Edge detection

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www.AlisMath.com

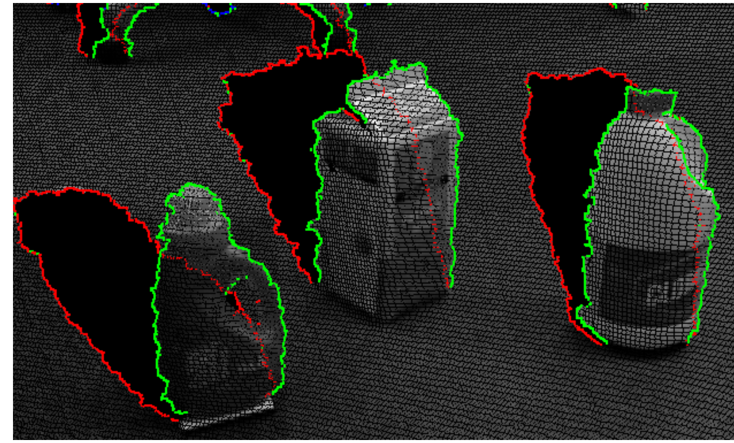
References

- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
- http://www.cs.cmu.edu/~16385/

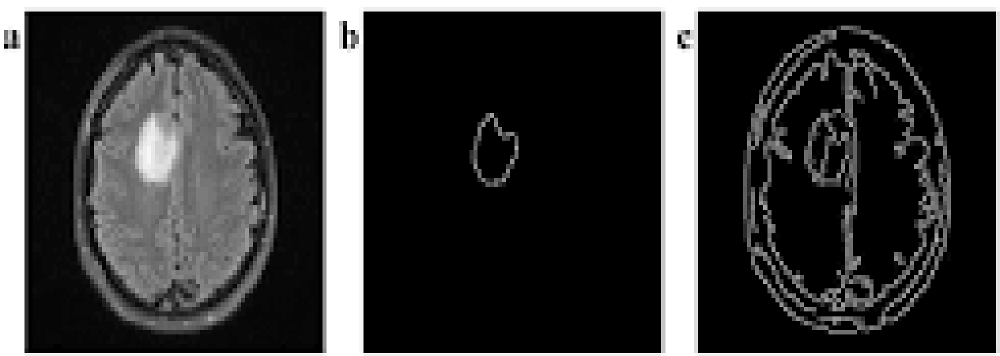
Contents

Some motivation

Art
(Instagram filters)



Robotics
(scene understanding)



Medicine
(tumor detection)

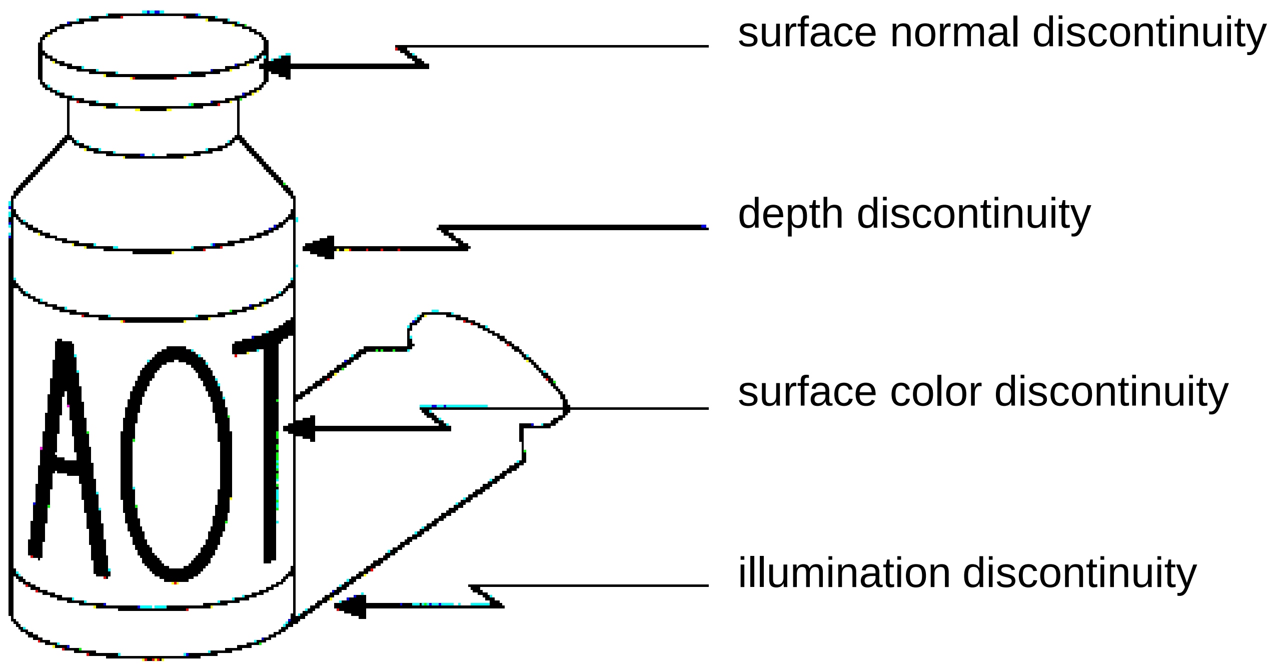
Autonomous vehicles
(license plate detection)

Why edges?

- Representation of objects can be done without full image representation- more compact.
- Edges are salient features (salient- “most noticeable or important”).

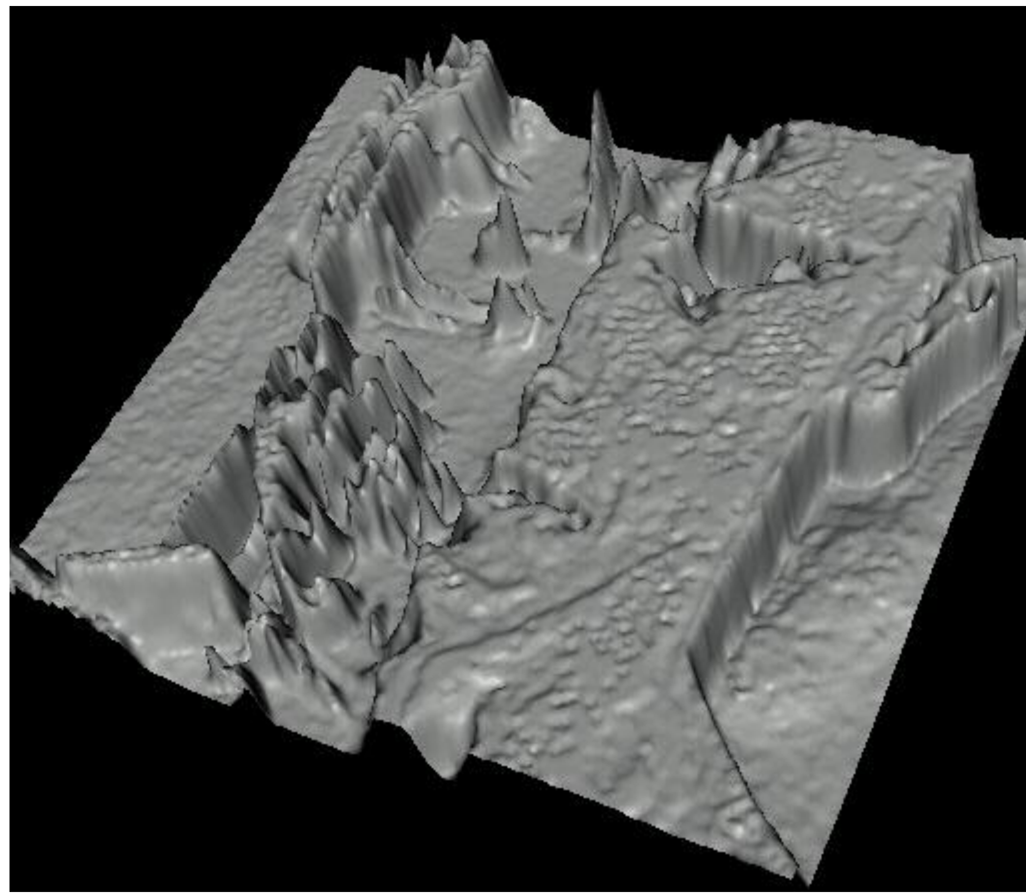
What are edges?

- "The outside limit of an object, area, or surface; a place or part farthest away from the center of something."
- Edges can be caused from many reasons in images:



Representation in images

- Rapid changes in colors.
- Looks like steep edges if represented as a surface:

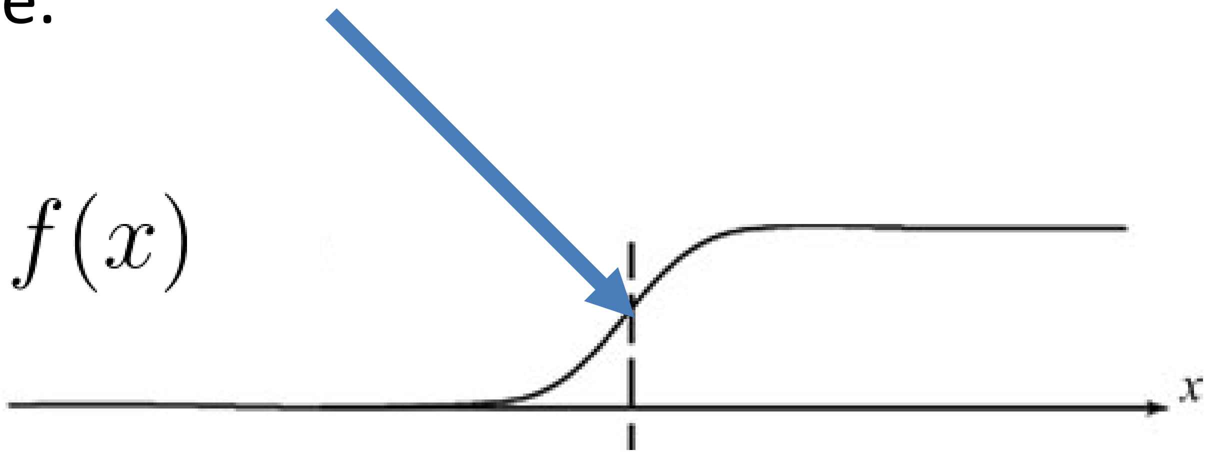


Contents

- Intro to edges
- **Basic edge image**
- Edge thinning
  - LoG
  - NMS
- Edge mask
- Canny edge detector
- Other edge related topics
  - Frequency representation
  - Unsharp filter

How to find edge image?

