Research on Adaptive Quadrature Algorithm Based on Undamped Spring System

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Abstract

This paper investigates the use of adaptive quadrature algorithms for the numerical approximation of integrals, with a focus on their application to undamped spring systems. Specifically, the paper proposes and analyzes the performance of two adaptive quadrature algorithms - the adaptive trapezoidal rule and the adaptive Simpson's rule for the efficient and accurate approximation of integrals. The paper presents the mathematical foundations of these algorithms and evaluates their performance using a set of benchmark functions. The results demonstrate that both algorithms offer significant improvements in accuracy and efficiency compared to traditional fixed-step quadrature methods. Furthermore, the paper explores the potential applications of these algorithms in the context of undamped spring systems, where they can be used to solve complex equations of motion and determine system parameters. Overall, this research provides valuable insights into the use of adaptive quadrature algorithms and their potential applications in engineering and scientific domains.

Problem

Consider Forced Vibrations without Damping then the differential equation is:

$$mu''(t) + ku(t) = F_0\cos(\omega t)$$

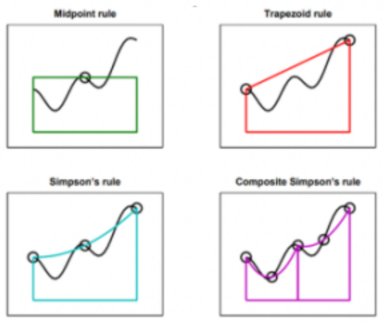
$$u = \frac{F_0}{m(\omega_0^2 - \omega^2)}(\cos\omega t - \cos\omega_0 t)$$

where $\omega = \sqrt{\frac{k}{m}}$ is the natural frequency of the system.

- Sketch the graph of u when $m = 1, k = 9, F_0 = 1, \omega = 2$, and $t \in [0, 2\pi]$.
- Study the Adaptive Quadrature Algorithm 4.3.
- Implement the Adaptive Quadrature Algorithm in MATLAB.
- Approximate $\int_0^{2\pi} u\,dx$ to within 10^{-4}.
- Investigate how many subintervals needed to perform the Simpson's rule with n=4.
- Discuss the disadvantage and advantage of Adaptive Quadrature method.

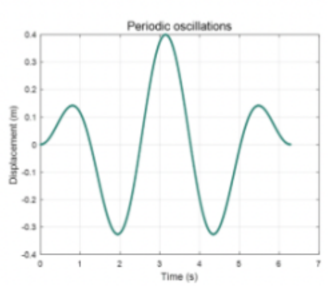
Results and Discussion

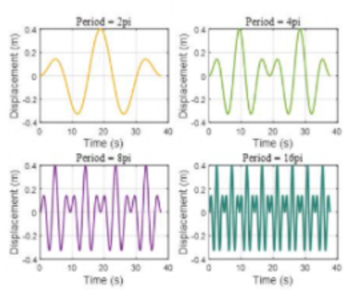
The accuracy of a quadrature rule can be predicted in part by examining its behavior on polynomials. The order of a quadrature rule is the degree of the lowest degree polynomial that the rule does not integrate exactly. There are four quadrature rules:



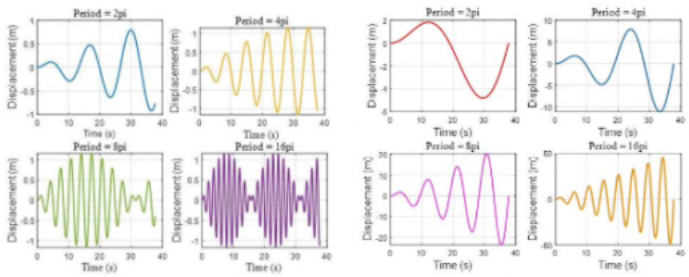
- When $m = 1, k = 9, F_0 = 1, \omega = 2$, and $t \in [0, 2\pi]$.

$$u(t) = \frac{F_0}{m(\omega_0^2 - w^2)}(\cos wt - \cos w_0 t) = \frac{1}{5}(\cos 2t - \cos 3t)$$





- The image on the left indicates that the frequency ω is close to the natural frequency ω0, and the solution exhibits a phenomenon called beating (also known as near resonance behavior). The image on the right indicates that the frequency ω is equal to the natural frequency ω0, and the solution exhibits a phenomenon called resonance.



- Apply the Compsite Simpson's rule with n=4 and step size $\frac{b-a}{4} = \frac{h}{2}$, we have

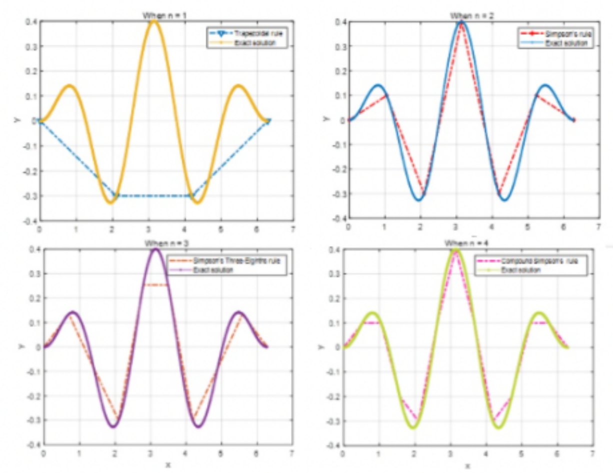
$$\int_a^b f(x)dx = S\left(a, \frac{a+b}{2}\right) + S\left(\frac{a+b}{2}, b\right) - \left(\frac{h}{2}\right)^4 \frac{(b-a)}{180} f^{(4)}(\vartheta), \text{ from some } \vartheta \in (a, b)$$

$$S\left(a, \frac{a+b}{2}\right) = \frac{h}{6}\left[f(a) + 4f\left(a + \frac{h}{2}\right) + f(a+h)\right]$$

$$S\left(\frac{a+b}{2}, b\right) = \frac{h}{6}\left[f(a+h) + 4f\left(a + \frac{3h}{2}\right) + f(b)\right]$$

$$\int_0^{2\pi} u\,dx = 0.00001$$

- Simpson's rule is a numerical integration technique that approximates the value of an integral by using quadratic polynomial functions to interpolate the function being integrated. The accuracy of the approximation depends on the number of subintervals used, which is determined by the value of n. In this investigation, we will explore how the number of subintervals affects the accuracy of the approximation when using Simpson's rule with n=4. We will also compare the results for n=1 and n=2, n=3, to see how much improvement is gained by increasing the number of subintervals.



Conclusion

This study explores an adaptive integration algorithm based on an undamped spring system. By analyzing the difficulties of traditional numerical integration algorithms when facing highly nonlinear integration functions, we propose an algorithm based on adaptive trapezoid method and adaptive Simpson method to improve the accuracy and efficiency of numerical integration. We also studied the convergence characteristics of different integration functions and the number of sub intervals required for the algorithm under different accuracy requirements. Experimental results show that our algorithm is superior to the traditional numerical integration algorithm in accuracy and computational efficiency, and can be applied to a wider range of integration functions. Therefore, this adaptive integration algorithm based on undamped spring systems has great application prospects in practical engineering.

References

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