to the equation of the population gravity model, the amount of interactions between people from two cities is inversely proportional to their distance and proportional to the size of their respective populations.

Through the SIR model we established, we now understand the rate and relationship of virus transmission among counties and cities. If we encounter another pandemic in the future, we will have a model readily available to which we can refer. With this model at hand, governments can adjust contact restriction policies according to each counties’ contact frequency with one another to minimize virus impact on Taiwan.

# 3. Research Process

[6] 郭錦婷（2000），《「引力模式」應用在雙邊貿易之分析─以亞太國家為例》，國立政治大學國際貿易學系碩士班學位論文，臺北。

To evaluate and visualize the dissemination of COVID-19, we constructed a model with adjustable input parameters using R language. Our model is built on a constant beta value (transmission rate) of 0.38/day[7] and our population data of every county is collected from the SEGIS platform, Ministry of Interior.[8]

The model we made is based on the interactions of people between cities and counties, so we must know where these interactions will occur and how many people would possibly participate in these interactions. The first input of our simulator is the number of cases that appear initially and the county where the initial cases emerged. Although changing the number of initial cases evidently affects the amount of people infected but does not have a significant effect on the proportion distribution of the infected peoples, we focus on the proportion of cases distributed across counties and cities. For example, since the population flow between Taipei and New Taipei City is the highest among others, so whenever Taipei has a new case, New Taipei City will be affected the most regardless of the amount infected. In other words, because of its high population flow, we can perceive the combination of Taipei City and New Taipei City as a district of its own.

The second of our input setting is theta. To better understand the meaning of the theta value itself, an explanation on our population flow calculations and how it is implemented into our model is required. After acquiring the population flow value between each county, we find that the population flow between Taipei City and New Taipei City is the highest value. Then, we divide every population flow value by the population flow between Taipei City and New Taipei City to get a value that can be better implemented into our SIR model. This act of division is represented by the constant K in our formula. Before inputting our newly obtained values into the model, we multiply them by theta. The actual value of theta is not more important than the percentage change of theta which correlates with a percent change in population flow. Values that exceed one after multiplying by theta will automatically be set to one.

The final input value of our simulator is days, which represents the number of days after the initial infections. This value will be utilized to observe the size of the infected population at certain points in each of our simulations.

## 4. Findings and Discussions

According to the results of our initial experiments, we can have reached a few conclusions about how each of the four input values of our simulator affect the size and distribution of the infected population:

1. The speed at which the coronavirus spreads will be proportional to the size of the initial infected population. However, the distribution of the infected population throughout Taiwan will not be affected by the size of the initial infected population.
2. As the number of days after the initial infection increases, the number of cases in each city steadily increases, without changing the distribution of the infected population throughout Taiwan.

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[7] 社會經濟資料服務平台(https://segis.moi.gov.tw/)

[8] Syed, F., & Sibgatullah, S. (2020). Estimation of the Final Size of the COVID-19 Epidemic in Pakistan. *medRxiv*.

3. The speed of the spread of the coronavirus increases as the initial size of the infected population increases.
4. The city at which the initial infected population emerges doesn't affect the distribution of the infected population.

We will be simulating three different scenarios by adjusting our input values in order to fit each scenario. Because we want to better understand the effects of government policies on disease prevention, we will be comparing the results of 3 scenarios: normal population flow, restricted population flow, and minimized population flow. By comparing the results of these three simulations, we can gain a greater understanding of how the four input values will influence the spread of the coronavirus together as well how important governmental policies promoting disease prevention are.

### 4.1. Scenario 1: Simulating normal population flow between counties

The first scenario we want to simulate is the spread of the virus under the normal population flow between counties and cities in Taiwan. In order to make our simulation as close as possible to the actual population flow between counties and cities, we set theta to 100. The results of this simulation can be used to evaluate the spread of the virus if the Taiwan government chose not to take any anti-epidemic measures.

In a scenario where theta is set as 100 and the initial number of cases is 10, after 30 days, about 230 people in Taiwan will be infected; after 60 days, the number of infected people would've increased by nearly 10 times, and about 2,000 people would have been infected; 90 days later, the number of infections would increase once again, and about 15,000 people would be infected.

### 4.2. Scenario 2: Simulating population movement restriction policies between counties

In our second scenario, we set theta to 50, meaning that the population flow between counties and cities has been reduced by 50%. With the results gained from this simulation, we can compare it to Situation 1 to observe the efficiency of limiting population flow numerically.

In a scenario where theta is set as 50 and the initial number of cases is 10, after 30 days, about a hundred people in Taiwan will be infected; after 60 days, the number of infected people tripled, about three hundred people would have been infected; 90 days later, the number of infections would've tripled once again, and about nine hundred people would be infected.

### 4.3. Scenario 3: Simulating lockdown policies

Our last scenario we will be simulating is the lockdown policy, where theta is set to 0. This is because in an ideal lockdown policy, there will be no interaction and population flow between counties. By analyzing the spread of the virus in this simulation, we can speculate whether lockdown policies are an effective policy if we encounter a large-scale epidemic or pandemic such as COVID-19 again in the future.