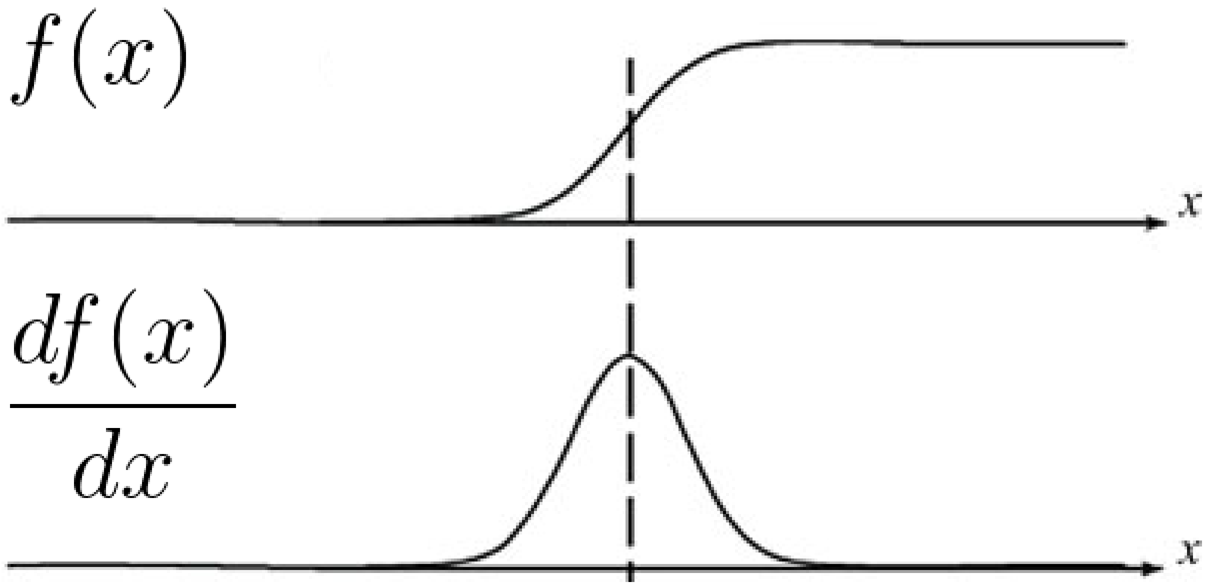
- Wanted result: image of a binary mask of where there is an edge.



- How to do so?

First order derivative

- Derivative of an edge:



- **Finding maximum points in the derivative of an image is a possible way to find edges!**

Deriving the derivative

- Definition of derivative in continues functions:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Deriving the derivative

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- In discrete space we can set $h = 1$:

$$f'[x] = f[x+1] - f[x]$$

Deriving the derivative

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$$f'[x] = f[x+1] - f[x]$$

- And in 2D space (derivative along x axis):

$$f'_x[x, y] = f[x+1, y] - f[x, y]$$

1st derivative filter

$$f'_x[x,y] = f[x+1,y] - f[x,y]$$

- We can mimic this derivative as a convolution operator:

$$f'_x = f * \boxed{+1 \;|\; -1}$$

  – Note 1: when a kernel size is even in some dimension, the center of the kernel needs to be specified (above the center is -1).

  – Note 2: remember that in the convolution operation **the kernel is flipped in both directions**.

Symmetric 1st derivative

- A more common approach is using the symmetric 1st derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

- How it's written in a discrete form?

Symmetric 1st derivative

- A more common approach is using the symmetric 1st derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

- Which translates to this kernel:

$$f'_x = f * \frac{1}{2}\boxed{\begin{array}{c|c|c} +1 & 0 & -1 \end{array}}$$

- We'll use the kernel above without the $\frac{1}{2}$ constant, since we only care about the ratio between gradients.

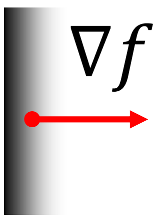
Y direction

$$f'_y = f * \begin{array}{|c|}\hline +1 \\ \hline 0 \\ \hline -1 \\ \hline \end{array}$$

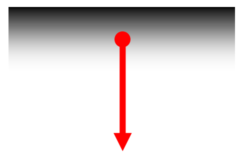
- The convolution kernel above is true for python/matlab/opencv image axis convention, where the positive y direction is down.
  - Let's use this convention in the below derivation.

Image gradient

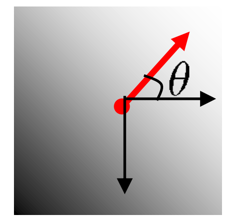
- The **gradient** of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$
- The gradient points in the direction of most rapid increase in intensity:



$\nabla f = [+1, 0]$



$\nabla f = [0, +1]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

Image gradient

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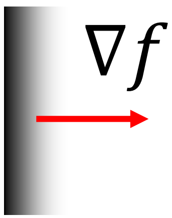
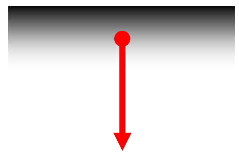
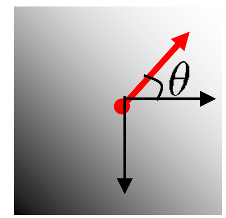


$\nabla f = [+, 0]$



$\nabla f = [0, +]$



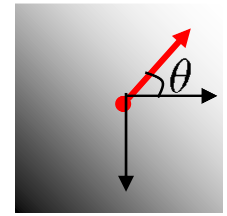
$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

- The edge **strength** is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Gradient direction

- The gradient direction is given by:



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\theta = atan2(-f'_y, f'_x)$$

  - $\theta \in (-\pi, \pi]$
  - $-f'_y$ because of the inversed y direction.
  - unlike regular $\tilde{\theta} = \arctan(\frac{y}{x})$ in which $-\frac{\pi}{2} < \tilde{\theta} < \frac{\pi}{2}$

$$\operatorname{atan2}(y, x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

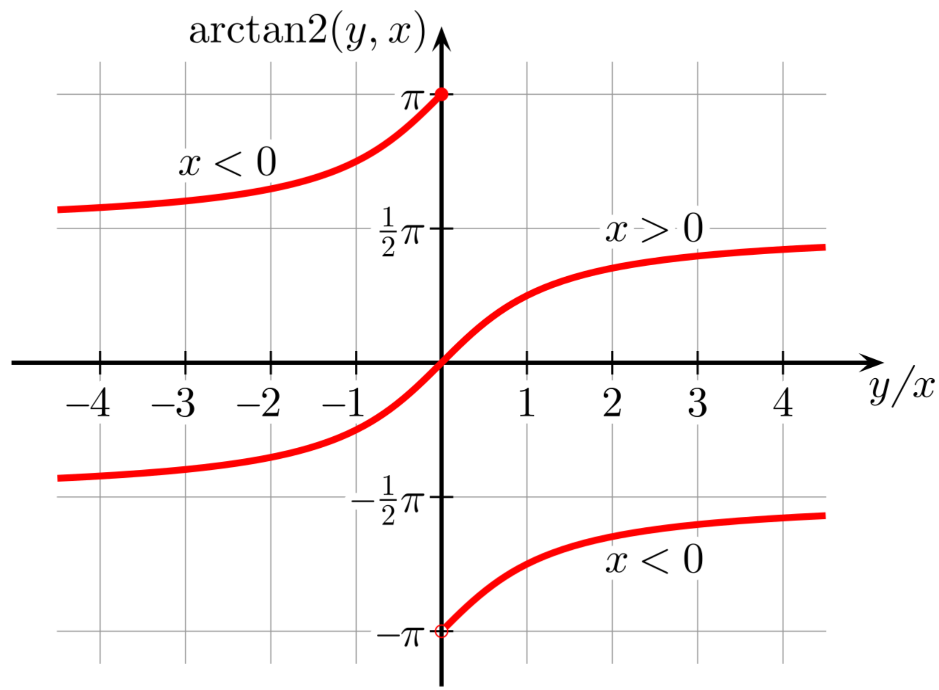
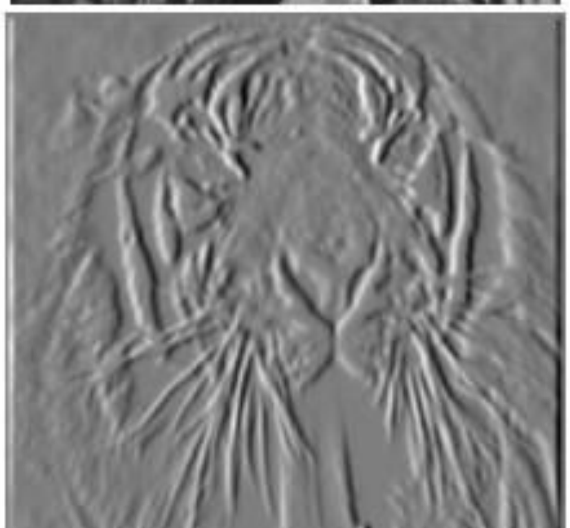


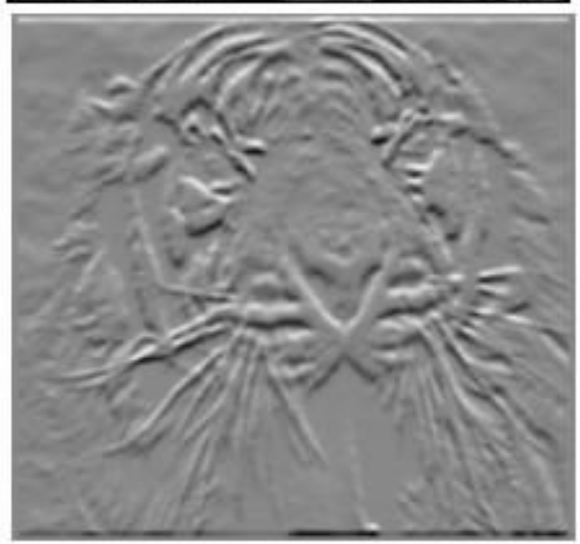
Image gradient example

Original image

$||\nabla f||$



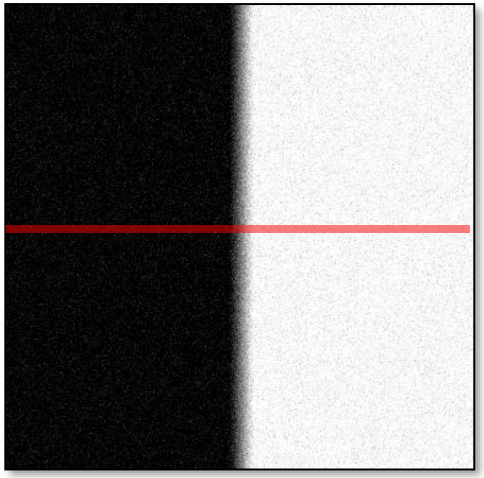
∇f_x



∇f_y

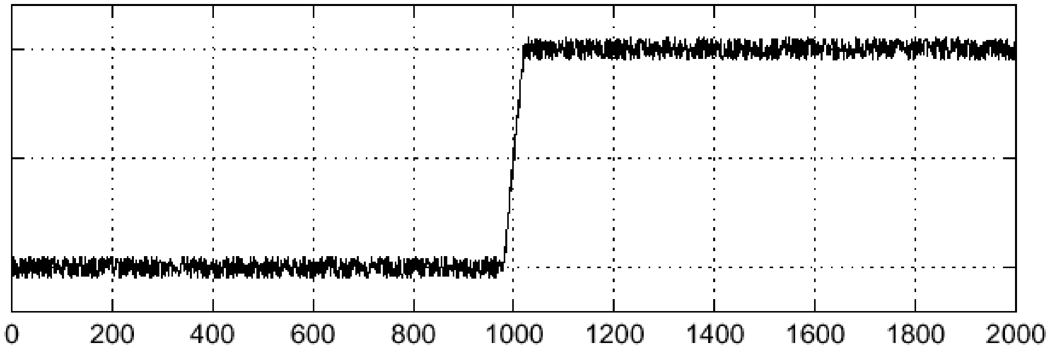
Noise effects

- How to find maximum of derivative in noisy environment?

