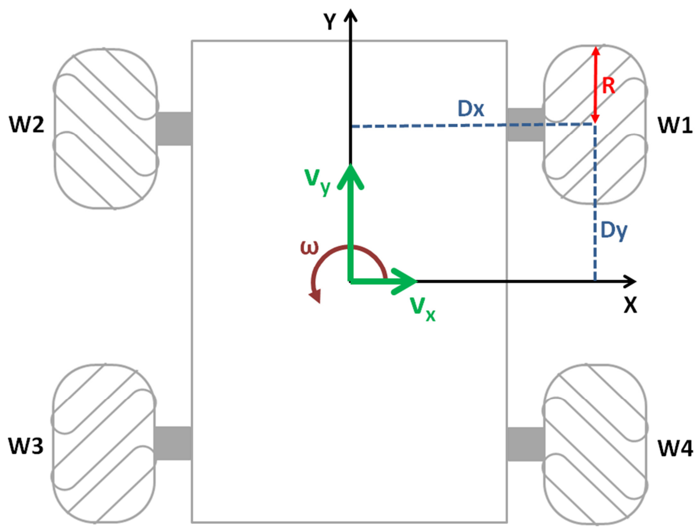
AGV Kinematic Model

The kinematic model for the AGV is a velocity based transformation between the vehicle coordinate system and the wheels coordinate system.

While developing the model we set the following basic assumptions:

- The movement only happens on a horizontal plane so that all wheels are in contact with the surface at all times.
- No slipping or sliding happens between the wheels and the surface. This assumption is not very realistic and causes most of the tracking errors depending on the surface of movement. An external tracking device (e.g. laser guide) can be added for higher quality tracking.
- No friction at contact point. This assumption is clearly an approximation, since motion only happens through friction. However, friction forces can be taken into account in the dynamic model, if required.
- All four wheels have the same *radius R* and are placed at symmetrical distance (*Dx,Dy*) from the center of the vehicle as shown in the drawing below. This assumption simplifies the mathematical notation and is satisfied by the vast majority of practical applications.



AGV coordinate system definition

The actual velocity of the vehicle is the time derivative of its position along the three degrees of freedom of its own coordinate system:

$$\begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} X \\ Y \\ \theta \end{bmatrix}$$

Note that these values are not equal to the velocities in the world coordinate system. A simple roto-translational transformation must be considered between the two systems.

The actual linear velocity of each wheel v_i is equal to its angular velocity ω_i times the radius of the wheel:

$$v_i = R\omega_i$$

Given these definitions, the kinematics direct and inverse transformations are now presented.

Inverse Transformations

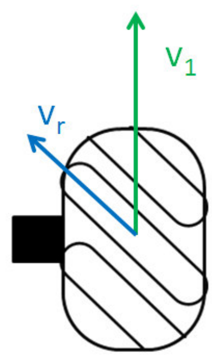
The wheels velocities can be calculated from the vehicle's velocity using the inverse Jacobian matrix:

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{bmatrix} = \frac{1}{R}\boldsymbol{J}^{-1}\begin{bmatrix}v_x\\ v_y\\ \omega\end{bmatrix}$$

where

$$\boldsymbol{J}^{-1} = \begin{bmatrix}-1 & 1 & Dx+Dy\\ 1 & 1 & -Dx-Dy\\ -1 & 1 & -Dx-Dy\\ 1 & 1 & Dx+Dy\end{bmatrix}$$

We show here the result for one wheel (W1). The others can be derived similarly.



Wheel W1 seen from the bottom

Looking at the wheel from the bottom we consider its linear speed v1 which requires an angular speed $\omega_1 = v_1/R$.

The problem consists in finding v1 as a function of the vehicle speed components v_x,v_y and ω.

We first observe that not all the magnitude of the wheel's speed v1 will cause the vehicle to move. That is because the rolls are free to rotate around their axes and some of the wheel's power will be inevitably lost.

Only the v_r component parallel to the rolls will cause motion, while the perpendicular component will not. Since the angle between the rolls and the wheel is 45 degrees we derive

$$v_r = v_1 \sin\frac{\pi}{4}$$

which means that the ideal efficiency of the Mecanum wheel is about 70%, while the remaining 30% is lost around the free rollers.

The active speed v_r causes a movement along both the x and y axes of the vehicle, precisely

$$v_x = -v_r \cos\frac{\pi}{4}$$

$$v_y = v_r \sin\frac{\pi}{4}$$

By subtracting the two previous equations

$$v_y - v_x = v_r\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) = v_r\sqrt{2} = v_1 \sin\frac{\pi}{4}\sqrt{2} = v_1$$

we obtain the relation between the linear components of the vehicle's speed and the wheel's rotational speed

$$\omega_1 = \frac{1}{R}\left(v_y - v_x\right)$$

The final step consists in adding the rotational component of the vehicle's speed. It is easy to observe that a rotational speed ω modifies the linear components v_x and v_y respectively by

$$v_x = v_x - \omega D_x$$

$$v_y = v_y + \omega D_y$$

By inserting these compensations in the previously derived formula we find the final transformation:

$$\begin{aligned}\omega_1 &= \frac{1}{R}\left(v_y + \omega D_y - v_x + \omega D_x\right) \\ &= \frac{1}{R}\left(-v_x + v_y + \omega\left(D_x + D_y\right)\right)\end{aligned}$$

The extension to the other wheels follows a similar pattern.

Direct transformations

The vehicle’s velocity can be calculated from the wheels velocities using the following transformations:

$$v_x = \frac{R}{4}\,(-\omega_1 + \omega_2 - \omega_3 + \omega_4)$$

$$v_y = \frac{R}{4}\,(\omega_1 + \omega_2 + \omega_3 + \omega_4)$$

$$\omega = \frac{R}{4(Dx + Dy)}\,(\omega_1 - \omega_2 - \omega_3 + \omega_4)$$

The derivation is quite straightforward as a linear combination of the inverse transformations equations.

For example, by adding ω_1 with ω_2 and ω_3 with ω_4, we find:

$$v_y = \frac{R}{2}(\omega_1 + \omega_2) = \frac{R}{2}(\omega_3 + \omega_4)$$

which immediately leads to the direct transformation equation.

However, it is important to notice that the four actuators are not completely independent. In fact, only three of them can be freely controlled, while the fourth must be derived accordingly.

The following relation must be always satisfied to avoid conflicts between the wheels:

$$\omega_1 + \omega_2 - \omega_3 - \omega_4 = 0$$

In case this equality does not hold true the direct transformations cannot be correctly calculated and any odometry strategy will invariably fail.