In a scenario where theta is set as 0 and the initial number of cases is 10, after 30 days, only 14 people in Taiwan will be infected; after 60 days, the number of infected people is maintained at 14; 90 days later, the number of infections still remains at 14. It can be seen that the efficiency of the lockdown policy is very high in suppressing virus spread, and the number of infections in this area can be stably maintained around the number of infections at the time of the initial outbreak.

**Table 1. Total infected number of three scenarios**

| days / scenario | 30 | 60 | 90 |
|---|---|---|---|
| Scenario 1 $\theta = 100$ | **228** | **1942** | **15215** |
| Scenario 2 $\theta = 50$ | **101** | **332** | **893** |
| Scenario 3 $\theta = 0$ | **14** | **14** | **14** |

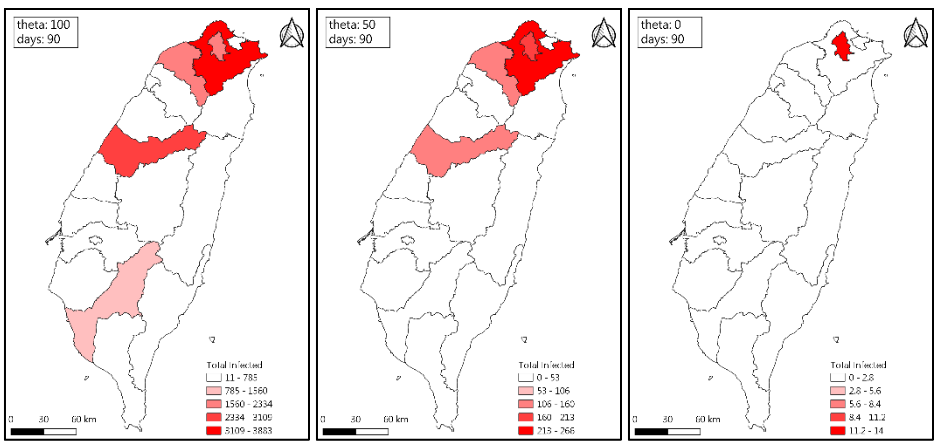


**Figure 1. Total infected number in counties of three scenarios**

### 4.4. Summary

Based on the data in Figure 1 and Table 1, we can see that when the population flow parameters change, both the number of cases and the extent of the spread of the virus will change accordingly. Therefore, if population movement restrictions are not adopted after the outbreak of the pandemic, the number of infected people in Taiwan will increase substantially within a few months as in Scenario 1. At the start of the pandemic, some countries didn't immediately implement policies limiting the spread of the virus, which may have eventually lead to the proliferation of the infected population throughout their respective countries. On the contrary, the Taiwanese government issued numerous policies requiring people to wear masks, sit separately from one another, and maintain social distancing, effectively reducing population movement and contact rates, just as in Scenario 2. Lastly, we can observe that the number of people infected in the lockdown policy indicates that the policy significantly suppressed the spread of the virus. Because there were no interactions with other counties and cities, the virus could not spread to other regions. It can be observed that the immediate adoption of a lockdown policy can greatly reduce the impact of the virus in the region.

### 4.5. Model Restrictions

The population gravity model is a simple way of modeling population flow and interaction; however, it only considers the population size and distance when calculating interactions between two cities. In reality, population flow will be affected by the presence of railways, geographic relations, as well as many other variables. Also, we used the centroid of county contours to calculate the distances between counties, hence, there will be some mild errors when calculating population flow. Another restriction of our research model is our assumption that people who have recovered from the virus will not be infected by the virus again. However, there have been instances where people who have recovered from the coronavirus are reinfected.

## 5. Conclusion

The current coronavirus pandemic proved to be devastating in various aspects, whether through the health and safety of the people, instability of political governments, unstable fluctuations of the economy, or the cessation of education. The critical challenge that countries are facing currently is to come up with solutions to stop the virus from spreading. Hence, knowing how to effectively prevent the spread of the virus while lowering the damage dealt as much as possible will be extremely crucial.

Responding to current events, our research seeks to answer the severe situation with both epidemiology and computational perspectives. Using a double-model mechanism simulator, we investigated the relationship between population flow and virus dissemination. Our double-model simulator consists of the SIR model to simulate the infected population and the population gravity model to calculate the parameter of population flow magnitude.

In our research, we simulated three different scenarios:

1. Normal population flow between counties without any regulations
2. 50% population flow between counties with some regulations
3. 0% population flow between counties, assuming Taiwan is under total lockdown

We observed that if we made infection time as our controlled variable, the relationship between initial cases and final cases showed a linear growth relationship, whereas if we set our initial cases as a control variable, the relationship between infection time and final cases show an exponential growth relationship. This highlights how essential it is for assertive and swift pandemic prevention demands.

In this tumultuous time, we advocate for the cooperation between civilians and governmental actions so that the dissemination of the virus can be effectively stopped, and we genuinely hope for the well-being of the global community.