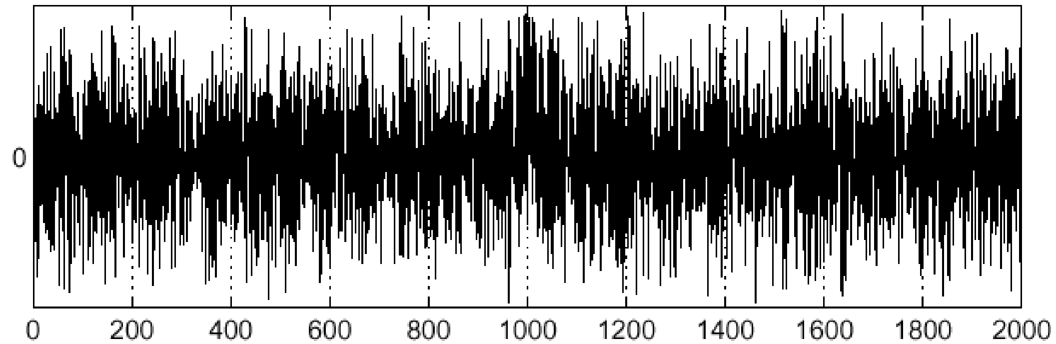


Noisy input image

$f(x)$



$\frac{d}{dx}f(x)$

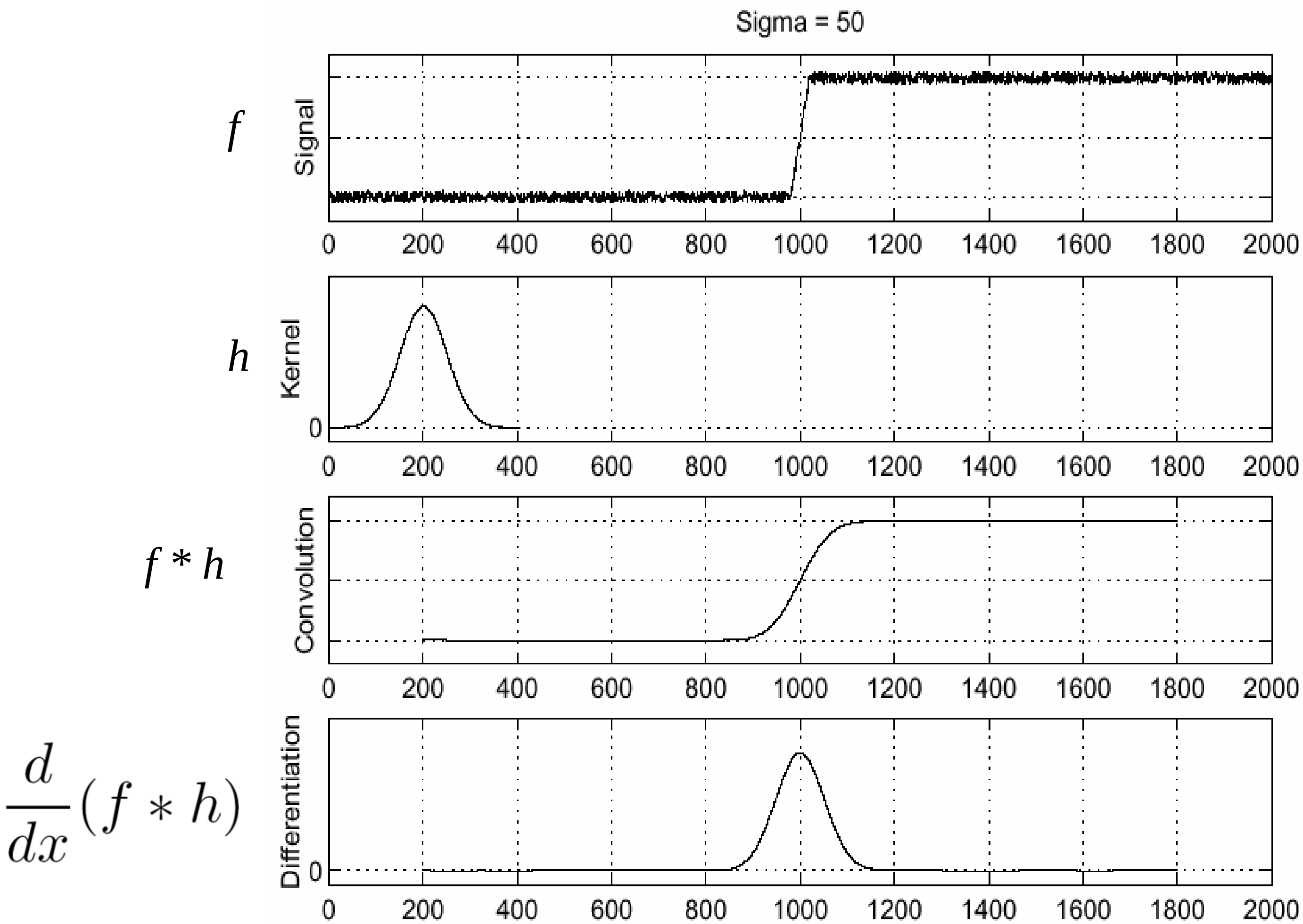


Solution 1: Prewitt filter

$$f'_x = f * \begin{array}{|c|c|c|}\hline +1 & 0 & -1 \\ \hline +1 & 0 & -1 \\ \hline +1 & 0 & -1 \\ \hline \end{array}$$

- The same as before but more robust to noise since it uses the diagonal neighbors as well (as a kind of directional mean).

Solution 2: smoothing the noise



- Search for the maximum in the smooth image!

Gaussian derivative kernel

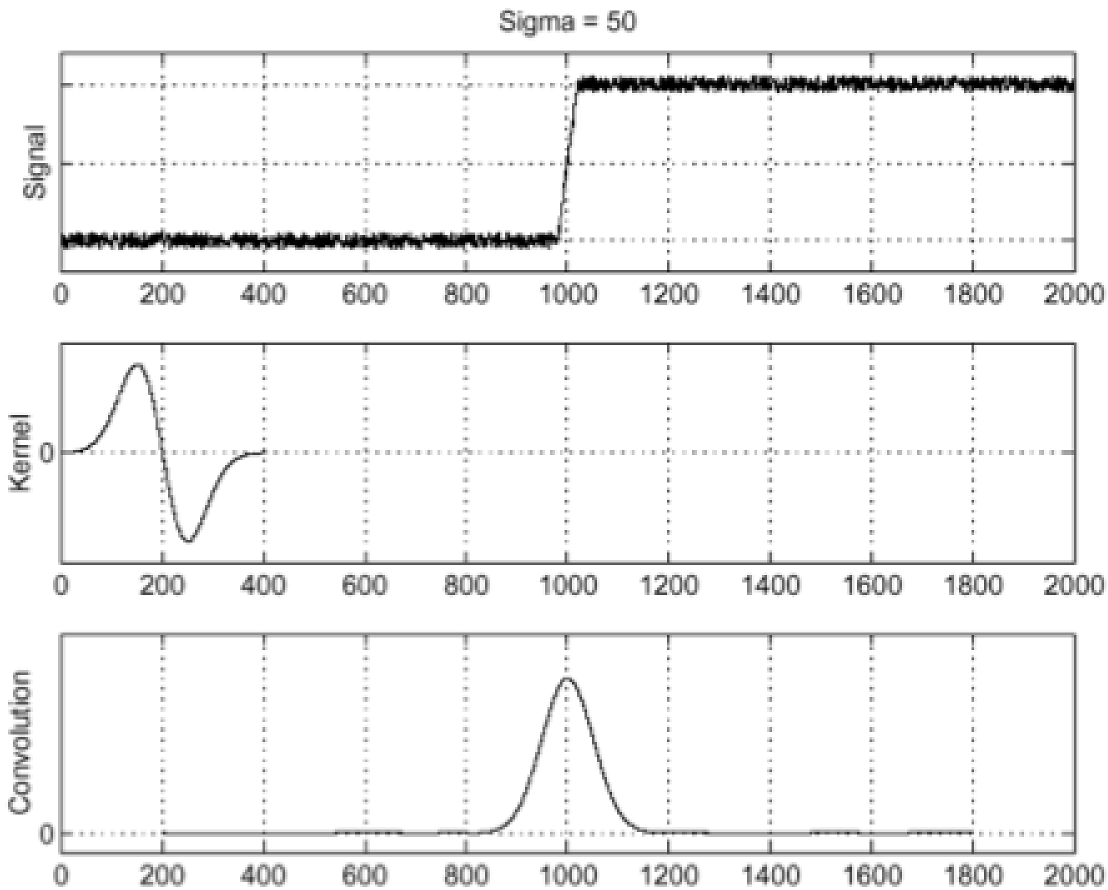
- Using this convolution trick:

$$\frac{d}{dx}(h * f) = (\frac{d}{dx}h) * f$$

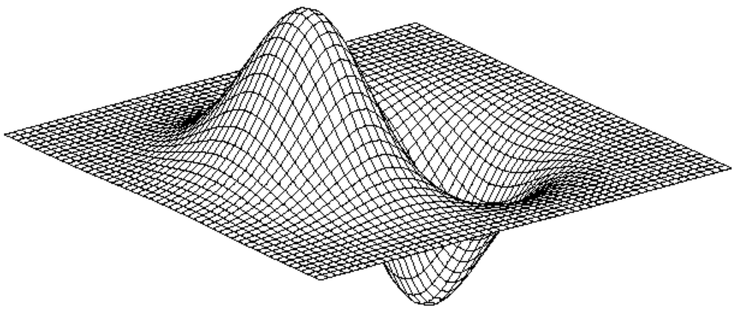
input

derivative of Gaussian

output (same as before)

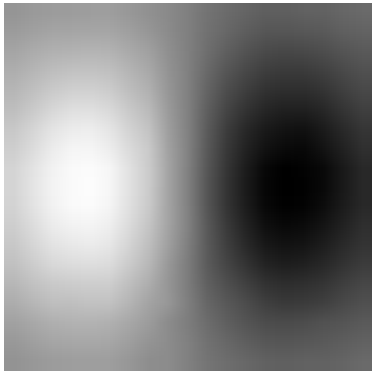


Gaussian derivative kernel 2D

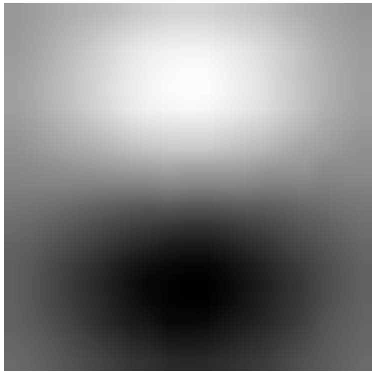


Derivative of Gaussian

x-direction

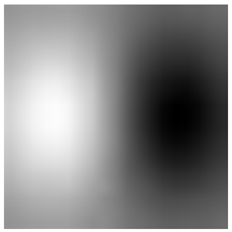


y-direction



Sobel filter

- Common approximation of derivative of Gaussian

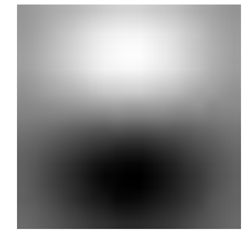


1	0	-1
2	0	-2
1	0	-1

s_x

1	2	1
0	0	0
-1	-2	-1

s_y



- Can also be thought of as Prewitt with higher weighting for closer neighbors.
- **In theory-** should give better performance relative to Prewitt since it includes smoothing effect.
- **In practice-** non definitive superiority to Prewitt (3X3 kernel is a rough approximation…).

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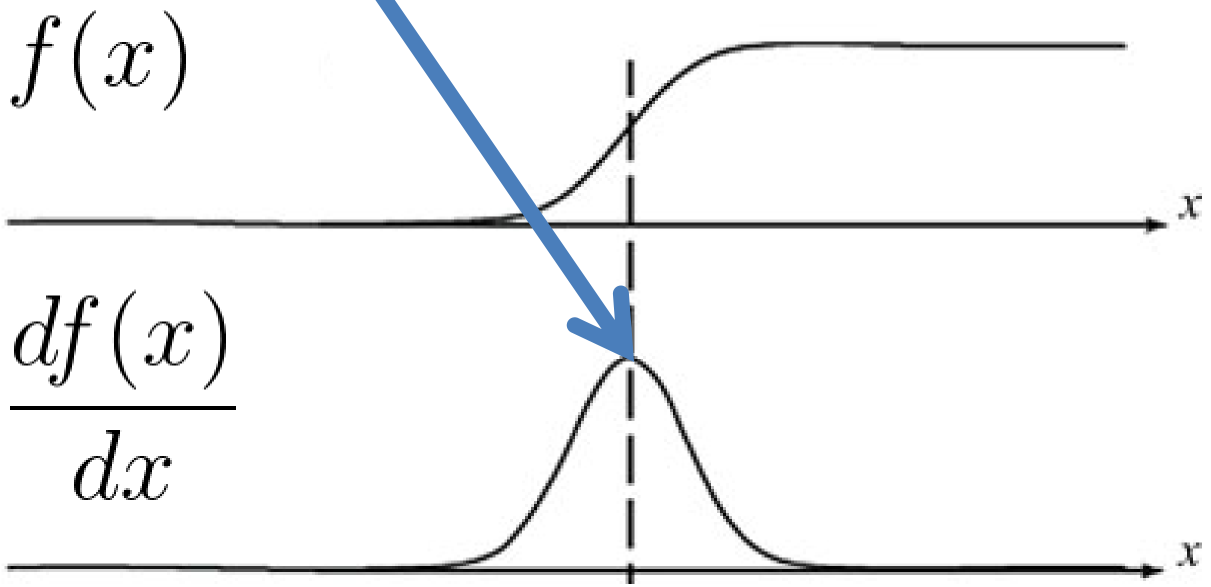
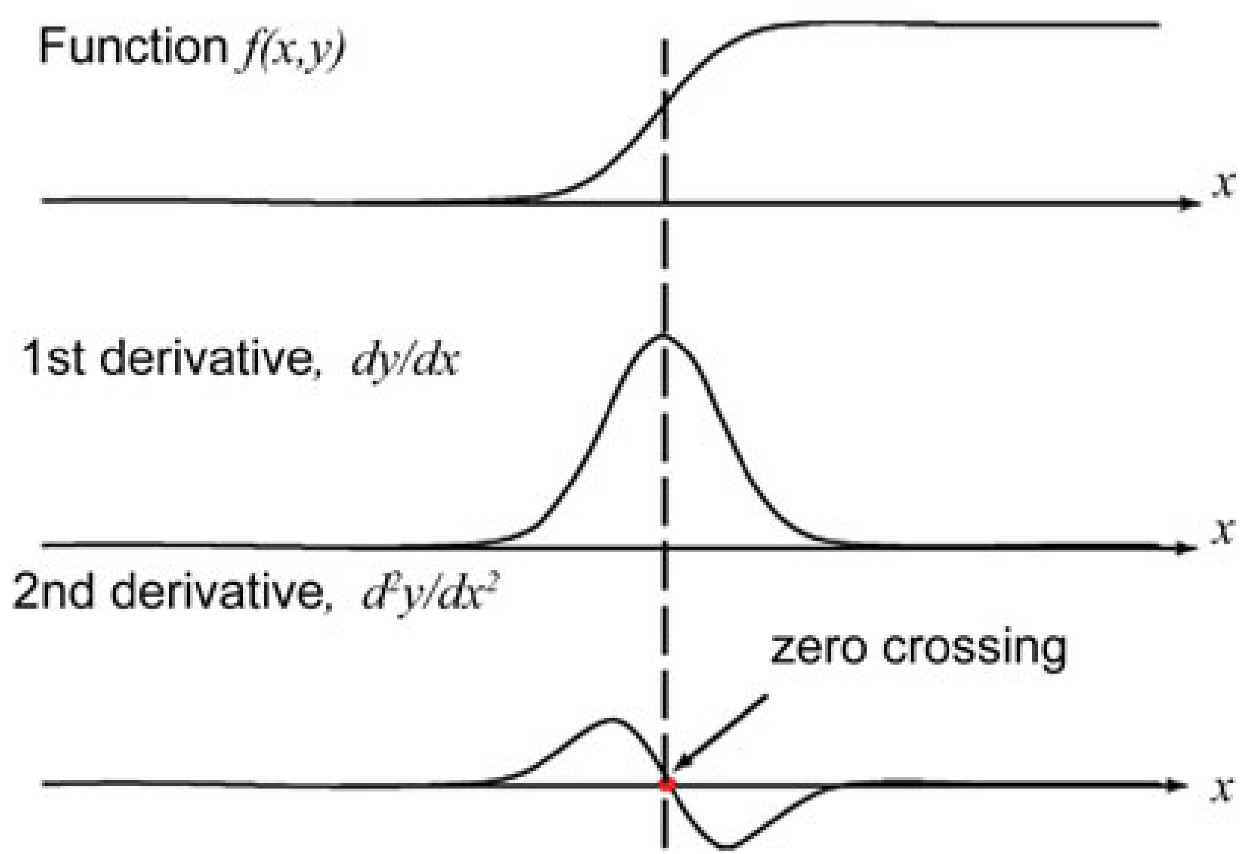
Edge thinning

- I have the edge filter result, but I want only one pixel to represent the edge in a binary mask.
- How do I find this?



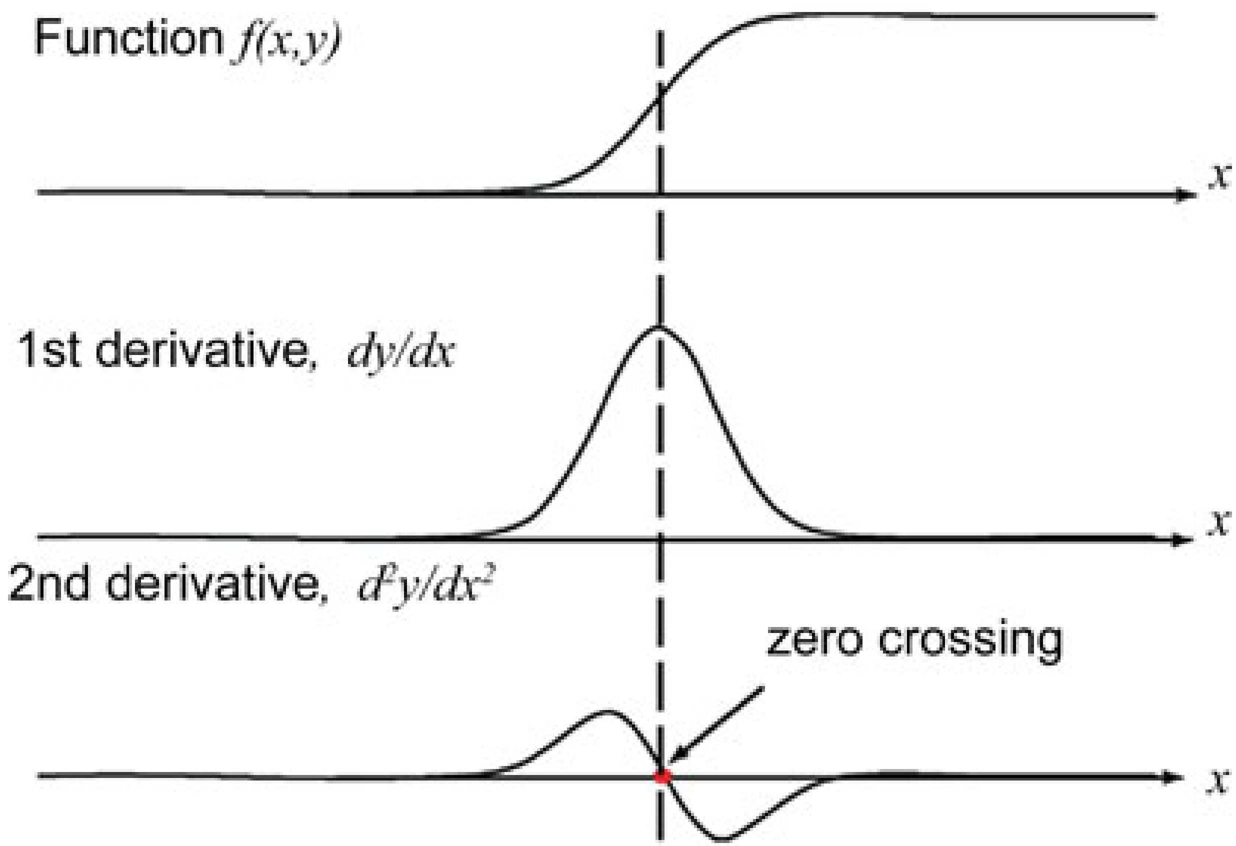
Naïve approach: 2nd derivative

- Let's try to find the zero crossing of the 2nd derivative.
- Only single zero crossing- should produce thinner edge



Naïve approach: 2^{nd} derivative

- **In practice:** this approach is very susceptible to noise!



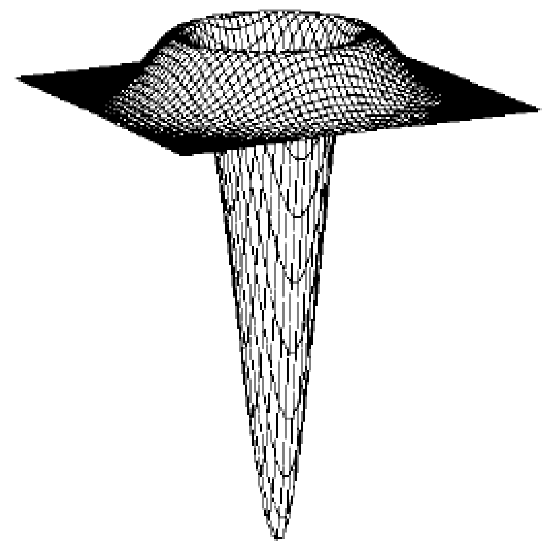
A better approach: LoG

- Let's take the 2nd derivative of the Gaussian (Laplacian of Gaussian: LoG) kernel so smoothing will help with noise reduction:

$$\Delta f = \nabla^2 f = \nabla \cdot \nabla f = \nabla \cdot [\frac{df}{dx}, \frac{df}{dy}] = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$$

$$\nabla^2 h_\sigma(u, v)$$

$$\mathrm{LoG}(x, y) = -\frac{1}{\pi\sigma^4}\left[1 - \frac{x^2 + y^2}{2\sigma^2}\right] e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



Laplacian of Gaussian

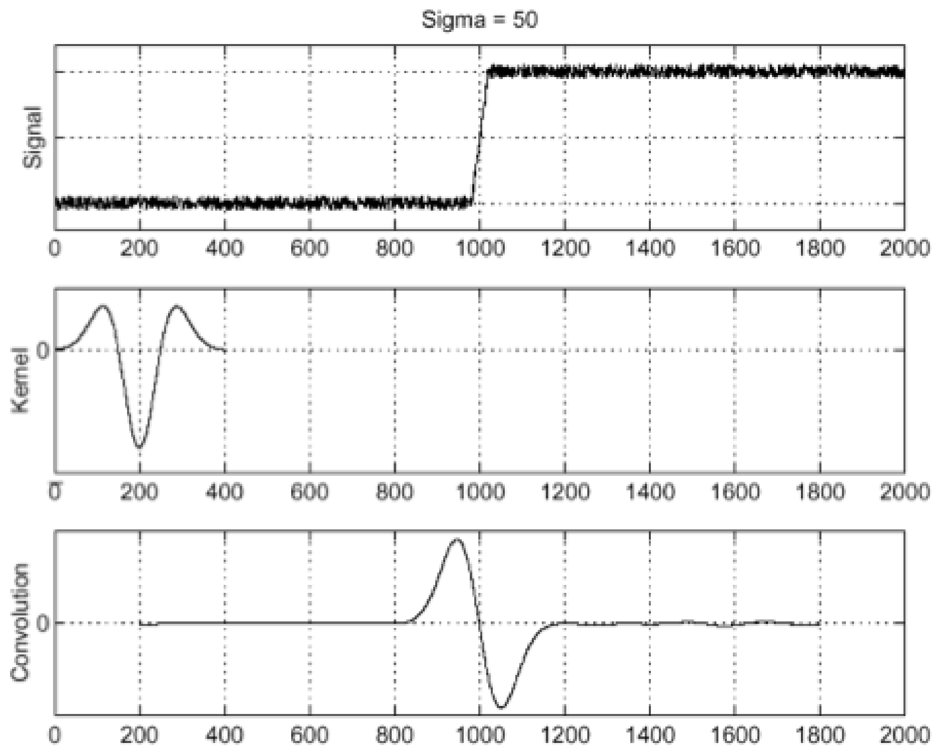
Find edge in noise signal: LoG

- Input is noisy step signal; output is zero crossing at the step.

input

Laplacian of Gaussian

output



LoG quantization

- Can be filter of different sizes:
  - 3X3:

0	1	0
1	-4	1
0	1	0

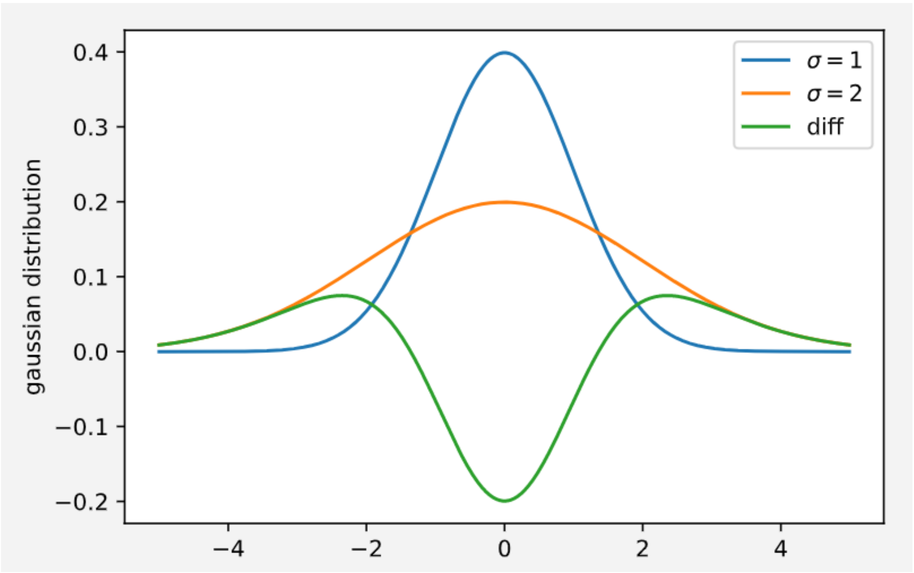
1	1	1
1	-8	1
1	1	1

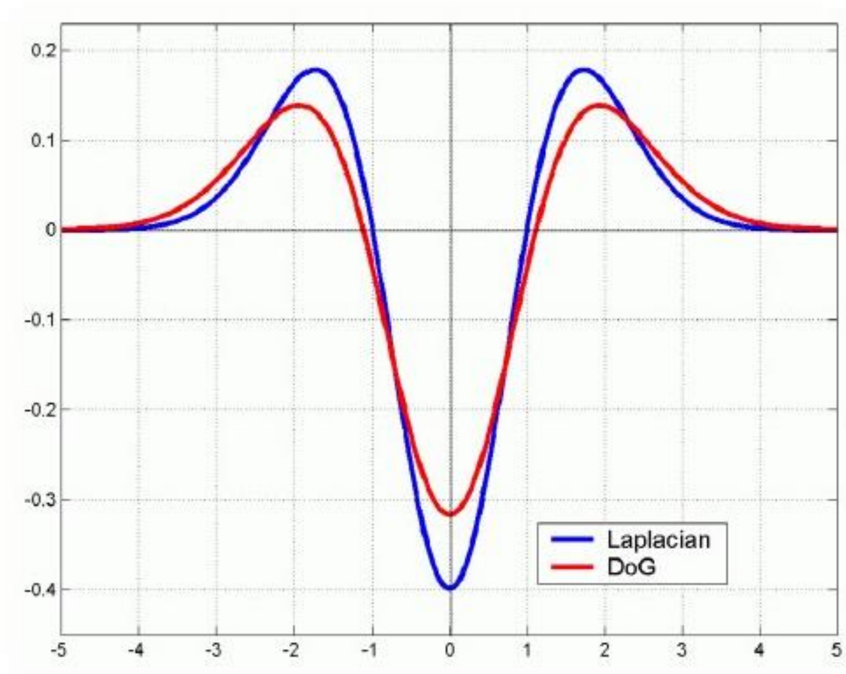
  - 9X9:

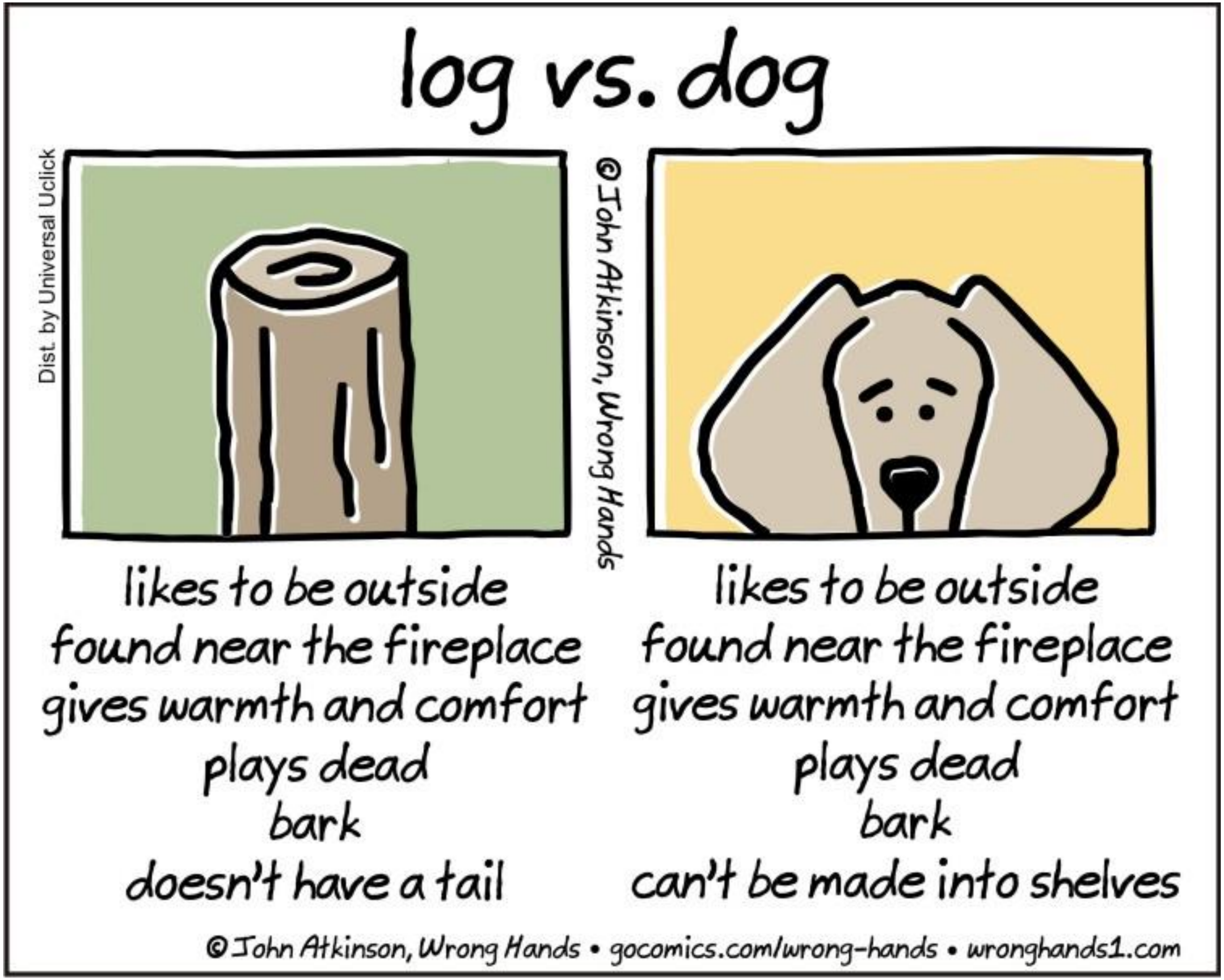
$$\begin{pmatrix} 0 & 0 & 3 & 2 & 2 & 2 & 3 & 0 & 0 \\ 0 & 2 & 3 & 5 & 5 & 5 & 3 & 2 & 0 \\ 3 & 3 & 5 & 3 & 0 & 3 & 5 & 3 & 3 \\ 2 & 5 & 3 & -12 & -23 & -12 & 3 & 5 & 2 \\ 2 & 5 & 0 & -23 & -40 & -23 & 0 & 5 & 2 \\ 2 & 5 & 3 & -12 & -23 & -12 & 3 & 5 & 2 \\ 3 & 3 & 5 & 3 & 0 & 3 & 5 & 3 & 3 \\ 0 & 2 & 3 & 5 & 5 & 5 & 3 & 2 & 0 \\ 0 & 0 & 3 & 2 & 2 & 2 & 3 & 0 & 0 \end{pmatrix}$$

DoG

- Can also use difference of Gaussians (DoG) to mimic LoG.
- Why do we want to do this? Faster computationally (explained here: https://dsp.stackexchange.com/a/37675)






log vs. dog
Dist. by Universal Uclick
© John Atkinson, Wrong Hands
likes to be outside
found near the fireplace
gives warmth and comfort
plays dead
bark
doesn't have a tail
likes to be outside
found near the fireplace
gives warmth and comfort
plays dead
bark
can't be made into shelves
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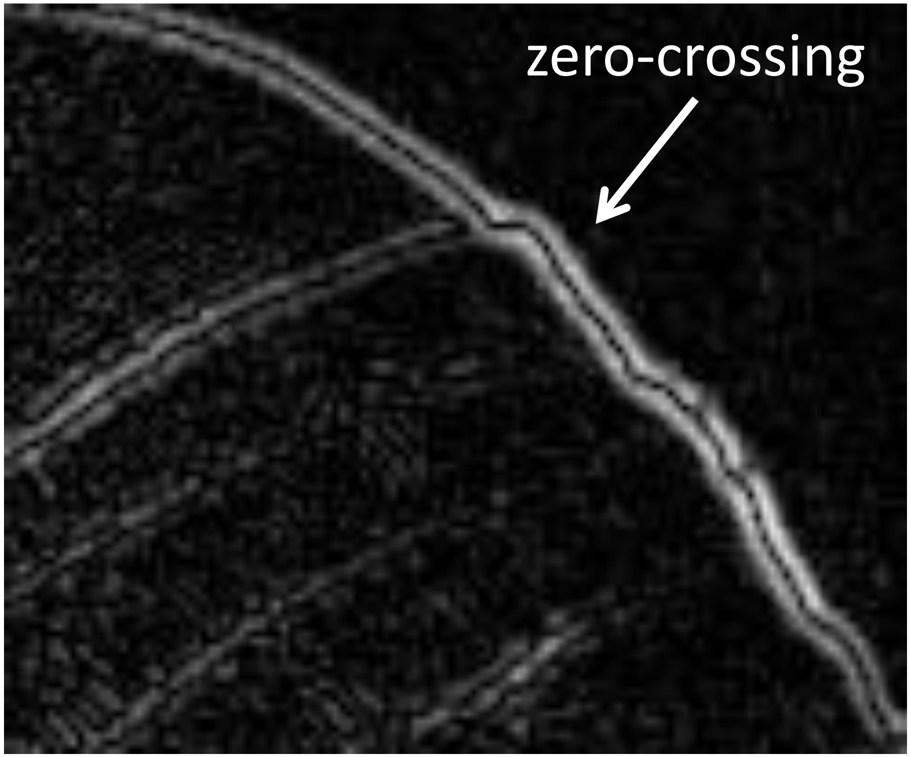
Example: LoG

$|LoG(x)|$

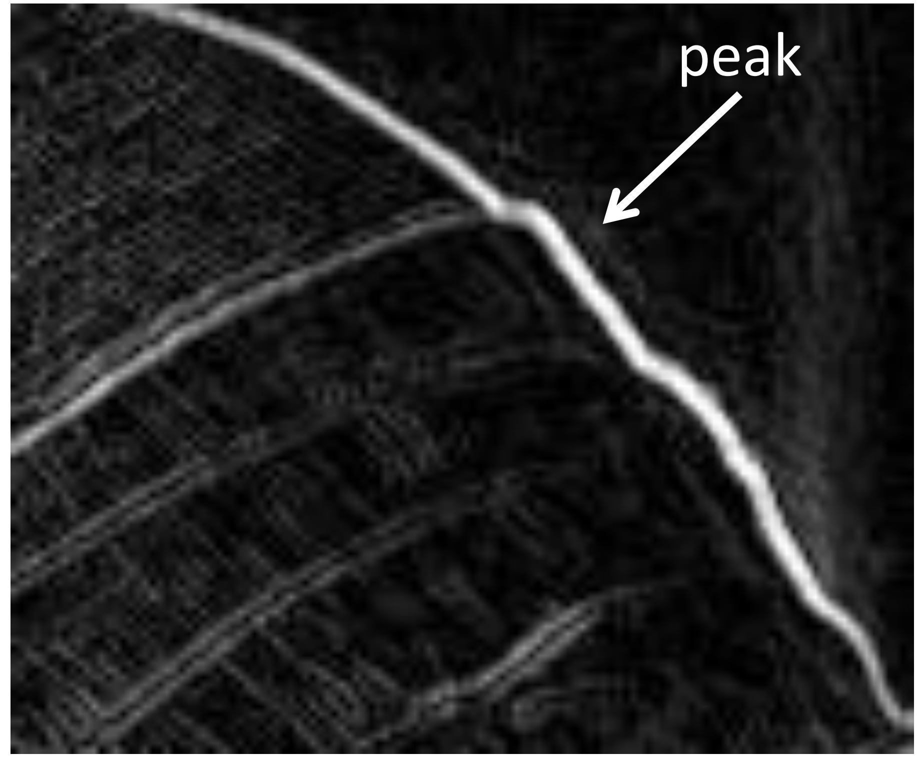
Absolute magnitude 1st derivative gaussian filter

Example: LoG

- Note: both images are after absolute value.



$|LoG(x)|$



Absolute magnitude 1st derivative gaussian filter

Zero crossing

- The new problem arising from the LoG filter is: how to mark the zero crossings?
- Answer: no easy algorithm to detect zero crossings.
  - E.g.: planes with minor noise will also produce zero crossing artifacts.

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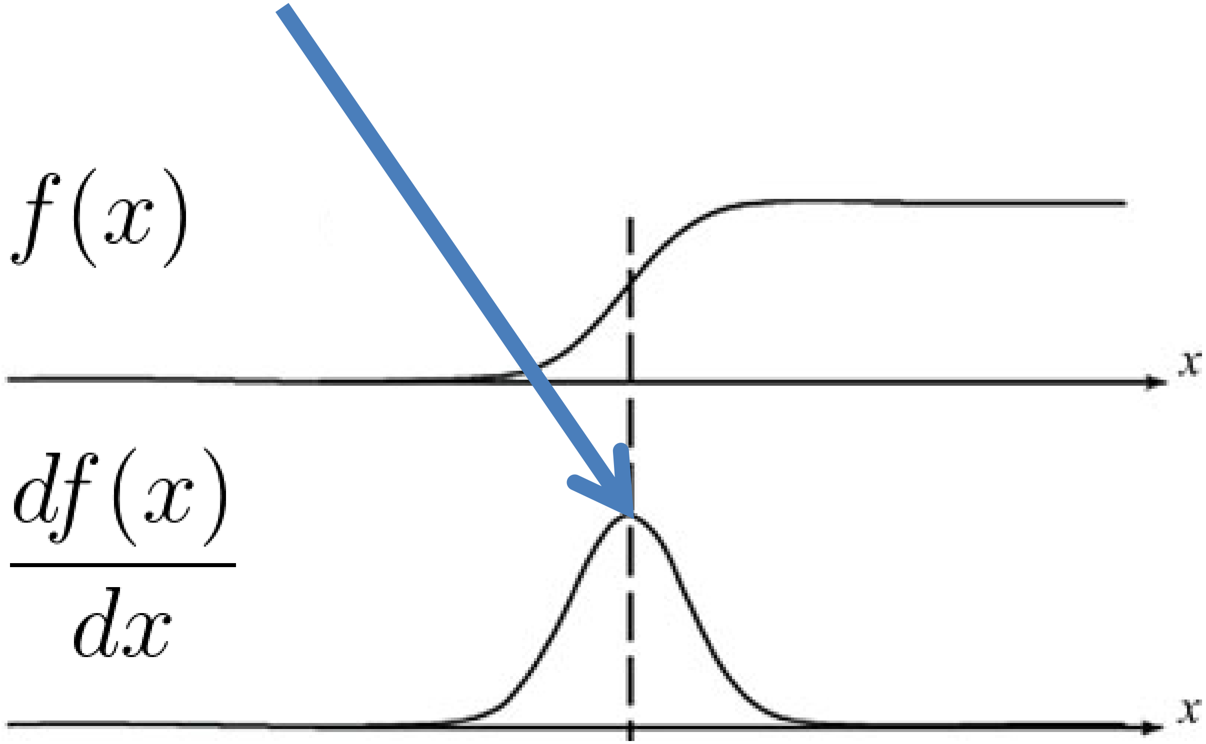
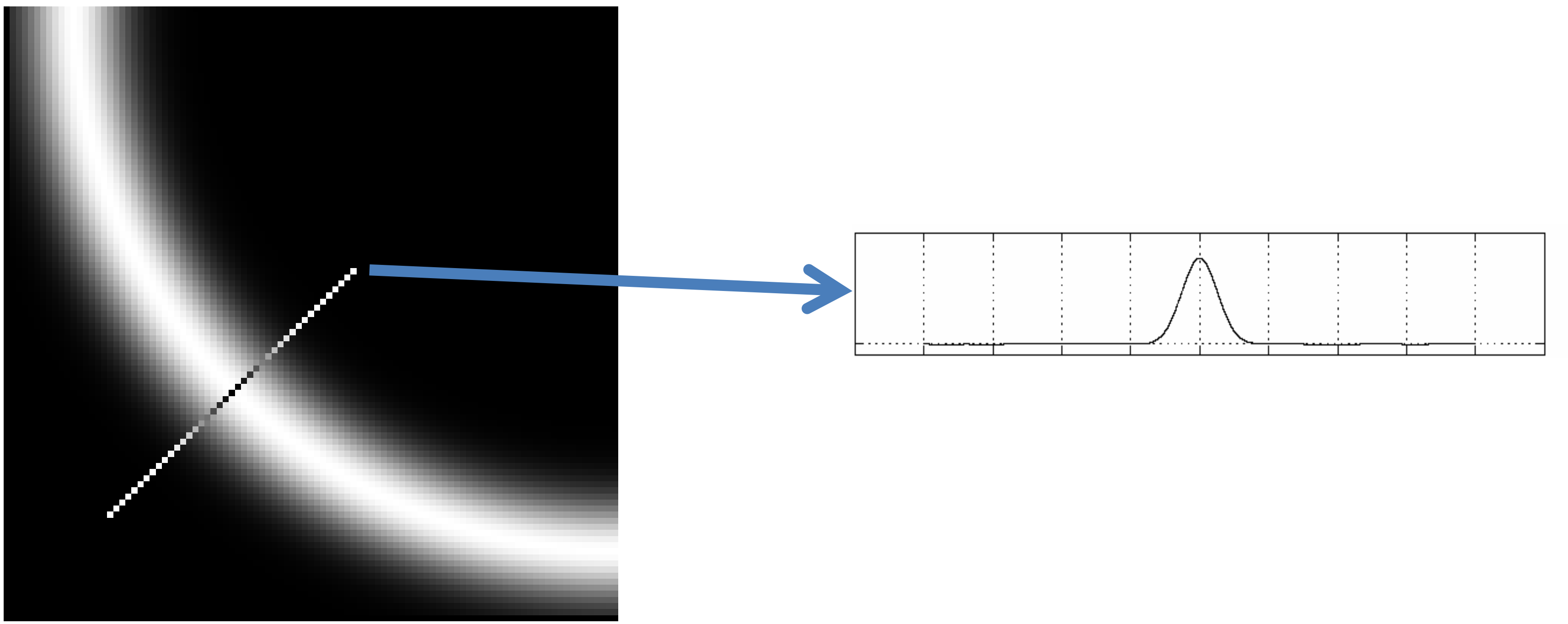
- How do I find this?



Non maximum suppression

- NMS
- Find the gradient magnitude + orientation of each pixel and search on this 1D line for maximum point.



NMS algorithm

```
get image gradient magnitude + orientation using
1D 3X3 gradient filter (e.g.: Sobel).
for each pixel p_0:
   Quantize ∢p_0 to one of four possibilities:
   [0°, 45°, 90°, 135°].
   In 3X3 neighborhood of p_0, find two neighbors
   in quantized gradient orientation {p_1, p_2}.
   If ||p_0|| < ||p_1|| or ||p_0|| < ||p_2||:
      ||p_0|| ← 0
```

NMS results

Before NMS

After NMS

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Edge mask

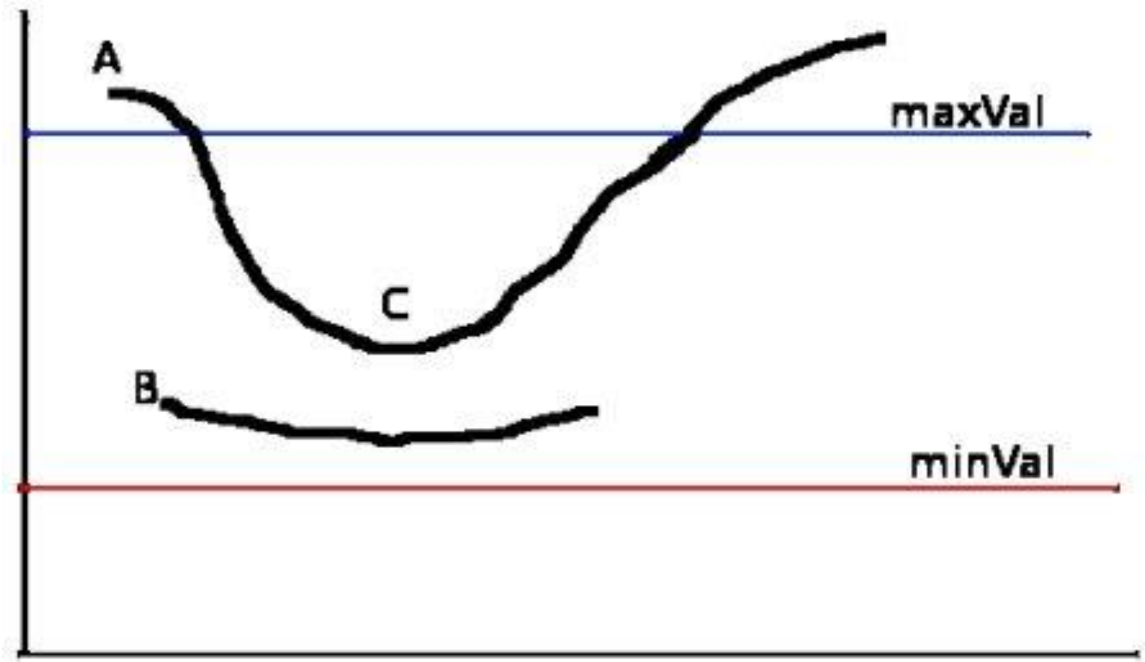
- How do we transform this integer image to a binary mask of where there is/ isn't an edge?

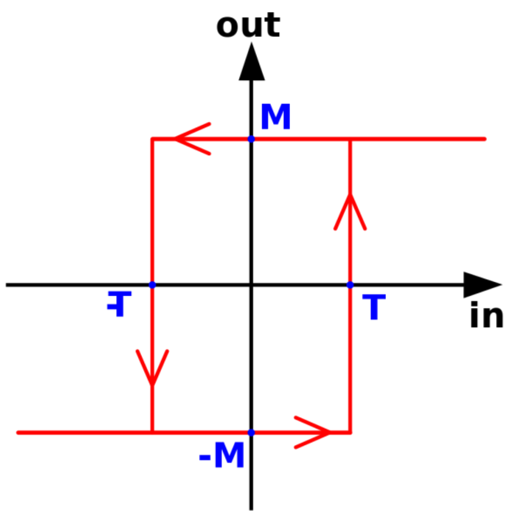
First try: single threshold edge mask

- Mask == binary image.
- Possible 1st solution- thresholding:
  - Choose an TH edge value, above which the pixel mask is 1, 0 otherwise.
  - The value can be a constant or percentile of the maximum edge value exists in the image.
  - Low TH: will get extra edges, but also input noise.
  - High TH: can miss lower valued edge pixels, less noise.
- How can we difference between low value edge pixels and noise?

Hysteresis motivation

- Weak edges are usually neighbors of strong edges, while noise can be at any pixel.
  - Usually “neighbors” means 3X3 square of adjacent pixels.
- If we know that a neighbor of a weak edge is a strong edge, then **the weak edge is a strong edge**!





Schmitt trigger hysteresis example plot

hysteresis

```
Choose two thresholds: {TH_h, TH_l | TH_h > TH_l}
For each pixel p_i:
   If p_i ≥ TH_h:
      p_i ← 1
   elif TH_l ≤ p_i < TH_h:
      p_i ← weak_edge_pixel
   Else: //p_i < TH_l
      p_i ← 0
While weak_edge_pixels that are neighbors of 1 exists:
    for each weak_edge_pixel_p_i:
      If weak_edge_pixel_p_i neighbor of 1:
           weak_edge_pixel_p_i ← 1
All remaining weak_edge_pixels ← 0
```

Contents

Canny edge detector

- Canny edge detector is one of the most known and used CV algorithms, still highly used even today (developed in 1986, cited 33000 times until 2019):

```
Gaussian filter
Find image gradient magnitude and orientation
NMS
Hysteresis
```

Example output

Important note: tradeoffs

- It's a common **misconception** in CV to think that one algorithm is **always** better than another.
- In CV, algorithms are highly dependent in the given environment in which they are executed. Each environment can vary in:
  - Noise.
  - Needed computation efficiency.
  - Overall problem variance.
  - Etc…

CV is the land Of tradeoffs

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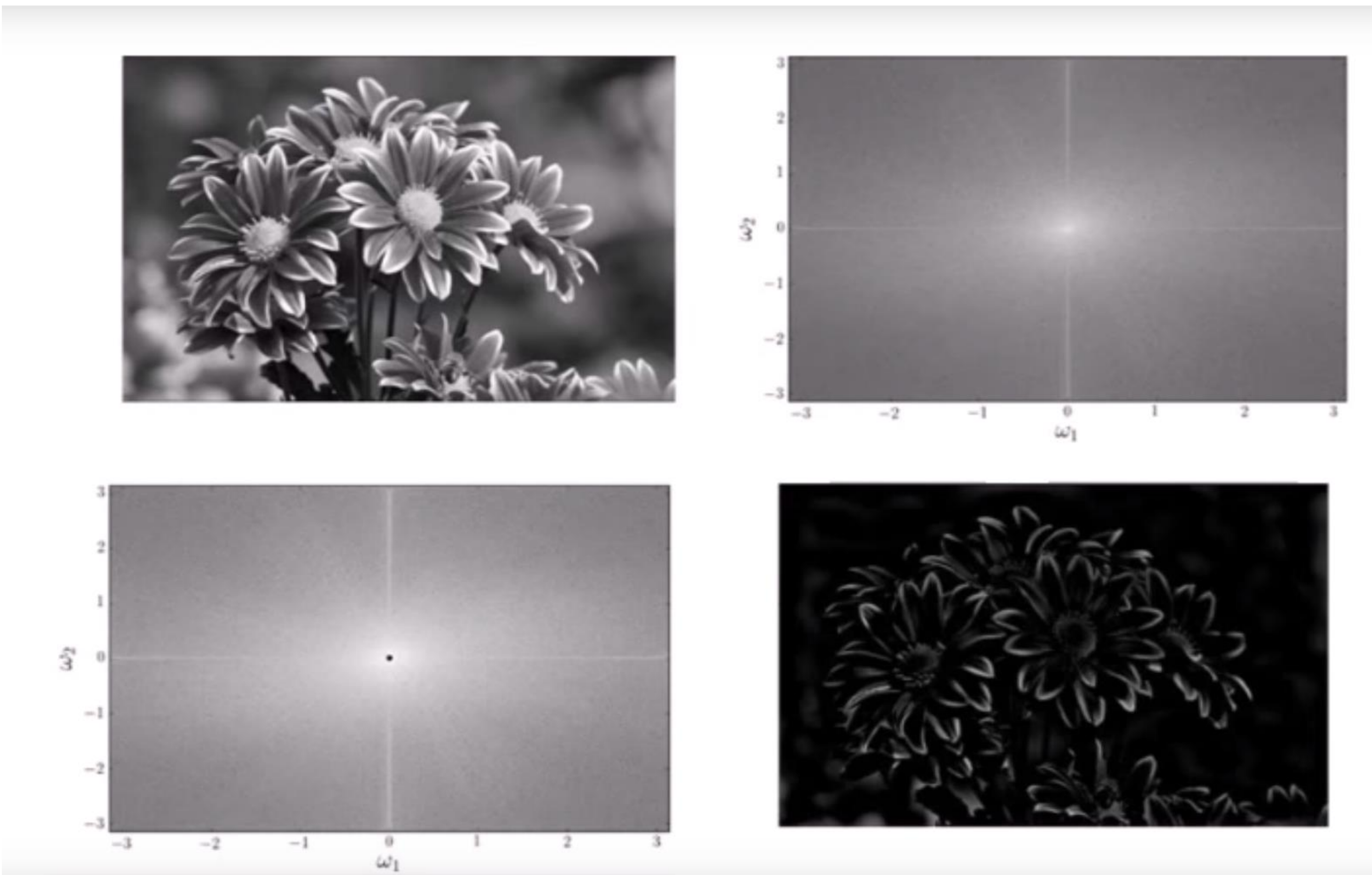
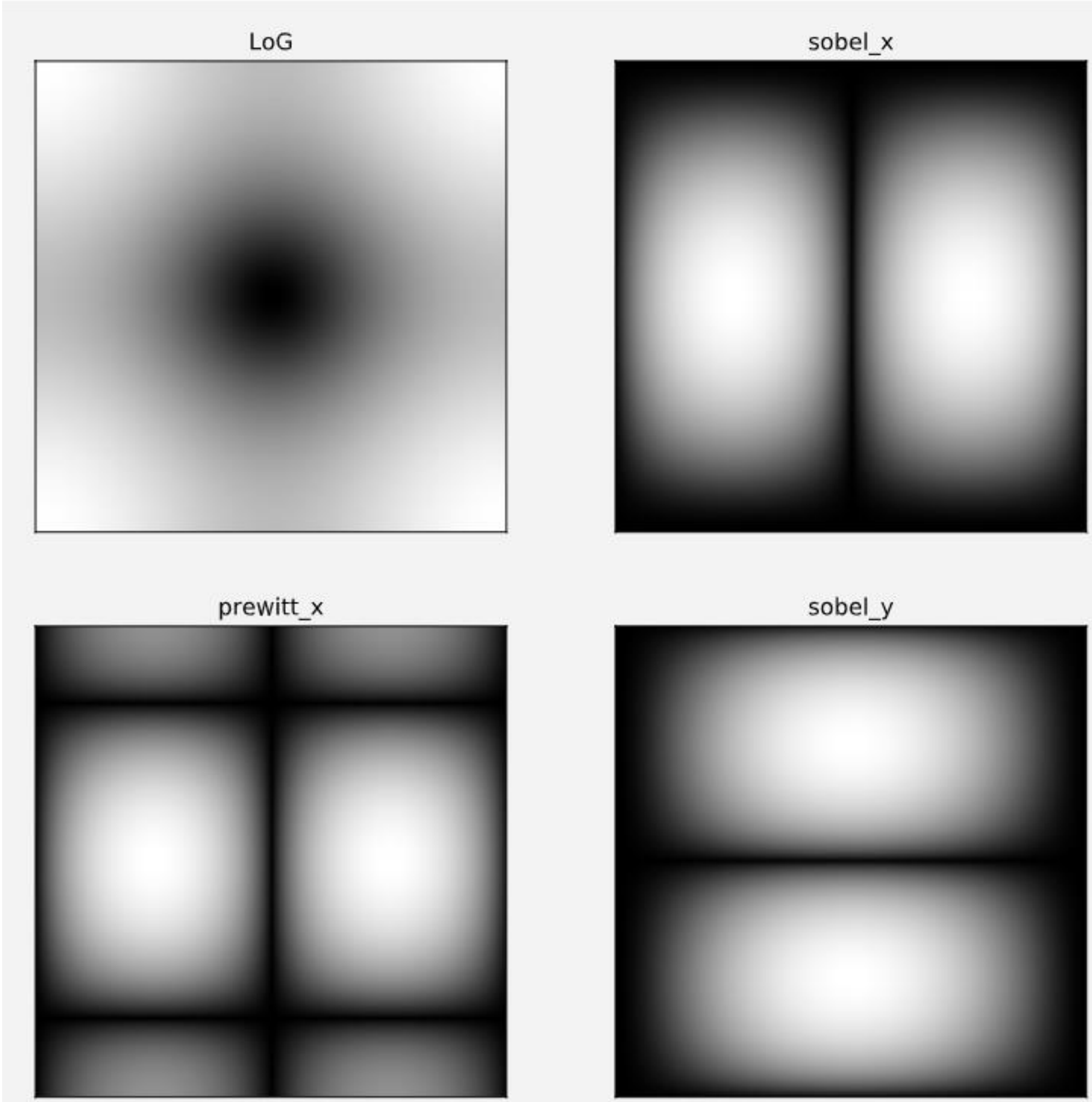
  - **Frequency representation**
  - Unsharp filter

HP filter

- Higher frequencies represents the edges of images.
- Removing the lower frequencies of an image will result in edge image!

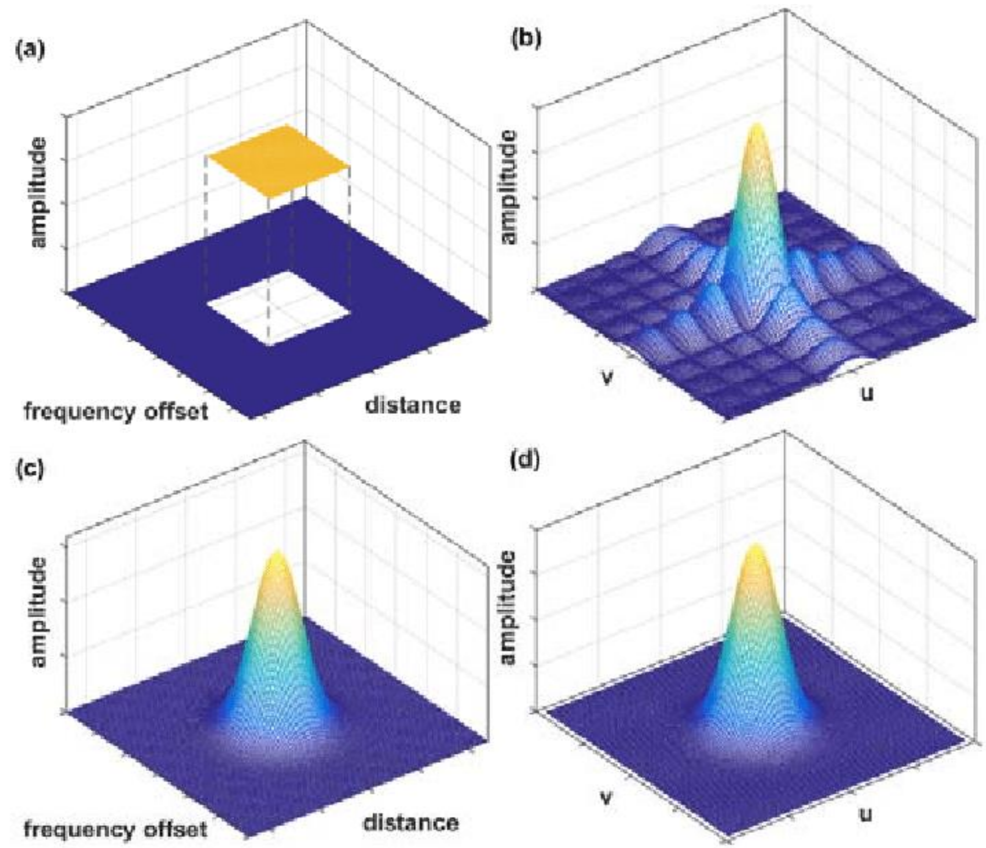


Edge filters- frequency representation



Why prewitt has waves?

- Recalling the mean filter – we can say that prewitt is like two side by side rectangles.
- Sobel is like two gaussians side by side!



Contents

- Intro to edges
- Basic edge image
- Edge thinning
  - LoG
  - NMS
- Edge mask
- Canny edge detector
- Other edge related topics
  - Frequency representation
  - **Unsharp filter**

Image sharpening

Obtain the high frequencies magnitude image.
Enhance the edges (e.g. by multiplying with a constant>1).
Add the enhanced edges back to the original image.

- Or- one liner:

$$f_{sharpen} = f + \gamma \cdot ||\nabla f||$$

Unsharp filter

- The former can also be done with only low pass filtering!

$$f_{unsharp} = f + \gamma(f - h_{blur} * f)$$

- This was also the way that photographers enhanced edges before CV (dates to the 1930s). More on this topic here: https://en.wikipedia.org/wiki/Unsharp_masking#Photographic_darkroom_unsharp_masking